The Education University of Hong Kong

Effectiveness of implementing inquiry-based learning in teaching Mathematics among pre-service teachers in Hong Kong

Honours Project Final Report

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Table of Contents

Declaration

I, *Wong Hiu Ching* , declare that this research report represents my own work under the supervision of *Dr. NG Cheuk Wing Margaret*, Senior Lecturer of Department of Curriculum and Instruction (C&I), and that it has not been submitted previously for examination to any tertiary institution.

Signed _____

Wong Hiu Ching

10.05.2020

Acknowledgements

I would like to express my deep gratitude to Dr. Margaret Ng for her patient guidance, enthusiastic encouragement, and valuable and constructive suggestions during the planning and development of this research project. Her willingness to give her time so generously has been very much appreciated.

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Abstract

This research study was aimed to investigate Hong Kong mathematics pre-service teachers' effectiveness in implementing inquiry-based learning in teaching, to examine their knowledge and skill level for developing and executing inquiry-based learning lessons, and to suggest various ways for future improvement on the IBL enrichment for pre-service teachers.

A questionnaire on the fulfillment of the 4D implementation model of mathematics inquiry by mathematics pre-service teachers was adapted from the inquiry instruction scale created by Learning Point Associates (Margolin, Brown & Miller, 2008) and the assessment checklist introduced by Fielding-Wells and Makar (2010), and an interview was designed to explore pre-service teachers' preference on teaching approaches and the difficulties they encountered with inquiry-based learning lessons. 37 mathematics pre-service teachers studying in the Education University of Hong Kong undertook the questionnaire and 4 mathematics pre-service teachers were selected to do the interview.

The findings of this research study showed that pre-service teachers were unfamiliar with the fundamental elements of inquiry-based learning in mathematics: the five major issues of inquiry-based learning in mathematics and the three design principles for developing an inquiry-based learning mathematics curriculum. Therefore, their effectiveness in each implementation phase of the 4D model is unsatisfactory. To improve the overall situation, pre-service teachers are recommended to have further enhancement on inquiry-based learning to elevate their knowledge, skills and experience with the implementation of IBL in mathematics teaching.

1. Introduction

In 2000, the Education Bureau has adopted a education reform. There is an increasing emphasis on the adoption of student-centered teaching approach in mathematics' education. Learners are encouraged to engage themselves in the knowledge developing process actively by building their knowledge through inquiry processes like communication, reasoning, problem-solving, and relation (Chin, Lin & Tuan, 2016; Education Bureau, 2017). The education reform's ultimate objective is to boost the mathematical proficiency of leaners, including mathematical conceptual understandings, the nine mathematics generic skills, and positive beliefs and attitudes towards mathematics learning (Education Bureau, 2017). To achieve the above goals, inquiry-based learning (IBL) is considered to be an essential tool.

IBL in mathematics is generally considered as a form of active, student-centered instruction (Hayward, Kogan & Laursen, 2016). It is facilitates learners' mathematical knowledge development and mathematical thinking (as cited in Chin, Lin & Tuan, 2016). IBL improves students' critical thinking skills by addressing loosely-defined problems and by establishing logical mathematical arguments and analyzing them (as cited in Hayward, Kogan & Laursen, 2016). It is defined by the development of an atmosphere by teachers in which students expand their mathematics competence by actively conjecturing, explaining and integrating ideas, gathering and analyzing data, rationalizing and drawing conclusions, and communicating results to other learners and teachers (as cited in Chin, Lin & Tuan, 2016).

According to the Education Bureau (2017), there are five aims of Hong Kong's mathematics education including students' a) abilities to think critically and creatively, to conceptualize, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines; b) ability to communicate with others, express their views clearly and logically in mathematical languages; c) ability to manipulate numbers, symbols and other mathematical objects; d) number sense, symbol sense, spatial sense, measurement sense, and the capacity to appreciate structures and patterns; and e) a positive attitude towards mathematics learning and an appreciation of the aesthetic nature and cultural aspect of mathematics.

The foregoing objectives of mathematics education can be accomplished with the characteristics of IBL. The five traits of IBL are process-oriented, analysis, community learning, discussion monitoring, and real-life application. When students solve the

inquiry mathematical problems on their own, they internalize the mathematical conceptual processes which are more prioritized in the IBL as well as mathematics education. Moreover, in IBL, the teacher would pose an inquiry problem arising from the questions of the mathematical content or the students. Then students would need to use mathematics to solve the inquiry problem which requires them to think creatively and critically. Besides, when exploring the inquiry problem, students are expected to work in pairs or groups. Throughout the learning process, they have to assist each other, enabling them to communicate, debate and expand on ideas as well as formulate a solution in mathematical languages. With the deep involvement with mathematics and cooperation with peers in IBL, it is an apt pedagogy for achieving the goals of mathematics education.

Despite there are plenty of teaching materials and resources for IBL in mathematics, there is a lack of research investigating Hong Kong teachers' adoption of IBL in mathematics education. Although the mathematical inquiry knowledge of teachers is vital for implementing IBL in mathematics teaching, there is an inadequate awareness of teachers' level of related inquiry knowledge and how they apply the relevant knowledge in lessons. Besides, researchers have not yet reached any consensus on whether pre-service mathematics teachers possess with knowledge and skills to implement IBL in their classroom effectively as well as handling students' inquiry learning by creating a favorable environment. Investigating the above questions would greatly expand the awareness of pre-service mathematics teachers' inquiry knowledge development and IBL in mathematics, and thus support the learning of the students.

Therefore, this research study aims to investigate the effectiveness of implementing IBL in teaching by pre-service mathematics teachers in Hong Kong. Specifically, pre-service teachers' knowledge for the implementation of and skills about the implementing IBL would be examined through questionnaires as well as follow-up interviews to address the following research questions:

I. What are Hong Kong pre-service mathematics teachers' experiences with the implementation of inquiry-based learning in their classrooms?
II. Do they possess the knowledge and skills to implement the 4D model of inquiry-based learning?
III. What concerns do they have with respect to the implementation of inquiry-based learning in their classroom?
IV. Do they encounter any gaps or inconsistencies in their knowledge and skills regarding the implementation of the 4D model of IBL when they implement the mathematical inquiry? If not, how do they prevent

inconsistencies when implementing IBL in teaching? If any, how do they respond to the inconsistencies to let the inquiry process continue?

Most importantly, recommendations regarding the future IBL related curriculum in Hong Kong and professional development would be made for further improvement.

2. Literature Review

2.1 Inquiry-based learning

IBL is a pedagogy that change from traditional deductive forms to a more inspiring and stimulating teaching and learning model (Engeln, Euler & Maass, 2013). It is a teaching method that provides an authentic learning atmosphere that connects the learning of generic skills with real-world problems (Education Bureau, 2017). The generic skills can be incorporated into teaching and learning mathematics with IBL's learning format. There are nine generic skills promoted in mathematics education, including collaboration skills, communication skills, creativity, critical thinking skills, information technology skills, mathematical skills, problem-solving skills, self-learning skills, and self-management skills (Education Bureau, 2017). With the arrangement of IBL in mathematics, the above generic skills are involved in the learning activities. In IBL, students discover new insights or causal relationships by posing questions, hypothesis formulation, investigations and conclusion construction in a collaborative manner (Pedaste et al., 2015; Lehtinen & Viiri, 2017). During inquiry, students need to learn to be responsible for their own part and respect others throughout discussion. Students have to apply problem-solving skills and critical thinking skills when they need to solve the problems by selecting only the relevant information (Engeln, Euler & Maass, 2013). It is believed that their creativity is likewise strengthened and evolved by formulating solutions, ideas presentation, and suggesting approaches of enhancing their solutions (as cited in Voet & Wever, 2018). Most significantly, inquiry learning activities increase students' interest in mathematics learning and awareness of mathematics' importance in daily life (Education Bureau, 2017).

2.2 Five major issues of IBL in mathematics

With reference to the systematic research conducted by Dreyøe, Larsen, Hjelmborg, Michelsen and Misfeldt (2018), they have condensed five major issues for inquiry-based learning (IBL) in mathematics: 1) communication in the mathematics classroom, 2) mathematical competence, 3) moving in and out of the mathematical domain, 4) tools and resources for planning and implementing inquiry-based learning, and 5) professional development and collaboration. To implement IBL in mathematics effectively, these five major issues have to be well addressed so as to

allow mathematics teachers to get a full picture of how IBL practice can take place in the classroom.

2.2.1 Communication in the mathematics classroom

In IBL, communication between instructors and learners, and among learners play an important role. Firstly, to facilitate interaction between the instructor, the learners and the mathematics content, an environment must be created in which students feel comfortable communicating their understanding in mathematics. As in IBL, students are preferred to work in collaborative groups rather than doing inquiry on their own. When students do inquiry collaboratively, they have to communicate with their groups on constructing and executing plans to address the inquiry problem. In the process, students would practice listening to others and receiving and providing feedback to others. Moreover, they would learn to negotiate ideas, develop a tolerance for different ways of thinking as well as resilience to continue contributing to the group when their ideas are rejected (Fielding-Wells & Makar, 2010; Education Bureau, 2017). Hence, students are believed to build their communication skills, deepen their learning through collaborative inquiry learning only if there is an environment that allows them to communicate and express their mathematical understanding freely and comfortably with their classmates. They would benefit from doing inquiry in collaborative groups as their communication skills are believed to be deepened through collaborative inquiry learning (Education Bureau, 2017).

