Kindergarten Children's Number Sense Development through Board Game

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Abstract

Number sense lays the foundation for children's later mathematical achievement. In practice, however, preschool children could mechanically count or even add and subtract as a result of practice and drilling, yet hardly understand what numbers mean and their relationships. In other words, children "do" math without understanding numbers. The current study explores the possibility of teaching number sense to 5-year-old kindergarten children using a strategically designed board game. A mixed design incorporating quantitative and qualitative methods was adopted. The effect of the board game on children's number sense improvement was examined using a small scale experimental design. Teacher's scaffolding and children's peer tutoring during the play sessions were discussed. The study shed light on how to implement number sense teaching with a play-based pedagogy.

Keywords: Number sense, board game, play-based learning, children

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Number Sense

Number sense is broadly defined as an understanding of what numbers mean and of numerical relationships (Malofeeva, Day, Saco, Young, & Ciancio, 2004). Based on an extensive literature review, Berch (2005) listed 30 presumed features of number sense that constitute awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or a mental number line. According to Berch, there are two general approaches to conceptualize the construct, the first of which is a lower order, perceptual sense of quantity; and the second is a higher order, conceptual sense-making of mathematics. Jordan, Kaplan, Olah, and Locuniak's (2006) factor analysis on children's number sense development revealed a similar two-dimensional model: basic number skills and conventional arithmetic. Between the two factors, it is argued that the basic number skills are the underpinning for higher order mathematical thinking and are likely to be important early indicators of numeracy proficiency (Jordan et al., 2006; Lembke & Foegen, 2009).

An intrinsic number sense is present early in life in human infants (Wynn, 1992; Xu, Spelke, & Goddard, 2005). With age and experience, children's number sense develops into a complex and interrelated set of concepts (Malofeeva et al., 2004). Facilitated by informal teaching in everyday interaction with adults and peers, children already possess fundamental number sense before entering formal schooling (Griffin, Case, & Siegler, 1994), although there are significant variations by social class and cognitive ability (Ginsburg & Golbeck, 2004; Jordan & Levine, 2009). Preschool children's number sense predicts later math achievement (Forget-Dubois et al., 2007; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Locuniak & Jordan, 2008). Deficits in number processing concepts were the primary reason for math disabilities (Mazzocco & Thompson, 2005).

Number sense measurements

As Berch (2005) reviewed, number sense is a broad construct including various components. Recent years have seen a growing number of measurements attempting to capture as many components as possible. One of the normed and widely used number sense tests is the Test of Early Mathematics Ability (TEMA), with the TEMA-3rd edition being the latest revision (Ginsburg & Baroody, 2003). The TEMA measures early math ability of children as young as 3 and 4 years of age. It includes both the informal number sense components such as numbering, number comparison, calculation and concepts, as well as the formal number sense components such as number literacy, number facts, and formal calculation and concepts.

Another well documented early number sense test is the Early Numeracy Test (ENT) designed for children from 4 to 7 years of age (Van de Rijt, Van Luit, & Pennings, 1999). Two major number sense components exist in the ENT: readiness and counting. Readiness includes comparison, classification, one-to-one correspondence, and seriation. Counting includes using number words, synchronous and shortened counting, resultative counting, and general knowledge of numbers. Norms for the ENT were established in different countries with large samples for cross country comparisons (Aunio, Ee, Lim, Hautamaki, & Van Luit, 2004; Aunio, Hautamaki, Heiskari & Van Luit, 2006; Aunio, Niemivirta, Hautamäki, Van Luit, Shi, & Zhang, 2006; Ee, Wong, & Aunio, 2006).

Based on their two-dimensional model of number sense, Jordan et al. (2006)’s Number Sense Battery is consisted of basic number skills including counting, number recognition, number knowledge, non verbal calculation, estimation, and number pattern; and conventional arithmetic including story problems and number combinations. Longitudinal data using the Number Sense Battery revealed diverse yet reasonable growth curves of kindergarten children’s number sense development.