Secondly, in an inquiry classroom, the teacher plays the role of a facilitator and communicates with students through providing guidance. Scaffolding students' learning through analysing their solution processes and deductive reasoning is crucial (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). Hence, teachers' major role is to guide the inquiry process to probe students' thinking. The most common way of providing guidance is through approach questioning. However, teachers have to consider the inquiry process through students' points of view in order to give effective questions (Fielding-Wells & Makar, 2010). Moreover, instructional scaffolding can be provided to advance learners' thinking through appropriate questioning such as asking higher level and cognitively based questions while students are actively exploring their learning process (as cited in Reaume, 2011). In addition, teachers can encourage learners to move towards higher levels of knowledge and understanding by giving affirming responses and feedback (Reaume, 2011). With cognitive conflicts promoted by the teacher, students' learning can be enhanced by constantly investigating the mathematical content involved.

2.2.2 Mathematical competence

The ultimate goal of IBL in mathematics is to develop and enhance students' generic skills which incorporate several essential mathematical competencies including problem-solving competence, mathematical creativity and so on. Students are encouraged to pose questions in inquiry-based activities, and answer them by investigation, and thus establish the relationship between mathematical generalizations and mathematical competence. Students' flexible thinking is believed to be increased by this kind of authentic mathematical inquiry (i.e. the problem-posing process). (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). Therefore, inquiry-based teaching that involves problem-solving is preferred as it build the problem-solving competence for students (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018).

Moreover, scaffolding for students in the inquiry processes is also important in elevating their mathematics competence. Scaffolding means the use of teaching aids that have characteristics such as "responsiveness" and "fading" (Jessen, Doorman & Bos, 2017). "Responsiveness" refers to the scaffolding is adapted to students' needs, while "fading" refers to the gradually disappearing scaffolding as the students advance with their inquiry. The level of scaffolding has to be adjusted to students' abilities. The teacher can vary to challenge high-achieving students or accommodate the needs of low-achieving students. With carefully designed unstructured inquiry tasks and sufficient scaffolding, the aim of elevating students' mathematics competencies can be achieved.

2.2.3 Moving in and out of the mathematical domain

All interactions in the classroom have to start with the students' prior knowledge. The mathematical knowledge of students and their understanding of their environment are the major aspects of IBL (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). When designing the mathematical inquiry units, teachers take learners' prior knowledge, learning objectives, the process for reaching the desirable knowledge, as well as where and how the new knowledge can be applied into consideration (as cited in Brune, 2010). This can enable students to develop and build new and deeper understandings through inquiry (Eick & Reed, 2001). Considering students' prior knowledge can ensure the inquiry question or problem can engage all learners in making and testing their mathematical hypothesis in the later phase (as cited in Brune, 2010). Most importantly, by connecting learners' prior knowledge and the experiences with the knowledge could offer new experiences to students to build further connections between different knowledge and skills. Hence,

with keeping students' pre-understanding in mind, scaffolding on their prior experience can be offered by teachers to imposes a significant influence on students' learning outcomes (Taylor & Bilbrey, 2011; Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018).

Other than considering students' prior knowledge, it is crucial to use a variety of learning activities. Diversified learning experiences at various levels can be arranged in the mathematics inquiry classroom (Education Bureau, 2017). In inquiry lessons, students would respond to problematic situations that engage them in mathematical problems in a real-life context in mathematical inquiry. Hence, a good inquiry question that underlies an authentic problem to give the investigation purpose is needed to reinforce or extend students' problem-solving abilities and to encourage their logical thinking (Fielding-Wells & Makar, 2010; Education Bureau, 2017). Hence, open-ended questions or non-routine problems can be introduced. This type of question has multiple correct answers since there are ambiguities in the problem or in the process of solving the problem which requires students to come up with various solutions (Fielding-Wells & Makar, 2010). Moreover, they require students to discriminate what knowledge is needed for certain situations to solve make the ill-structured questions to well-structured. Hence, they require students to put great emphasis on evidence, reasoning and judgments while justifying their answers. Moreover, less organized and more open-ended inquiry learning exercises would allow students to be versatile and prepare them to deal with conditions and issues outside school (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018).

2.2.4 Tools and resources for planning and implementing IBL

Technology-based tools and resources, and didactical tools are essential for help teachers in preparing and implementing IBL in mathematics. With the increasing advancement of information technology, the mathematics curriculum strongly emphasizes incorporating IBL into the teaching and learning of various topics (Education Bureau, 2017). Teachers can integrate technology with suitable pedagogy and mathematics topics to strengthen the learning and teaching effectiveness through IBL. For instance, the computer algebra system can be used by students in the inquiry process when the inquiry process involves handling large amounts of repetitive computations which conventional computing equipment could not handle smoothly. Appropriate use of tools and resources can enhance learning effectiveness and stimulate deep learning (Education Bureau, 2017). Before using the e-learning material in the inquiry lessons, teachers have to consider whether the following two aspects of the design of the e-learning materials are satisfied or not in

the planning process. Firstly, guidance on operation of the e-learning resources need to be developed and given in the classroom to ensure students can have a specific objective when engaging in problem-solving or exploratory activities. Secondly, the shortcomings of e-learning resources in aspects such as concept building, cognitive knowledge accumulation, and mathematical proofs should be figured out (Education Bureau, 2017).

2.2.5 Professional development and collaboration

Professional development involves the development of teacher competencies, and how IBL can be prepared and assessed, and how teacher collaboration can be developed and sustained (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). Since teachers are the central decision-makers who decide which level of IBL to be adopted and how it can be adapted in the classroom (as cited in Mupira & Ramnarain, 2017), teachers carry great responsibility for the success of IBL. Therefore, teachers have to be equipped with certain knowledge and skills before and during the implementation of IBL. Moreover, teachers' content knowledge, pedagogical knowledge and theoretical knowledge must act in concert before they can implement IBL (Crawford, 2000). Teachers' knowledge is important to make them an effective teacher (as cited in Reaume, 2011). On one hand, teachers with less content knowledge tend to be less confident and preferred to stick to the textbook (Reaume, 2011). On the other hand, teachers with a strong background in their subject are better prepared for offering authentic learning experience to learners (as cited in Reaume, 2011). Most importantly, the teacher's specialized content knowledge also play a crucial role, as the teacher has to consider the inquiry process through the students' point of view (Fielding-Wells & Makar, 2010). With the aim to strive for better development of inquiry knowledge and practices, teachers are encouraged to take professional development programs (Chin, Lin & Tuan, 2016).

Other than individual professional development enhancement, teachers can collaborate with colleagues to strive for improvement in the planning and implementation of IBL. Teachers can gather, collect and utilize those small, already planned, or already implemented teaching units in an IBL context to evaluate the difference between each other's teaching approaches (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018).

2.3 Three design principles for developing IBL mathematics curriculum

Based on the five central issues in IBL in mathematics, the complexity and natures of IBL in mathematics are being discussed. Hence, teachers should follow the three principles proposed by Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt (2018), which reflect the five important associated with IBL in mathematics, to develop and implement their inquiry curriculum. The first principle is it has to be an exploratory, dialogical, and application-oriented teaching method with room for student participation increases the effects of the student's understanding of mathematical concepts and develops appropriate ways of working (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). This principle focuses on the dialogic climate, exploration, application of knowledge, students' degree of participation, as well as the mathematical concepts. Teachers have to create a dialogue about the specific mathematical concepts and knowledge which allow room for teacher-students communication. The communication between teachers and students has a major impact on the investigation which determines whether the investigation can be conducted or not. Moreover, the mathematical concepts involved are of great importance and teachers should emphasize the specific mathematical concepts and knowledge during the inquiry process.

The second principle is that teachers should prioritize students' experience of the teaching and the content should be meaningful both from an internal mathematical perspective and from the perspective of the situation of application or inquiry so as to enhance motivation and learning (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). It is crucial for all activities to be meaningful for students and involve dialogue which makes students wonder, think, reflect, and be curious so they would develop the motivation to participate and learn. Hence, teachers should design and construct inquiry activities with students' prior mathematical experience and knowledge as bases. A meaningful starting point is an authentic problem situation in students' surroundings or in daily life. Students need to analyze the problems or situations properly and clearly as they use arithmetic to solve authentic issues. They have to select the more appropriate strategies or methods and apply them to solve the problem systematically. Consequently, students can appreciate mathematics' importance and its beauty which makes their learning meaningful (Education Bureau, 2017).

The last principle is it has to be an exploratory, dialogical, and application-oriented teaching with room for student participation increases the possibility of implementing mathematical competencies (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). This principle emphasized the students' participation in the inquiry activities.

Since it is crucial to allow every learners to carry out the excellence of their abilities (Education Bureau, 2017), teachers need to offer teaching differentiation depending on whether the student is talented or challenged in mathematics to ensure all students are participating in the activities. Moreover, the inquiry activities should aim at getting students to understand the specific mathematical subject matter (a skill or an understanding of a concept), as well as gaining mathematical competencies such as problem-solving or reasoning.

2.4 4D model of implementation of IBL in mathematics

With the three principles as the fundamental basis of the development of the inquiry–based mathematics curriculum, the 4D model should be adopted. According to Fielding-Wells and Makar (2010), there are four phases in inquiry: Discover, Devise, Develop and Defend. During the Discover phase, teachers will make the students become excited about the learning unit and problems by immersing them in the context of the problems and letting students draw on their own experience to grasp the overall issue. In the Devise phase, students would develop their overall direction and create a plan with a preliminary solution to the given problem. Moreover, students would be assisted to determine the needed evidence to answer the inquiry question. During the third phase, students would implement their plans and develop different data collection methods to record the required evidence. After data collection, students would have to analyze the data as well as select the most suitable way(s) to represent their mathematical thinking which can be used to communicate with others effectively. In the last phase, students would show their comprehension of the relation between the problem, the proof and the solution. In addition, students would also have to explain their solutions and communicate their conclusions about the question to the audience. Finally, they would reflect on the whole inquiry process as well as the knowledge gained throughout the process.