The Number Sense Test by Malofeeva et al. (2004) was designed on the basis of two previous tests created by Griffin et al. (1994) and Clement (1984). It includes six components:

counting, number identification, number-object correspondence, ordinality, comparison, and addition-subtraction. The advantage of this test is that the items were designed into three levels, level one concerns numbers from 1-6, level two concerns numbers from 7-12, and level three concerns numbers from 13-15, so that the testing could be tailored to meet each child's developmental stage.

Promoting number sense through games

It has been documented that educational games could function as effective and engaging teaching tools in promoting early number sense development (Broody, Eiland, & Thompson, 2009; Griffin, 2004; Tournaki, Bae, & Kerekes, 2008). In a controlled trial, Young-Loveridge (2004) found teaching a group of low achieving 5-year-olds using commercial number books and games for 30 minutes a day for 7 weeks significantly improved their numeracy ability compared to the contrast group. The effect diminished with time after the intervention ceased but remained significant after a year.

Another well implemented randomized controlled trial by Ramani and Siegler (2008) found that playing number board games improved numeracy skills of low-income preschoolers. The study included 136 4- to 5.5-years old preschool children from 10 urban Head Start centers. Seventy-two children were randomly selected to play a number board game with a trained experimenter while the other 64 played a different version of the same game using colors instead of numbers. The game included a liner number board with numbered squares, two game pieces, and spinner. Players took turns spinning and moving the game pieces around the board. They read numbers from the board aloud while moving game pieces. Each child completed 4 game sessions for 15 to 20 minutes each over a 2-week period. The results showed that at the end of the 2-week period, children who played the number game had better counting and number identification skills than those who played the color game. They were also better at picking the higher number from a pair and estimating positions on a number line. The effects persisted 9 weeks after the game sessions ended, with effect size ranging from .55 to .80.

Teacher Facilitation and Peer Tutoring

To maximize the effect of the game pedagogy, children need efficient support from the teacher and their peers. Vygotsky's (1978) learning theory specifies scaffolding as the intentional, strategic support that an expert, either a teacher or a more capable peer, provides that allows children to complete a task they could not accomplish independently. To successfully scaffold, the teacher needs to determine what kind, how much, and when help is needed for a child to achieve independent problem solving, which leads the teacher to become a keen observer of children's learning and an effective facilitator to create conditions for students to direct their own learning and development (Gregory, 2006). According to Rogers (1983), a teacher facilitator sets the initial climate for group experience, elicits and clarifies the learning purposes, and makes self available as a flexible resource; she also responds to intellectual and emotional expressions, takes the initiative to share, and recognizes own limitations.

Peer tutoring is a kind of scaffolding that involves an experienced peer assisting an inexperienced peer in completing a task (Tomasello, Kruger, & Ratner, 1993). Whilst the tutee is faced with the problem of accomplishing a task, the tutor is faced with the problem of understanding the tutee's particular problem, and developing effective ways to respond and help the tutee. According to Tabacek, McLaughlin, and Howard (1994), peer tutoring is one of the most effective instructional techniques that promote both academic and social development. On the one hand, the symmetric nature of the peers' social status facilitates children to take greater responsibility in their own learning than when they are interacting with an adult (Ashley & Tomasello, 1998). On the other hand, the asymmetric nature of the expert and the novice enables scaffolding to take place. In order to successfully tutor a peer, young children need to appreciate the cognitive conflict and knowledge difference between the pair and take each others' perspectives into consideration (Johnson-Pynn & Nisbet, 2002), which is a rather challenging cognitive attainment developed during early childhood (Frye & Wang, 2008). Empirical evidence has suggested successful peer tutoring could emerge in children as young as 6-year-old (Xu, Gelfer, Sileo, Filler, & Perkins, 2008).