The mathematical inquiry involved in the 4D model requires a good inquiry question which should be an authentic problem or topic that gives an investigation purpose and at the same with potential for mathematical depth (Fielding-Wells & Makar, 2010). An authentic problem provides the inquiry with a purpose to drive the investigation. This helps to keep the students focused on the purpose as the inquiry proceeds and ensure that the mathematics has meaning. In addition, ambiguous words are recommended when it comes to developing a good inquiry question. Words such as ‘Best’, ‘Good’, ‘Typical’ which require students to debate the meaning and then determine how this might be measured or assessed are considered as ambiguous words. When the debate takes place, this is where mathematics can be linked. The

process of clarifying an ambiguous question provides many opportunities to learn mathematics (Fielding-Wells & Makar, 2010). With a good inquiry question, the mathematical inquiry process starts with a good foundation.

3. Methodology

3.1 Participants

In this research, participants were year 3 to final year students from the Mathematics Education program at EdUHK who have completed their teaching practicum at least for one time. During their teaching practicum, they would have the opportunity to apply and implement IBL into their teaching and evaluate whether they were effective enough to deliver IBL teaching and how the university can better prepare them for implementing IBL into their teaching in the future. Moreover, the reason for Mathematics as the subject of focus for this research is because the mathematical knowledge is important for every person as mathematics pervades every part of life. Hence, it would be valuable to investigate whether Mathematics pre-service teachers are equipped with skills and knowledge to adopt various pedagogies to help students learn mathematics.

Eventually, there were 37 pre-service teachers undertook the questionnaire, with 26 females and 11 males. They completed two sections regarding their background information with IBL and their level of fulfillment in the implementation of the 4D model inquiry model in the questionnaire. After the completion of the questionnaire, 4 participants, with 2 males and 2 females were chosen conveniently to undertake the interview. All the interviewees had expressed their view towards IBL and shared the challenges they encountered during their IBL lesson. They also suggested possible ways of improvement for the IBL education in the university.

3.2 Instruments

3.2.1 Questionnaire

The questionnaire was designed based on the five major issues of IBL in mathematics and the three principles for inquiry lesson development. In the questionnaire (see Appendix 2), there are two major sections including pre-service teachers' background information with IBL, and their level of fulfillment in the development of their inquiry lessons and the implementation of the 4D model inquiry model in the questionnaire. The first section concerns participants' demographics, their means of learning knowledge about IBL as well as their frequency in using IBL to teach mathematics. The second section concerns the participants' fulfillment of the 4D mathematics inquiry implementation model. The

participants would imply their views on the 28 statements using the 6-Likert scale, ranging from 1 to 6 which represents "Strongly Disagree" to "Strongly Agree" respectively. These 28 statements are adapted from the inquiry instruction scale created by Learning Point Associates (Margolin, Brown & Miller, 2008) and the assessment checklist introduced by Fielding-Wells and Makar (2010).

For the results of the second section, the quantitative data were analyzed through Statistical Package for the Social Studies (SPSS) and the analysis was coded with marks, mean scores to show an overview of participants' responses.

3.2.2 Interview

The interview aims to providing more information about interviewees' concerns they had while considering whether to develop and implement IBL in teaching Mathematics or not in their teaching practicum and the challenges they encountered when they implemented IBL in their teaching. The interviews were semi-structured individual interviews that lasted for twenty minutes each. There was a set of open-ended questions with some follow-up questions depending on the interviewees' responses (See Appendix 3). The interviews were audio-recorded for data analysis (See Appendix 4).

Relevant parts of the interview data would be transcribed for qualitative analysis. The transcriptions were reviewed to draw substantial discussions on the efficacy of IBL implementation in teaching mathematics by pre-service teachers and some suggestion for the IBL related enrichment in the future.

4. Results and Discussions

The data analysis of the questionnaire was performed using SPSS to quantify the participants' responses on their fulfillment of the four stages of the 4D model of mathematics inquiry for calculating the corresponding mean scores. The mean scores would be used to illustrate the degree of effectiveness of pre-service teachers' implementation of IBL in their teaching.

The data of the interview was analyzed through transcending the respective sections of the audio-recordings to explore the issues and difficulties faced by pre-service teachers in the implementation of IBL, to evaluate their effectiveness of implementation and to make recommendations for future course improvement. Responses from pre-service teachers in the interview will be labeled as pre-service 1, pre-service 2, pre-service 3

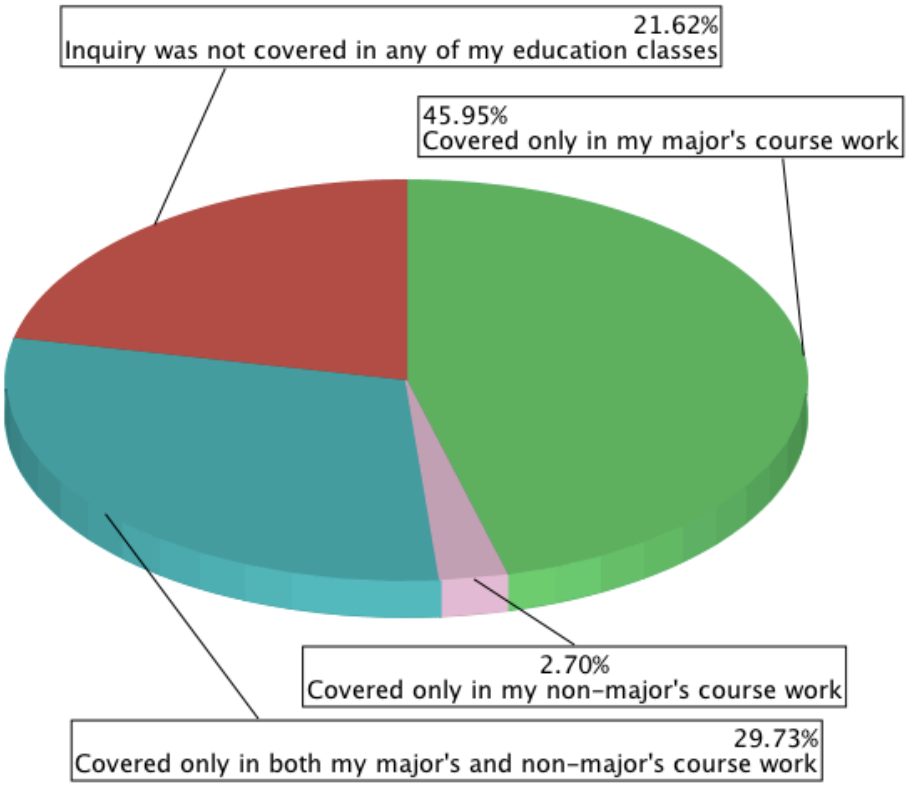
and pre-service 4 (PT1, PT2, PT3 and PT4) to indicate their individual responses. Detailed interview transcriptions can be referred to Appendix 5.

The collected data from both questionnaires and interviews were reviewed in light of the two major foci discussed in the literature review, i.e. the related IBL knowledge and skills of pre-service teachers in (a) developing; (b) implementing IBL curriculum. The findings included pre-service teachers' fulfillment score for each phase of the 4D implementation model (see Section 2.4).

4.1 Pre-service teachers' backgrounds

Aiming at knowing more about pre-service teachers' experience(s) in the teaching practicum and their past learning experience and use of IBL, participants were required to answer several questions regarding their background in the first part of the questionnaire.

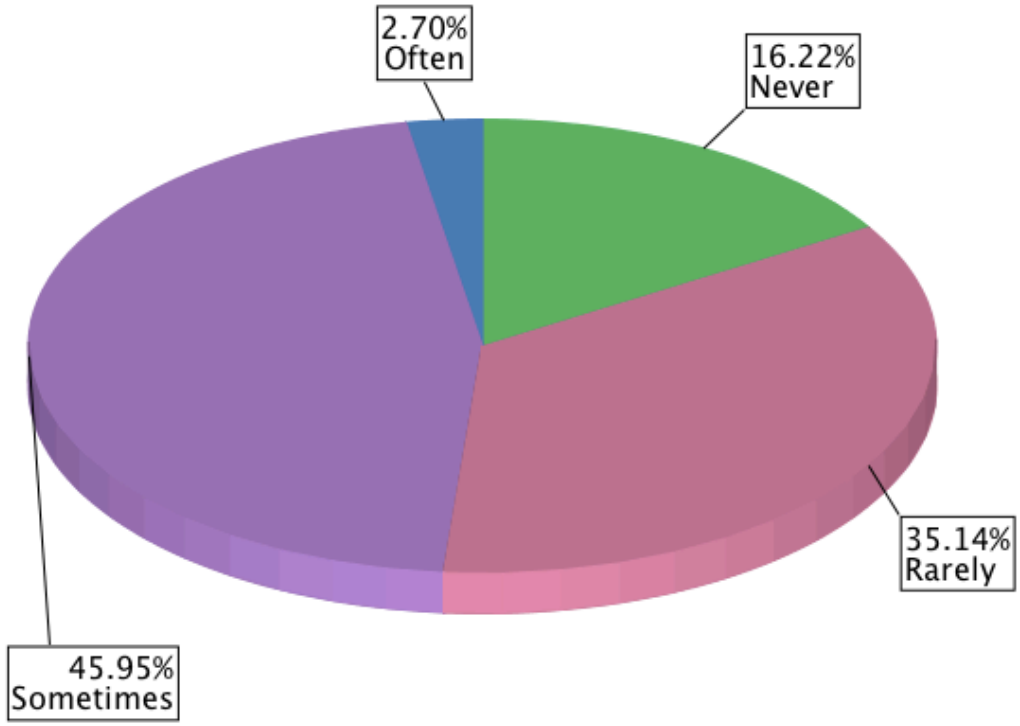
Graph 1. IBL coverage in teacher education (mathematics major)



From Graph 1, it indicates 78% of the participants had IBL content covered in their major's course work. However, there is a small number of participants which consists of 18.9% reported that none of their courses in professional education covered any content of IBL. Since all the participants were from the Mathematics Education program at EdUHK, they should receive the same major courses only differ in the

focus of primary and secondary level. Hence, this shows that even the participants were from the same department under the same education program, they had different impressions on the IBL coverage in their teacher education course(s). This inferred that that IBL contents covered in their major course work might not be sufficient and adequate so that the pre-service teachers had opposite views on whether they had learnt about IBL or not. Hence, pre-service teachers' mastery of the three design principles for developing IBL mathematics curriculum (as stated in Section 2.3) is being doubted.