Context of Current Study

Hong Kong kindergarten mathematics teaching focuses on drilling and practice on counting, number combination, and computation (Cheng, Chan, Li, Ng, & Woo, 2001). Gersten and Chard (1999) cautioned that students could be drilled to number facts and taught algorithms for computation, but never develop much number sense. This is exactly the second author of the current paper, who is a teacher researcher, observed in the classroom. For example, a child could do 5 + 1 +1 = 7 without understanding one needed a 5-dollar coin and two 1-dollar coins to buy a 7-dollar item. Another child knew from memory that 2, 4, and 8 are even numbers, but could not make the inference that 12 is also an even number since she did not understand what even number meant. In other words, children "do" math without understanding numbers.

To meet the challenge of developing young children's number sense, the latest version of Hong Kong Education Bureau's Guide to the Pre-primary Curriculum (2006) advocates that the learning objectives of pre-primary mathematics should focus on developing interests in math through play and experiment, developing concepts of counting, sequencing, ordering, sorting, and comparing, developing concepts of space and time, and solving problems through observation, analysis, and discussion. In terms of teaching practice, it promotes teaching math through play and activities, providing opportunities for children to explore and discover, encouraging children to discuss math, and more importantly, avoiding one-way teaching and repetitive drilling.

Board games give teachers a tool to engage children in math through play-based learning. As informal as playing games is, it involves intensive practice on various basic number sense concepts, such as counting, number line, one-to-one correspondence, and space and time. When designed by teachers, higher level number sense concepts, such as number transformation, computation, and story problem, could be integrated easily based on the teachers' evaluation of her students' number sense level, which makes the games highly adaptable and sensitive to children's developmental needs. Furthermore, since the games are played by a group of children simultaneously, it is possible and desirable to mix children with different levels to encourage social interaction and peer tutoring.

Method

Design

The current study is aimed to examine the effectiveness of a teacher-designed board game in facilitating children's number sense development, as well as the teacher's scaffolding and children's learning process while playing the board game. The study is a mixed design incorporating both qualitative and quantitative methods in a small scale experimental study. It examines the effectiveness of the board game by comparing the results from a pretest and a posttest on number sense developmental level from an experimental group and a contrast group. Qualitative data include transcriptions of videotaped game sessions, anecdotal observations of teacher's scaffolding and children's learning process, and the teacher's own reflective journals on issues emerged from playing the game.

Participants

Eight 5-year-old children (4 girls and 4 boys) were randomly recruited from a K3 classroom in a working class neighborhood kindergarten in Hong Kong. Based on their performances on the pretest, the 8 children were assigned to either the Experimental Group (EG) or the Contrast Group (CG) with matching abilities. There were equal numbers of boys and girls in the two groups.

Materials

The Chinese version of Malofeeva et al.'s (2004) Number Sense Test was adapted from Zhao (2006). Based on Chinese children's number sense ability, Zhao suggested the three levels of the test to be set as 1-10 for level one, 11-20 for level two, and 21-30 for level three. Pilot testing found ceiling effects for items in level one, therefore this study used items from level two and level three only. There were 95 items in total in level two and level three, with maximum possible points of 173. Materials used for the test included a Teddy bear, number cards, wooden counting beans, and scoring sheets. The pretest and posttest were administrated by the teacher researcher with each individual child in a quiet room in the kindergarten.

The teacher-designed board game was a snake-shaped number line with 60 numbered squares. There were 4 game pieces allowing 4 players at a time, two dice, 20 Wise Cards, and coins of different values. The sequence of the players and the number of steps of each move were determined by throwing the dice. Children counted while moving their game pieces. Some of the numbered squares on the board had math problems. Children needed to solve the problem when they stopped at one. Other children would determine whether the answers were correct. If the answer was correct, the child was rewarded by moving 2 steps forward; otherwise 3 steps backward. The player who arrived at the finishing point first won the game.