Graph 2. Preference on the use of IBL in teaching mathematics



Graph 2 displays that 52% of the participants prefer and would sometimes or often put IBL into teaching mathematics. However, at the same time, the other 48% of the pre-service teachers showed that they would rarely or even never use IBL in teaching mathematics. The IBL approach is still not very popular and preferred when it comes to real teaching practice. The possible underlying cause for this phenomenon might be pre-service teachers' conflicting core beliefs with inquiry practices which hindered them to choose IBL as a pedagogical strategy (Ramnarain & Hlatshwayo, 2018). Pre-service teachers' beliefs influence their interpretation and decision, which in turn affect their teaching choices and actions in the classroom (as cited in Ramnarain & Hlatshwayo, 2018). Moreover, another reason would be pre-service teachers' questionable mastery of the three design principles for IBL mathematics curriculum development (as stated in Section 2.3). Without solid mastery of the IBL curriculum

design principles, pre-service teachers would less likely to incorporate IBL elements and hence not even involve the 4D mathematics inquiry model in their teaching designs.

4.2 Pre-service teachers' fulfillment of implementation of the 4D model of IBL in mathematics

To find out pre-service teachers' fulfillment of the 4D implementation model of IBL in teaching mathematics, they were required to indicate their views on 28 statements using the 6-Likert scale, ranging from 1 to 6 which represents "Strongly Disagree" to "Strongly Agree" respectively. The scores were calculated and shown with regards to every stage of the 4D model to investigate pre-service teachers' degree of fulfillment.

Apart from the questionnaire, aiming to explore pre-service teachers' concerns about and challenges encountered in the implementation of IBL in teaching mathematics, 4 participants were selected conveniently after the completion of the questionnaire to undertake the interview for sharing their deeper thoughts and insights on the IBL experiences they had during their teaching practicum as well as suggestion for the university can take to better prepare them for implementing IBL into their teaching.

Table 1. Mean fulfillment score of pre-service teachers in every stage of the 4D model

		Fulfillment score of Discover Stage (S1 - S4)	Fulfillment score of Devise Stage (S5 - S12)	Fulfillment score of Develop Stage (S13 – S18)	Fulfillment score of Defend Stage (S19 – S23)	Fulfillment score of Overall (S24-S28)
Total	Mean	15.7838	32.4595	22.4865	19.8649	22.1351
	Mean per quest-ion	3.9460	4.0574	3.7477	3.9730	4.4270
	N	37	37	37	37	37

Table 1 illustrates the mean fulfillment scores of participants in all four stages and the overall arrangement had the greatest mean score where the four stages were having a rather lower mean score. Among the four stages, the Devise stage among was indicated to be the most positive with a mean score of 4.06; the least positive stage was having a mark of 3.75 which belongs to the Develop stage. The Discover and

Defend stage shared a similar mean per question, with a score of 3.95and 3.97 respectively.

The discussion would be divided into four sections, including pre-service teachers' level of fulfillment and the challenges encountered in the four implementation stages of IBL. Significant and relevant results from the questionnaire data and parts of the interview transcription would be used to draw significant discussions.

4.2.1 Discover phase

As mentioned in Section 2.2.3 and 2.4, in the Discover phase of the 4D model of IBL, it is reflected that pre-service teachers commonly unable to make students understand the inquiry question or get the meaningfulness of it. In this paper, the findings from questionnaires and interviews both confirmed that pre-service teachers experienced similar problems in the implementation of IBL.

Table 2. *Statement 4: My students are able to make meaning of the inquiry question and understand why the inquiry question is important.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.0270**
	Disagree	12	32.43	35.14	
	Slightly Disagree	12	32.43	67.57	
	Slightly Agree	9	24.32	91.89	
	Agree	3	8.11	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

The finding from the questionnaires, in Table 2, exhibits that there are nearly 70% of the pre-service teachers disagreed with the statement "My students are able to make meaning of the inquiry question and understand why the inquiry question is important". The mean score of this statement is the lowest when compared to other statements for the Discover phase (see Appendix 3). Moreover, another finding from the interviews showed that pre-service teachers were concerned about students' understanding of the inquiry question and its meaningfulness in the interviews. Hence, they would consider the nature of the mathematical topics and the students' prior knowledge to ensure the students could learn with successful and meaningful inquiry experience.

"Since the topic is very common in daily life and students might have seen it in their daily life. Since they already had some knowledge or experience about volume, they might be able to learn through IBL. Therefore, I tried to teach this topic by IBL. It is easier for them to understand the inquiry question and its meaningfulness."—PT 2

According to the respondents, they think the mathematical topics which are comparatively concrete (i.e. topics about Shape and Space or Measure) would be more suitable for IBL. Moreover, they emphasize the importance of students' prior knowledge. If they are sure that the students have adequate prior knowledge and familiarity with the topic, the students are more motivated to learn and to get involved and engaged in the IBL activities. These findings confirmed that most of the students could understand the inquiry question but not all of them get its meaningfulness in the first phase. Without getting the importance of the inquiry question, the students might be less motivated in participating in the later phases of IBL.

4.2.2 Devise phase

As stated in Section 2.2.4 and 2.4, in the Devise phase of the 4D model of IBL, it is reflected that students are commonly incapable of creating their inquiry plan with a preliminary solution and determining the needed evidence without the use of well-designed IBL tools. In this paper, the findings from both questionnaires and interviews confirmed students experienced similar difficulties.

Table 3. *Statement 11: My students are able to see the problems with their plan.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.0270**
	Disagree	10	27.03	29.73	
	Slightly Disagree	15	40.21	70.27	
	Slightly Agree	9	24.32	94.59	
	Agree	2	5.41	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

From the questionnaire findings, in Table 3, the majority of pre-service teachers (70.3%) showed disagreement with the statement "My students are able to see the problems with their plan".

Table 4. *Statement 12: My students are able to revise their plan through my questioning when they encounter problems.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.5136**
	Disagree	1	2.70	5.41	
	Slightly Disagree	4	10.81	16.22	
	Slightly Agree	8	21.62	37.84	
	Agree	18	48.65	86.49	
	Strongly Agree	5	13.51	100.00	
	Total	37	100.00	100.00	

On the other hand, in Table 4, over 80% of the participants responded that their students could revise their inquiry plan with the assistance and guidance from the teacher to cope with the problems they encountered. Moreover, pre-service teachers had expressed their concerns about finding appropriate IBL teaching tools in the interview findings.

> *"Secondly, the sticks were not very suitable for the activity. I spent a long time finding the more appropriate materials for the activities. However, I couldn't find the sticks that I wanted. As the lengths of the stick have a huge difference, the difficulty of the task would increase and it affects the effectiveness of my lesson."—PT1*

Without the use of suitable teaching and learning tools, students experienced more difficulty in solving the inquiry problem. Hence, they have to rely much on the teacher, with often interference from the teacher, the process of IBL would be affected and the effects imposed by IBL may be less than expected. All in all, the above findings confirmed that the major problem in this phase might be the need of using suitable IBL tools. Students might lack the capacity to discover the problem with the tools. Hence, teachers' assistance and guidance are essential and crucial to help students to experience successful IBL when there are problems with the tools.

4.2.3 Develop phase

As seen in Section 2.2.2 and 2.4, in the Develop phase of the 4D model of IBL, it is reflected that students commonly lack the mathematical competence to do IBL. In this paper, both findings from the questionnaires and interviews confirmed that students experienced similar problems.

Table 5. *Statement 16: My students are able to analyze the evidence mathematically.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.2972**
	Disagree	5	13.51	16.22	
	Slightly Disagree	16	43.24	59.46	
	Slightly Agree	13	35.14	94.59	
	Agree	1	2.70	97.30	
	Strongly Agree	1	2.70	100.00	
	Total	37	100.00	100.00	

Table 6. *Statement 17: My students are able to sort their evidence to make it clear and organized.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	3	8.11	8.11	**3.2972**
	Disagree	3	8.11	16.22	
	Slightly Disagree	15	40.54	56.76	
	Slightly Agree	13	35.14	91.89	
	Agree	2	5.41	97.30	
	Strongly Agree	1	2.7	100.00	
	Total	37	100.00	100.00	

From the questionnaire findings, in Tables 5 and 6, the positivity in pre-service teachers' responses regarding students' achievement in data handling and analysis is quite low as there are only approximately 40% of them showed agreement toward the related statements (Statement 16 & 17). Furthermore, pre-service teachers showed similar concerns in the interview as the learning mode of IBL lesson is very different from what they experience in their usual learning since IBL is often conducted through experiments, activities, discussion, data collection and presentations which are not usually included in the traditional lessons.

> *"Since the learning mode of the IBL lesson was very different from the usual lessons. In the usual lessons, students would remember everything the teacher said. However, in the IBL lesson, the conclusion was made by the students."—PT 1*

Hence, they could predict that students might struggle with some inquiry tasks in which the questionnaire results echoed with their comments. However, there are slightly more positive responses from the participant towards the statement

regarding students finding appropriate ways to present their findings mathematically.

Table 7. *Statement 18: My students are able to represent their mathematical thinking through finding ways to present their evidence.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.5405**
	Disagree	4	10.81	13.51	
	Slightly Disagree	10	27.03	40.54	
	Slightly Agree	18	48.65	89.19	
	Agree	4	10.81	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

With reference to Table 7, over 60% of the pre-service teachers agreed their students could prepare for the presentations of the findings in appropriate ways. The above findings showed the students performed quite well in the plan execution part and hence they could collect the desired evidence. However, from the participants' responses, it was shown that the students' performance in the data analysis and presentation preparation part was slightly inferior. This might because students were not familiar with this kind of practice as data handling and presentation preparation was less prevalent in other teaching approaches as reflected by respondents. Therefore, pre-service teachers need to provide more chances for students to practice their mathematical generic skills such as data handling and data analysis.

4.2.4 Defend phase

As mentioned in Section 2.2.1 and 2.4, in the Defend phase of the 4D model of IBL, it is reflected that students commonly unable to comment or criticize their classmates' presentations. In this paper, the findings from both questionnaires and interviews confirmed that students experienced similar problems.

Table 8. *Statement 23: My students are able to critically examine the mathematical explanations of other groups.*

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	4	10.81	10.81	**3.0270**
	Disagree	5	13.51	24.32	
	Slightly Disagree	16	43.24	67.57	
	Slightly Agree	10	27.03	94.59	
	Agree	2	5.41	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

From the questionnaire findings, Table 8 reveals pre-service teachers' comparatively less positive reaction (32.4% of agreement) with the statement "My students are able to critically examine the mathematical explanations of other groups" while there were over 60% of the participants disagreed with the statement which is a higher disagreement rate than the other statements. Moreover, pre-service teachers also revealed their concerns about students' performance in criticizing their classmates' presentations.

> *"It is usually the teacher who does the commenting or criticizing. Hence, when it comes to the students to comment or criticize others' presentations, they often have no comments or not being able to criticize. I think it is normal as they did not have much experience in that. Therefore, they might be too shy or not really able to see the problems of others' presentations."—PT1*

The underlying cause might be students were not having enough confidence and ability to make a critique as this is not a common practice in the classroom (Fielding-Wells & Makar, 2010). Nevertheless, qualities such as communication, meaning making and in-depth mathematical thinking are highly valued in mathematics which are can be achieved and experienced in the Defend stage of IBL (as cited in Makar & Fielding-Wells, 2018). Therefore, pre-service teachers need to encourage students to engage in meaningful communication processes such as commenting or criticizing others' presentations in order to promote mathematical communication between students.

4.2.5 Overall

All in all, from the above data interpretation, the pre-service teachers involved in the questionnaire were quite conserved about their effectiveness in implementing IBL

into their teaching. Although there exists a rather high positivity in some of the statements regarding the implementation of the 4D model, the comparatively high level of agreement towards some of the statements is not significant to show that the pre-service teachers were effective in implementing IBL. The relatively high mean scores in the overall classroom arrangement on the other hand display a fact that the pre-service teachers were capable to create an environment to facilitate the high level of student engagement in IBL. However, it is crucial for pre-service teachers to have the ability to guide the students in participating in the IBL activities as well as achieving the task requirement in each phase which are seemed to be inadequate at the moment.

There are two possible underlying causes for the pre-service teachers' low effectiveness in the IBL implementation. The first reason might be students are unfamiliar with the natures of the IBL lesson (as stated in PT1's response in Section 4.2.3). Since most pre-service teachers might be the first time to have IBL with their students during the teaching practicum, the students were not used to this kind of learning style. They were used to follow what the teacher instructed or assigned and therefore they seldom had the power to design what to do in their learning. Hence, a comparatively active and student-centered learning like IBL might be overwhelmed to the students. The second reason might be pre-service teachers lack experience, skills, and knowledge in the implementation of IBL (i.e. findings regarding Graph 1 in Section 4.1). As students might not have much experience with IBL, teachers need to be responsible for leading and guiding students to walk through the process of IBL as well as help them fulfill the task in each stage. Since collaboration and discussion are central elements of IBL, teachers should have the confidence in guiding student collaboration and discussion while the high intellectual focus is maintained (Makar & Fielding-Wells, 2018). However, if the pre-service teachers themselves were not equipped with adequate soft power for handling an IBL lesson, they might therefore need further professional development to enhance their effectiveness in implementing IBL into their teaching.

4.3 Suggestions on pre-service teachers' overall fulfillment of the 4D model for the implementation of IBL

This paper has revealed pre-service teachers' responses regarding their students' performances and the effectiveness of their IBL mathematics lessons. This reflected that pre-service teachers ought to be equipped with essential and sufficient knowledge and skills to support the implementation of the 4D IBL model in their future

mathematics lessons. The following would be suggestions made according to the five major issues of IBL for pre-service teachers.

4.3.1 Suggestions on uplifting the abilities to move in and out of the mathematical domain

In the Discover phase of the 4D model of IBL, it is reported that pre-service teachers were unable to make students understand the meaningfulness of the inquiry question. As mentioned in Section 4.2.1, this problem is caused by students' lack of ability to move in and out of the mathematical domain to relate the inquiry problem to students' daily lives. With reference to the major issue of IBL mentioned in Section 2.2.3, one possible way to encourage students to move in and out of the mathematical domain is to use a variety of learning activities. More open-ended and less-structured tasks could be introduced to allow students to think flexibly and therefore and train them to deal with situations and problems they encounter in real-life. Pre-service teachers could encourage the use of modeling activities in their inquiry-based teaching. Regarding Greer, modeling can be viewed as the link between the aspects of reality, and the development of abstract formal structures in mathematics (as cited in Philippeaux-Pierre, 2009). The task for the students is to understand the question situation through the written description and to make reasonable assumptions and construct one or more than one models (solutions and strategies) to solve the problem. Hence, mathematical modeling could enable students to better reason and think mathematically which would aid students in making connections between mathematics in school and their out-of-school mathematical knowledge (as cited in Philippeaux-Pierre, 2009). This permits students to properly use arithmetic to formulate and solve issues in everyday life, mathematical contexts and other disciplines (Education Bureau, 2017). By using these activities to connect the mathematics world with reality, students are believed to get the meaningfulness of the inquiry activities. This helps to make the learning content meaningful for both the mathematical perspective and the situation of application which is one of the principles of developing IBL mathematics curriculum (i.e. the second principle in Section 2.3). Therefore, pre-service teachers are suggested to utilize well designed modeling activities in their IBL lessons to uplift students' abilities to move in and out of the mathematical domain.

4.3.2 Suggestions on developing tools and resources for planning and implementing IBL

In the Devise phase of the 4D model of IBL, it is reported that students' inabilities to execute and evaluate their inquiry plan. As mentioned in Section 4.2.2, this problem

might be caused by the lack of suitable teaching and learning tools for IBL. Referring to the major issue mentioned in Section 2.2.4, one possible solution is for pre-service teachers to establish various teaching and learning tools and resources for IBL at the school level. It is recommended to develop a collection of tools with other teachers so that teachers can work collaboratively and share the materials afterward. If inquiry is expected to be the norm, schools must understand that tools and resources are integral elements of teaching, and they should devote adequate resources to purchase suitable materials. Teachers should not be required to have the necessary teaching materials (National Research Council, 2000).

Since developing teaching materials is time-consuming, other than developing new teaching materials for IBL, teachers can adapt traditional materials to support IBL. Traditional materials such as textbook tasks often present precisely the information needed to solve a certain problem which only requires the application of a formula instead of learning to apply mathematics outside the classroom. Teachers can transform and adapt the structured version of teaching materials into unstructured versions. One possible way to transform a structured task to an unstructured task is to take all sub-questions and pose them in random order, or as parts of a puzzle and ask the students to find the original order (Jessen, Doorman & Bos, 2017). All in all, it is suggested that pre-service teachers could promote the development of an IBL resources bank at the school level to ensure there are enough suitable tools for conducting IBL activities.

4.3.3 Suggestions on elevating the level of mathematical competence

In the Develop phase of the 4D model of IBL, it is reported that students struggled with inquiry skills such as data handling and presentation preparation in the IBL activities. As mentioned in Section 4.2.3, this problem is caused by students' low level of mathematical generic skills such as data handling and presentation preparation as they were less prevalent in other teaching approaches. Regarding the major issue of IBL mentioned in Section 2.2.2, pre-service teachers could elevate students' mathematical generic skills by incorporating inquiry-related processes into classroom practices. The tasks of the IBL lesson should provide opportunities to learn about the inquiry skills in mathematics which can be achieved by unstructured tasks. Unstructured tasks would give students many opportunities for students to investigate, objectively evaluate, collaborate and communicate outcomes. Unstructured problems could place students in an active role in mathematical problem-solving.

Moreover, IBL empowered by unstructured tasks which allow multiple strategy solution will enhance students' mathematical competence. The students' approaches, their interpretations of the inquiry problem, their estimations, representations, conclusions and collaboration give chances to reflect on the mathematics inquiry processes (Jessen, Doorman & Bos, 2017). Teachers are proactive throughout the process. They help and inspire students who fail and expand those who excel by using carefully chosen strategic questions. This can ensure all students have the opportunities in participating in the inquiry activities and increase their mathematical competencies. This echoed with one of the principles of developing IBL mathematics curriculum (i.e. the third principle in Section 2.3). They respect the experiences of the student, including mistakes, and scaffold learning using students' logic and knowledge. Hence, it is suggested that pre-service teachers could impose unstructured inquiry tasks with multiple possible solutions to elevate students' mathematical competence.