The problems on the board included the sum of the dots on the two dice (addition), the difference of the dots on the two dice (subtraction), adding 2 to or subtracting 2 from the dots on one die, and making a larger or smaller 2-digit number out of the dots on the two dice. In addition to the problems written in the squares, the players had the chance to draw a Wise Card from a pile at certain stops. The problems on the Wise Card ranged from comparing 2 numbers (e.g., which number is larger, 26 or 19?), comparing the quantity of 2 piles of dots (e.g., which pile has more dots?), identifying a missing number from a sequence (e.g., which number is missing, 14, 15, _, 17?), identifying an even or odd number from 4 numbers (e.g., which number is an even number, 23, 67, 34, 89?), identifying which number from 2 numbers is closer to a 3rd number (e.g., which one is closer to 24, 23 or 28?), to story problems (e.g., I have 22 candies, I give you 3, how many do I have left?) and money combination questions (e.g., A train ticket costs $3, how would you combine coins of different values?).

Intervention

While the CG children were taught by their regular classroom teacher number combinations and computational equations, the EG children played the board game once a week with the teacher researcher for 8 weeks. Each session lasted about 30-50 minutes, and ended as soon as there was a winner. During the game, the teacher played the role of a facilitator and observer. She also scaffolded children's learning by asking children to clarify questions, probing new solutions, and explaining and summarizing strategies when needed.

The intervention effect is a combination of the board game and teacher scaffolding and peer tutoring. Although the CG children also had the opportunity to be scaffolded by the teacher and peers during their regular classes, the board game sessions the EG children participated in provided more teachable moments such as cognitive conflict and competitive problem solving for teacher scaffolding and peer tutoring to take place.

Results and Discussion

Number sense improvements

Table 1 listed the percentage of children's correct responses on the Number Sense Test items. Although there are 6 components in the test, rote counting alone counts for 30 points out of the total 173 possible points, therefore it was separated from counting and listed here as an independent category.

Children from the EG and the CG were matched based on their pretest results. Their performances on the pretest items were almost identical, except for *Ordinarity,* on which the CG children did better than the EG children. During the posttest, however, the EG children did better than the CG children on almost all items, especially *Addition-subtraction* and *Comparison*. Children from both groups scored 58.3% at *Addition-subtraction* in the pretest;

but the EG children managed to achieve 93.7% in the posttest, compared with 72.9% from their counterparts. For *Comparison*, the CG children scored 87.5% in both pretest and posttest without improvement; meanwhile the EG children improved from 87.5% to 98.7%.

Table 1. *Percentages on the Number Sense Test items before and after the intervention of the Experimental Group (EG) and the Contrast Group (CG)*

	EG	CG
	Pretest	
Rote counting	96.5	97.9
Counting	87.7	86.7
Number identification	100	100
Number-object correspondence	81.9	80.5
Ordinarity	72.9	83.3
Comparison	87.5	87.5
Addition-subtraction	58.3	58.3
	Posttest	
Rote counting	100	99.3
Counting	96.4	91.2
Number identification	100	100
Number-object correspondence	91.6	88.8
Ordinarity	93.7	87.5
Comparison	98.7	87.5
Addition-subtraction	93.7	72.9

Note. n = 4.

Teacher's role

The teacher participated in the game sessions as a facilitator and observer. She introduced the game to children at the beginning of the 8 weeks. During the first couple of weeks, the teacher spent considerable time discussing the rules of the game with children and made rule revisions based on children's suggestions. Once the children agreed and were familiarized with the rules, they started to sustain the game sessions themselves. The teacher remained actively engaged through asking questions to facilitate math reasoning. For example, when other children decided the player gave a wrong answer to a question, she often asked them to explain to the player why it was wrong. At times she would stop the players and ask who was the closest to the finishing point and who was the second closest to prompt seriation reasoning and number comparison. In her reflective journal, the teacher wrote:

> *Children seemed to have mastered the rules of the game after the previous sessions. The flow of the game was much better since they did not need my explanation of each and every step anymore. I took on a facilitator's role by encouraging children to take charge of the game. By asking the right questions at the right time, I gave children more opportunities to express their thinking strategy.*