4.3.4 Suggestions on improving communication in the mathematical classroom

In the Defend phase of the 4D model of IBL, it is reported that the communication between students about criticizing others' presentations was lacking. As mentioned in Section 4.2.4, this problem is caused by students' lack of confidence and incapability to criticize their classmates. With regard to the major issue of IBL mentioned in Section 2.2.1, pre-service teachers need to build an atmosphere in which students feel confident almost sharing their understanding of mathematics to improve the problem. Under a comfortable environment, students are empowered to master the content and to believe that the instructor takes their revelations and supposition truly (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). Moreover, the culture and norms within the classroom have a critical effect on the students' learning and their mathematical creativity (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018). Making a critique is not a common practice in class, the teacher can establish a new norm by demonstrating how to make a critique about students' presentations. After that, the teacher can encourage students to try to criticize others without being judged. With more experience, students are believed to become more confident and capable to provide feedback to others. This helps to create a dialogue about the specific mathematical concepts and knowledge both between students and the teacher which fulfills one of the principles for developing IBL mathematics curriculum (i.e. the first principle in Section 2.3). Therefore, it is suggested that pre-service teachers need to build up a comfortable mathematical classroom with sufficient communication either of teacher-student communication or student-student communication.

4.3.5 Suggestions on pre-service teachers' professional development and collaboration in IBL

In the overall phase of implementing the 4D IBL model, it is reported that pre-service teachers' overall effectiveness in the IBL implementation is quite low. As mentioned in Section 4.2.5, this problem is caused by pre-service teachers' lack of skills, knowledge and experience in the development and implementation of IBL. Regarding the major issue of IBL mentioned in Section 2.2.5, continuous professional development and collaboration are needed for pre-service teachers to improve in the time of being an in-service teacher. From the perspective of individual professional training, on-job training could be one possible option to further develop teacher competencies. In on-job training, in-service teachers would learn about effective IBL pedagogies with realistic practices close to what teachers are supposed to do in their classrooms. The professional development programs are intended to bring in "a critical questioning attitude towards practice and knowledge in practice that follow reflection on the practice of teaching" in which teachers participate in inquiry as a way of improving their expertise. In the training, teachers can redevelop their identities as mathematics teachers through internalizing new means of thinking, acting and speaking. Moreover, teachers can establish practical mathematical inquiry analysis and effectively incorporate IBL in their classroom by having first-hand experience working on mathematical inquiry tasks as learners, as well as observing an actual inquiry lesson taught by an exemplary teacher. The mathematical inquiry tasks are intended to involve teachers to improve their comprehension of IBL's pedagogic awareness.

From the perspective of collaboration, a learning community plays significant impacts on teacher development and learning. Collaboration could be a form of professional learning community that facilitates, promotes and sustains teacher change (Lewis, Perry & Hurd, 2009). With reference to Levin and Marcus (2010), the social dimension of teacher learning draws attention to the fact that teachers learn more effectively when working with others instead of working alone. The professional learning community can be consisted of teachers with various backgrounds. With the constant exchange of ideas, it helps contribute to the capacity of a teacher to reflect and evaluate their own teaching on the planning and implementation of IBL. Hence, continue teacher collaboration can contribute to support the teachers' professional development on IBL at the school level. As a result, it is suggested that pre-service teachers need to have on-job training for continuous professional development on IBL as well as develop collaboration with their colleagues when they become in-service teachers.

Overall, from the analysis in sections 4.3.1 to 4.3.5, pre-service teachers' knowledge and skills regarding the five major issues of IBL in mathematics are still lacking and need to be enhanced.

5. Limitations

In the research study, there are several limitations identified. They would be discussed and explained in detail in the following.

Firstly, this research study's sample size is quite small which cannot represent all the Hong Kong mathematics pre-service teachers. The participants of this research were all from one university and the majority was from the primary section. The gap exists between the number of participants from the primary section and secondary section might hinder the study to provide a comprehensive picture of all Hong Kong mathematics pre-service teachers.

Secondly, the data collected from both instruments might not be conclusive. Although the questionnaire questions were adapted from previous related research tools, the design had insufficient items to investigate pre-service teachers' fulfillment of the 4D inquiry implementation model. Thus, the data obtained would be not statistically significant.

Thirdly, the evaluation made from the challenges encountered by pre-service teachers in their implementation of mathematical inquiry might be shallow. Since not all interviewees had implemented IBL in their teaching, some of their responses were from the IBL lesson observation. Without being the instructor themselves, their experiences with IBL might be less significant. Hence, the evaluation would only be based on the interviewees who actually had instructed the IBL lesson themselves.

6. Conclusion

In conclusion, this research study was conducted with the aim to investigate Hong Kong pre-service teachers' effectiveness in implementing IBL in teaching mathematics, to discuss whether they have adequate knowledge and skills to develop and execute IBL lessons, and to make suggestions for enhancement on the IBL enrichment for pre-service teachers. The following are some implications made to address the four research questions of this research study.

Regarding the experiences with the implementation of IBL (as stated in the research question 1), only half of the pre-service teachers were found to have the experience of

implementing IBL in their teaching. Hence, this represents that IBL is still not a popular pedagogy among pre-service mathematics teachers when it comes to the real teaching practice. The underlying cause for this phenomenon would be either pre-service teachers hold a conflicting belief with the pedagogic theories behind IBL or pre-service teachers' poor mastery of the three design principles for IBL mathematics curriculum development hinders them from incorporating IBL elements into their lessons.

With regard to the level of knowledge and skills about IBL for developing IBL lessons (as stated in the research question 2), pre-service teachers were found to be unfamiliar with the fundamental elements of IBL: the five major issues of IBL mathematics and the three design principles for developing IBL mathematics curriculum. Hence, it is shown in the questionnaire and interview analysis that pre-service teachers were found quite conservative towards their effectiveness of implementing IBL. Although they showed positive responses towards part of the implementation statements, they were not satisfactory about their performance in implementing IBL in their teaching. They seemed not to be equipped with adequate skills, knowledge and abilities to organize and hold an IBL lesson. If they do no acquire certain qualities, they might not be able to provide a fruitful inquiry learning experience which allows students to acquire those important generic skills in mathematics.

With reference to the concerns about the implementation of IBL in mathematics classroom (as stated in the research question 3), pre-service teachers had expressed two major concerns regarding the implementation of IBL in mathematics. The first concern is regarding students' familiarity with the IBL natures. Since IBL is not a common practice adopted by teachers, students might be overwhelmed by such a comparatively active and student-centered learning style. The second concern is pre-service teachers' own abilities for guiding and leading an IBL lesson. However, with inadequate acquisition of IBL related qualities, teachers were unconfident to handle an IBL lesson especially when students would require guidance from the teacher.

In the aspect of IBL related enhancement to prevent inconsistencies when implementing IBL in teaching (as stated in the research question 4), pre-service teachers are recommended to make reference to the five major issues of IBL mathematics (Dreyøe, Larsen, Hjelmborg, Michelsen & Misfeldt, 2018) and take corresponding actions. By referring to the five major issues, pre-service teachers could improve and enhance their knowledge and skills for IBL, their development for IBL activities, the execution of the IBL lessons.

The results of this research have the following significances for IBL development. Firstly, being a teacher is one kind of IBL. Since teachers need to the effectiveness of their IBL teaching, teachers would have to trial-and-error to test how to implement IBL and which kind of IBL activities suit their students the best. Secondly, teachers especially pre-service teachers need to self-evaluate constantly (before execution, during execution, after execution) to adjust and modify their IBL lessons. The self-evaluation process is a valuable opportunity for teachers to learn about the strengths and weaknesses of their teaching and to make further improvements. Lastly, as the writer herself is also a pre-service teacher, the major aim of this paper is to give feedback to pre-service teachers especially mathematics teachers regarding their fulfillment of the 4D implementation model of IBL in their teaching practicum. With the results of the paper, pre-service teachers need to build up their IBL knowledge and skills with the base of the five major issues of IBL mathematics and the three design principles for developing IBL mathematics curriculum. If pre-service teachers can develop a strong base of IBL in the teacher education, it is believed that they would have a more successful and effective experience when they implement the 4D model of IBL when teaching mathematics.

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Appendix 1: Consent Form

THE EDUCATION UNIVERSITY OF HONG KONG
DEPARTMENT OF CURRICULUM AND INSTRUCTION

CONSENT TO PARTICIPATE IN RESEARCH (FOR INDIVIDUAL)

EFFECTIVENESS OF IMPLEMENTING INQUIRY-BASED LEARNING IN TEACHING MATHEMATICS AMONG PRE-SERVICE TEACHERS IN HONG KONG

I _____ hereby consent to participate in the captioned research supervised by Dr Ng Cheuk Wing Margaret and conducted by Wong Hiu Ching, who are staff / students of curriculum and instruction in The Education University of Hong Kong.

I understand that information obtained from this research may be used in future research and may be published. However, my right to privacy will be retained, i.e., my personal details will not be revealed.

The procedure as set out in the **attached** information sheet has been fully explained. I understand the benefits and risks involved. My participation in the project is voluntary.

I acknowledge that I have the right to question any part of the procedure and can withdraw at any time without negative consequences.

Name of participant _____

Signature of participant _____

Date _____

INFORMATION SHEET

EFFECTIVENESS OF IMPLEMENTING INQUIRY-BASED LEARNING IN TEACHING MATHEMATICS AMONG PRE-SERVICE TEACHERS IN HONG KONG

You are invited to participate in a project supervised by Dr Ng Cheuk Wing Margaret Senior Lecturer of Department of Curriculum and Instruction and conducted by Wong Hiu Ching, a year 5 undergraduate from Bachelor of Education (Honors)(Primary Mathematics)(Five-year Full Time) in The Education University of Hong Kong.

The introduction of the research

A) What does the research involve?

The aim of this study is to investigate how effective can Hong Kong pre-service teachers implement inquiry-based learning in teaching mathematics in their teaching practicum. Moreover, the research also investigates whether the pre-service teachers possess the knowledge and skills in implementing inquiry-based learning in teaching mathematics.

B) Why were you chosen for this research?

It is essential for pre-service mathematics teacher to consider what pedagogies to be adopted in their classroom to make the students to achieve the aims of mathematics education. Pre-service mathematics teachers' point of view, knowledge, skills and experience and recommendation towards the implementation of inquiry-based learning in mathematics classroom are valuable to this research.

The methodology of the research

A) Describe how many participants you will include in this study

The targeted participants for this study are 40 Hong Kong pre-service Mathematics teachers at the Hong Kong University, the Chinese University of Hong Kong and the Education University of Hong Kong. These participants have received mathematics educational training courses for either the primary and/or secondary sections and have completed their teaching practicum. Also, 8 pre-service teachers will be randomly invited to take an in-depth interview.

B) Procedure of the research

Mixed research method will be adopted in this study. In phase I, questionnaire designed based on the five major issues of inquiry-based learning in mathematics to investigate whether pre-service teachers have fulfilled the three principles when they develop their inquiry lessons and follow the 4D model of inquiry mathematics to implement the inquiry-based learning or not will be delivered to targeted participants. Hence, quantitative data collected can be used to assess their skills and knowledge level for the

implementation of IBL in the questionnaire, close-ended statements supported by a 5-likert scale will be used. The questionnaire will be consisted of three major parts: experience with inquiry-based learning, statements for skills level, statements for knowledge level. Participants will have to rate their level of agreement towards each statement by using the 5-likert scale.

In phase II, a 20-minute interview will be conducted with the 8 conveniently selected interviewees separately who participated in phase I questionnaire. The interview will be of semi-structure format with open-ended questions. Open-ended questions are used to explore and investigate interviewees' concerns they have while considering whether to implement inquiry-based learning in teaching Mathematics or not in their teaching practicum. Participants will need to elaborate how those challenging factors have hindered and impeded them from using inquiry-based learning in teaching mathematics with reference to their own experiences. Moreover, interviewees will share their ways of preventing inconsistencies between their inquiry plan and the implementation of inquiry-based learning if they did not encounter any inconsistencies in their teaching practicum. One the other hand, interviewees will share how they respond to or cope with inconsistencies existed in order to let the inquiry lesson progress.

As the consent form will be for approval in early January, the questionnaire will be distributed after the approval from the participants. After collecting the questionnaires, the in-depth interview which lasts for around 20 minutes will take place. Transcription will be started once the interview is conducted in which audio recording will be required with interviewees' permission. SPSS will be used for the quantitative data analysis in phase I whereas coding will be used for phase II qualitative data analysis.

C) Potential benefit

There will be no benefit for the participation but your answers are of great value to this study.

The potential risks of the research

This study involves no potential risk. Your participation in the project is voluntary. You have every right to withdraw from the study at any time without negative consequences. All information related to you will remain confidential, and will be identifiable by codes known only to the researcher.

Dissemination of the results

The results will be used in academic purpose for the honors project of the researcher. All information and interview content will be confidential.

If you would like to obtain more information about this study, please contact Wong Hiu Ching at telephone number or their supervisor Dr Ng Cheuk Wing Margaret at telephone number .

If you have any concerns about the conduct of this research study, please do not hesitate to contact the Human Research Ethics Committee by email at hrec@eduhk.hk or by mail to Research and Development Office, The Education University of Hong Kong.

Thank you for your interest in participating in this study.

Wong Hiu Ching
Principal Investigator

Appendix 2: A Sample Questionnaire

Effectiveness of Implementing Inquiry-based Learning in Teaching Mathematics Among Pre-service Teachers in Hong Kong

Introduction

A researcher from the Education University of Hong Kong would like to invite you to participate in a research study of "Effectiveness of Implementing Inquiry-based Learning in Teaching Mathematics Among Pre-service Teachers in Hong Kong". This research study is under the guidance of Dr. Ng Cheuk Wing Margaret.

The purpose of the study is about the level of effectiveness pre-service mathematics teachers possess with the implementation of inquiry-based learning in teaching mathematics. Inquiry-based learning is a teaching approach which create an authentic learning environment that bridge students' learning of the generic skills with real world issues. The objective of the questionnaire is to investigate to what extent are pre-service teachers have fulfilled the three principles of inquiry lesson development and follow the 4D model of inquiry mathematics to implement the inquiry-based learning. Your participation would contribute to the exploration of skill and knowledge levels of pre-service mathematics teacher for the implementation of inquiry-based learning. The decision to participate in this study is entirely up to you. You may refuse to take part in the study ***at any time***. Your decision will not result in any loss or benefits to which you are otherwise entitled.

The information collected will only be used for academic purposes. Research records will be kept strictly confidential. If you have any further questions about the study, at any time feel free to contact Miss Wong Hiu Ching at or by telephone . Thank you.

Background information

1. What is your gender?
   ☐Male ☐Female
2. How many times of teaching practicum experience do you have?
   ☐1 time ☐ 2 times ☐ 3 times or above
3. Which grade did you teach during your teaching practicum? (You can choose more than one option)
   ☐ P.1 ☐ P.2 ☐ P.3 ☐ P.4 ☐ P.5 ☐ P.6
   ☐ F.1 ☐ F.2 ☐ F.3 ☐ F.4 ☐ F.5 ☐ F.6
4. What is the average number of students of your mathematics class?
   ☐ 1-15 ☐ 16-20 ☐21-25 ☐25-30 ☐30-35 ☐more than 35
5. Was inquiry based learning covered in any of your professional education?
   ☐ Covered only in my major's course work
   ☐ Covered only in my non-major's course work

☐ Covered in both my major's and non-major's course work

☐ Inquiry was not covered in any of my education classes

6. Have you ever attended any workshop that covered inquiry-based learning?

☐ No

☐ Yes, I have attended one workshop that discussed inquiry

☐ Yes, I have attended two or more workshops that discussed inquiry

7. How often do you use inquiry when teaching mathematics?

☐ Never ☐ Rarely ☐ Sometimes ☐ Often

Please use the scale below to respond to the following statements about the 4D-model of inquiry mathematics.

1=Strongly Disagree

2=Disagree

3=Slightly Disagree

4=Slightly Agree

5=Agree

6=Strongly Agree

Discover Phase	1	2	3	4	5	6
1. My students are able to get excited about the mathematics unit.	○	○	○	○	○	○
2. My students are able to have enough prior knowledge regarding the inquiry question.	○	○	○	○	○	○
3. My students are able to understand the inquiry problem.	○	○	○	○	○	○
4. My students are able to make meaning of the inquiry question and understand why the inquiry question is important.	○	○	○	○	○	○
Devise Phase	1	2	3	4	5	6
5. My students are able to generate their own ideas, questions, and/or propositions.	○	○	○	○	○	○
6. My students are able to come up with a plan/investigation to solve the inquiry question.	○	○	○	○	○	○
7. My students are able to share ideas or work with each other mathematically.	○	○	○	○	○	○
8. My students are able to ask mathematically oriented questions about the phenomena they observe.	○	○	○	○	○	○
9. My students are able to know what mathematical evidence do they need to answer the inquiry question.	○	○	○	○	○	○
10. My students are able to gather the evidence they need.	○	○	○	○	○	○
11. My students are able to see the problems of their plan.	○	○	○	○	○	○

12. My students are able to revise their plan through my questioning when they encounter problems	○	○	○	○	○	○
Develop Phase	1	2	3	4	5	6
13. My students are able to do hands-on mathematical investigation.	○	○	○	○	○	○
14. My students are able to record the evidence accurately.	○	○	○	○	○	○
15. My students are able to record the evidence through appropriate methods.	○	○	○	○	○	○
16. My students are able to analyze the evidence mathematically.	○	○	○	○	○	○
17. My students are able to sort their evidence to make it clear and organized.	○	○	○	○	○	○
18. My students are able to represent their mathematical thinking through finding ways to present their evidence.	○	○	○	○	○	○
Defend Phase	1	2	3	4	5	6
19. My students are able to explain their findings and conclusions to other students.	○	○	○	○	○	○
20. My students are able to support their explanations with mathematical knowledge.	○	○	○	○	○	○
21. My students are able to use data to support their explanations.	○	○	○	○	○	○
22. My students are able to make formal presentations to the class and tell others what they have learnt.	○	○	○	○	○	○
23. My students are able to critically examine the mathematical explanations of other groups.	○	○	○	○	○	○
Overall	1	2	3	4	5	6
24. My students are able to be arranged in seating which facilitates student discussion.	○	○	○	○	○	○
25. My students are able to be assigned to work in groups.	○	○	○	○	○	○
26. My students are able to be given opportunities for student-to-student interaction.	○	○	○	○	○	○
27. My students are able to be engaged in whole class discussions.	○	○	○	○	○	○
28. My students are able to engage in small group discussions.	○	○	○	○	○	○

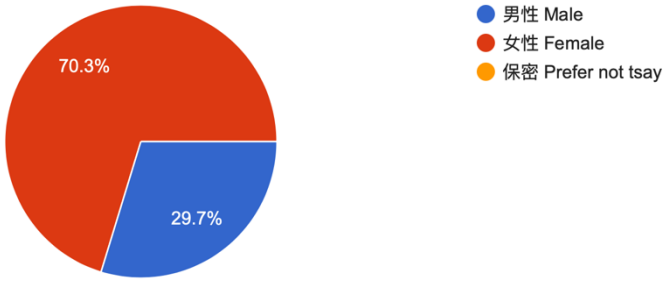
~Thank you for your participation~

Appendix 3 Questionnaire data

Demographics

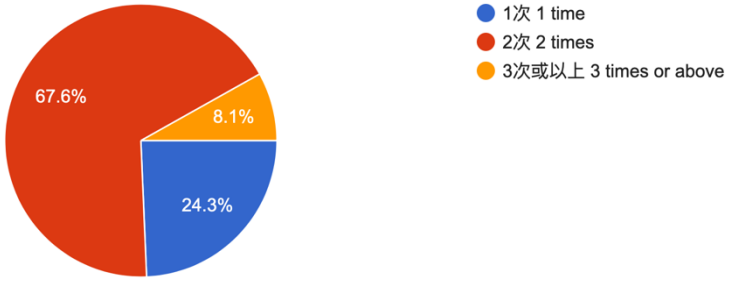
1. 您的性別是? What is your gender?