To scaffold children's learning process, the teacher needed to be a keen observer of children's developmental level. She wrote in her journal about her observation of children's counting strategy:

> *I noticed children used different strategies of counting when I asked how they came out with the answer. Some said they used fingers to add; some said they counted one by one in their mind; some said they kept the larger number in mind and used fingers to add up to the total; and some said they simply remembered the answer.*

The teacher constantly reflected on her role in the game and consequentially enhanced her teaching. For example, by reviewing the videotapes of the play sessions at the early stage of the intervention, she realized that she did not always give enough time and space for children to process the information:

> *I found myself in the video providing help to children without thinking through. For example, I read the Wise Cards out of first reaction; I used close ended questions without giving children the opportunity to express the strategy they used. Need to stand back a little.*

The teacher managed to scaffold children's learning through suggesting strategies according to the child's zone of proximal development, refocusing on problems, and eliciting multiple solutions. Different number sense developmental levels existed among children in the EG. Some more advanced children were already mastering 2-digit numbers; while others were still struggling with basic number relations. The teacher derived her instructions based both on the problem at hand and on the child's number sense level. For example, a boy threw a 6 and a 4 when the question was to figure out the difference between the 2 numbers. Seeing the boy's hesitation, the teacher suggested him to use his fingers to count forward to 6, and then backward to 4. The boy used the strategy and figured out the difference was 2. She then asked the boy to explain to others why so. In another case, the question was that out of 22 and 28, which number was closer to 26. The following is the transcript of this learning episode:

Girl 1 (looking at teacher): Is it 22? Looks like it… (Guessing)
Teacher: Don't know. Count them yourself. How would you count them? (Suggesting a strategy)
Girl 1 (pointing to Boy 2): You count them yourself. (Encouragement)
Boy 2: It's 25. (Guessing)
Teacher: Which one is closer to 26, 22 or 28? (Reiterating the question)
Girl 1: Which one is closer to 26, 22 or 28? (Reprocessing)
Boy 2: Then it is 22. (Estimating)
Teacher: Is it 22? Is it closer to 26? (Probing)
Boy 2: But 28, 27, 26. (Starting to use the strategy, counting backward)
Teacher: How about 22 then? (Asking for comparison)
Girl 1: 22, 23, 24, 25, 26. (Participating in problem solving)
Boy 2: Then it's 28. (Compared and problem solved)
Girl 1: 28. (Compared and problem solved)
Teacher: Sure? (Confirmation)
Boy 2 and Girl 1: Yes.

Children started by guessing the answer. The teacher suggested counting the numbers. "How would you count?" she asked. One child started to count backward from 28, and another child started to count forward from 22. After comparing the 2 counting sequences, both of them realized 28 was closer to 26 than 22 was.

There were times when children lost their focus on the challenging problems. In the previous example, when the boy gave a random guess of 25, the teacher did not judge or try to solve the problem for children; instead, she paused and read the question one more time to refocus children's attention and prompt thinking. The strategy worked, and the girl followed the teacher and reiterated the question again in a prolonged tone, indicating she was thinking through the problem.

At other times, the teacher tried to elicit multiple solutions on one problem to encourage flexibility in thinking. For instance, when a boy gave a solution to a question of how to combine coins to pay for a $9 item, the teacher asked whether there were any other ways to solve the same problem.

Peer tutoring

The teacher always gave children positive feedbacks. At the same time she encouraged children to show appreciation and support to fellow players by giving a round of applause to great or improved performances. Through modeling and encouragement, the teacher successfully established a supportive and respective culture in the group. Children proactively offered support and help to others when playing. There was an occasion when Boy 1 struggled to make a smaller 2-digit number with two dice while other children in the game already mastered the concept. He threw a 2 and a 3. The following is the transcript of children's conversation:

Girl 1: How to make a smaller 2-digit? (Already knows the answer, asking the question with teaching intention)

Boy 1: 3 plus 2…

Girl 1: Not 3 plus 2, it's not addition. The question was how to make a smaller 2-digit number. (Correcting mistake, reiterating the problem)

Boy 2: What's smaller 2-digit? (Also knows the answer, refocusing on the problem)

Girl 2: Do you need a piece of paper (to write it down)? (Suggesting a strategy)

Boy 2: Which one is smaller? (Small step scaffolding)

Boy 1: 3 is larger.

Boy 2: Which one is smaller then? (Small step scaffolding)

Boy 1: 2

Boy 2: Then you put them together; you will get it correctly then. (Suggesting next step, encouragement)

(Boy 1 puts 2 before 3.)

Girl 1: Correct! (Confirmation)
Teacher: What's the number then?
Boy 1: 23.

In this case, all three children were helping the boy with the problem. They either tried to correct the boy's mistaken understanding of the problem, or suggested a strategy of writing the possible number combinations down. Boy 2 used effective small step scaffolding to guide Boy 1 into solving the problem step by step.

Playing in a group, children also had the chance to learn from each other through collaboration and modeling. Again in the previous which-number-is-closer-to-26 problem, the two children collaborated on the problem solving by counting from different directions. Each of them made a unique contribution and together they solved the problem. Since working in a group requires children to communicate their thinking process in words, the game also provided opportunities for children to talk and discuss math. The more capable kids could discuss explicitly their methods in solving the problems. When the less capable kids encountered the same problems, they adopted the similar strategies their peers used. In her journal, the teacher reflected on the collaborative learning of the board game:

> *I found the board game is much more than just a math tool. It not only facilitates interaction among children, but also provides mutual scaffolding opportunities and an opportunity for social emotional education.*

Concluding Remarks

The board game was engaging and effective in terms of promoting early number sense development. As a form of informal education, game provided a genuine play-based learning experience for children to realize their zone of proximal development (Vygotsky, 1978). During the later stage of the intervention, children were highly engaged in the game and participated enthusiastically. The test results showed that the EG children's number sense improved, especially the more advanced arithmetic components like comparison and computation.

Play-based teaching and learning such as the teacher-designed game empowered teacher to engage in curriculum design and pedagogy reform. The game design took the curriculum objectives and children's current developmental levels into consideration, which made it adaptable and flexible. The teacher could potentially change the contents of the problems when children's ability improved, or when using the game with a different group of children. The teacher's professional development was enhanced by reflecting on and improving own teaching strategies in such a child-centered, play-based learning environment.

The game also empowered children to take charge of their own learning (Brown & Palincsar, 1989). Children started to ask for more challenging questions at the end when they learned all the concepts in the game. Finally, children were intrinsically motivated to learn math. The game was sensitive to children's individual differences. In playing the game, each child had the opportunity to learn on own pace. And since it was a social game played in groups, children also learned how to collaborate with others and exchange knowledge among each other through imitation and peer tutoring.

This current study is a small scale study with only one group of children playing the game. Future research should consider recruiting more children to examine the effect of the board game on number sense development, and to potentially benefit more children with the pedagogy. Like most of the experimental research in education, there is an ethical concern regarding to withholding the best teaching materials and pedagogy available to the CG children. For this study, the CG children showed curiosity towards their classmates' game sessions but did not have the opportunity to participate. The teacher acknowledged the issue and started to play the board game with the rest of the class after the study was completed. The other side of this issue is that the EG and the CG children were from the same classroom, in which there was more than one session devoted to regular math teaching each week. Since the EG children only played the game once a week, they also had exposure to their teacher's number combination and computational equation classes between the pretest and the posttest, together with the CG children, which might compromise the results of the experimental design. The number sense test used in this study does not measure some of the number sense components such as estimation. There were ceiling effects on some of the items even in the pretest for 5-year-old Hong Kong children in this study. Future study should consider using more complete number sense battery that reflects children’s ability more accurately.

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