37 responses



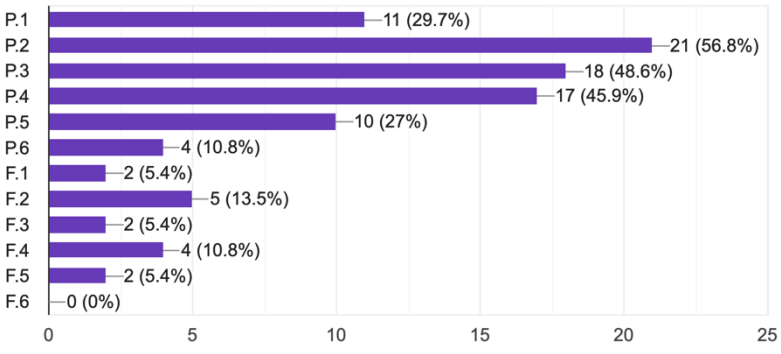
2. 您有多少次教學實習的經驗? How many times of teaching practicum experience do you have?

37 responses



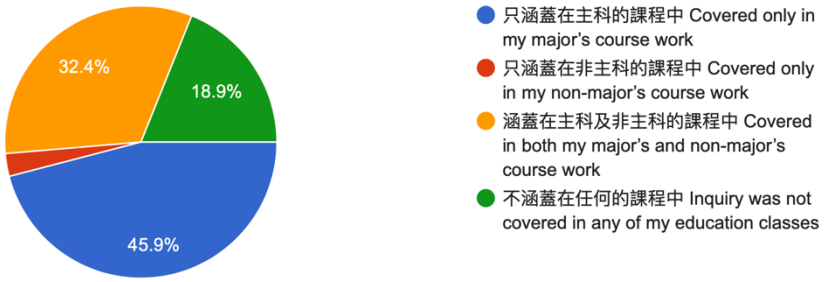
3. 您在教學實習時曾任教哪個/哪些年級? (多選多於一項) Which grade did you teach during your teaching practicum? (You can choose more than one option)

37 responses



4. 您的專科教育中有涵蓋探究式學習的內容嗎? Was inquiry based learning covered in any of your professional education?

37 responses

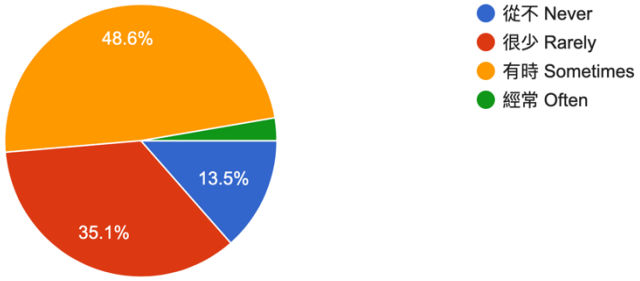


5. 您有否曾參與涵蓋探究式學習的內容的工作坊? Have you ever attended any workshop that covered inquiry-based learning?

37 responses

6. 您在數學教學時有多常利用探究式學習? How often do you use inquiry when teaching mathematics?

37 responses



Discover Phase

Statement 1: My students are able to get excited about the mathematics unit.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.4054**
	Disagree	1	2.70	2.70	
	Slightly Disagree	7	18.92	21.62	
	Slightly Agree	8	21.62	43.24	
	Agree	18	48.65	91.89	
	Strongly Agree	3	8.11	100.00	
	Total	37	100.00	100.00	

Statement 2: My students are able to have enough prior knowledge regarding the inquiry question.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.0541**
	Disagree	3	8.11	10.81	
	Slightly Disagree	4	10.81	21.62	
	Slightly Agree	16	43.24	64.86	
	Agree	11	29.73	94.59	
	Strongly Agree	12	5.41	100.00	
	Total	37	100.00	100.00	

Statement 3: My students are able to understand the inquiry question.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.2973**
	Disagree	2	5.41	8.11	
	Slightly Disagree	4	10.81	18.92	
	Slightly Agree	13	35.14	54.05	
	Agree	12	32.43	86.49	
	Strongly Agree	5	13.51	100.00	
	Total	37	100.00	100.00	

Statement 4: My students are able to make meaning of the inquiry question and understand why the inquiry question is important.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.0270**
	Disagree	12	32.43	35.14	
	Slightly Disagree	12	32.43	67.57	
	Slightly Agree	9	24.32	91.89	
	Agree	3	8.11	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

Devise Phase

Statement 5: My students are able to generate their own ideas, questions, and/or propositions.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.0811**
	Disagree	1	2.70	2.70	
	Slightly Disagree	9	24.32	27.03	
	Slightly Agree	15	40.54	67.57	
	Agree	10	27.03	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 6: My students are able to come up with a plan/investigation to solve the inquiry question.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.1081**
	Disagree	3	8.11	8.11	
	Slightly Disagree	8	21.62	29.73	
	Slightly Agree	10	27.03	56.76	
	Agree	14	37.84	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 7: My students are able to share ideas or work with each other mathematically.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.4054**
	Disagree	1	2.70	2.7	
	Slightly Disagree	6	16.22	18.92	
	Slightly Agree	12	32.43	51.35	
	Agree	13	35.14	86.49	
	Strongly Agree	5	13.51	100.00	
	Total	37	100.00	100.00	

Statement 8: My students are able to ask mathematically oriented questions about the phenomena they observe.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.7	**4.0811**
	Disagree	2	5.41	8.11	
	Slightly Disagree	7	18.92	27.03	
	Slightly Agree	13	35.14	62.16	
	Agree	11	29.73	91.89	
	Strongly Agree	3	8.11	100.00	
	Total	37	100.00	100.00	

Statement 9: My students are able to know what mathematical evidence do they need to answer the inquiry question.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.1622**
	Disagree	4	10.81	10.81	
	Slightly Disagree	6	16.22	27.03	
	Slightly Agree	9	24.30	51.35	
	Agree	16	43.24	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 10: My students are able to gather the evidence they need.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.7	**4.0811**
	Disagree	3	8.11	10.81	
	Slightly Disagree	5	13.51	24.32	
	Slightly Agree	14	37.84	62.16	
	Agree	11	29.73	91.89	
	Strongly Agree	3	8.11	100.00	
	Total	37	100.00	100.00	

Statement 11: My students are able to see the problems of their plan.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.0270**
	Disagree	10	27.03	29.73	
	Slightly Disagree	15	40.21	70.27	
	Slightly Agree	9	24.32	94.59	
	Agree	2	5.41	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

Statement 12: My students are able to revise their plan through my questioning when they encounter problems.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.5136**
	Disagree	1	2.70	5.41	
	Slightly Disagree	4	10.81	16.22	
	Slightly Agree	8	21.62	37.84	
	Agree	18	48.65	86.49	
	Strongly Agree	5	13.51	100.00	
	Total	37	100.00	100.00	

Develop Phase

Statement 13: My students are able to do hands-on mathematical investigation.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.4054**
	Disagree	2	5.41	8.11	
	Slightly Disagree	2	5.41	13.51	
	Slightly Agree	11	29.73	43.24	
	Agree	18	48.65	91.89	
	Strongly Agree	3	8.11	100.00	
	Total	37	100.00	100.00	

Statement 14: My students are able to record the evidence accurately.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.2700**
	Disagree	2	5.41	8.11	
	Slightly Disagree	4	10.81	18.92	
	Slightly Agree	13	35.14	54.05	
	Agree	13	35.14	89.19	
	Strongly Agree	4	10.81	100.00	
	Total	37	100.00	100.00	

Statement 15: My students are able to record the evidence through appropriate methods.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.0270**
	Disagree	2	5.41	8.11	
	Slightly Disagree	9	24.32	32.43	
	Slightly Agree	10	27.03	59.46	
	Agree	13	35.14	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 16: My students are able to analyze the evidence mathematically.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.2972**
	Disagree	5	13.51	16.22	
	Slightly Disagree	16	43.24	59.46	
	Slightly Agree	13	35.14	94.59	
	Agree	1	2.70	97.30	
	Strongly Agree	1	2.70	100.00	
	Total	37	100.00	100.00	

Statement 17: My students are able to sort their evidence to make it clear and organized.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	3	8.11	8.11	**3.2972**
	Disagree	3	8.11	16.22	
	Slightly Disagree	15	40.54	56.76	
	Slightly Agree	13	35.14	91.89	
	Agree	2	5.41	97.30	
	Strongly Agree	1	2.7	100.00	
	Total	37	100.00	100.00	

Statement 18: My students are able to represent their mathematical thinking through finding ways to present their evidence.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**3.5405**
	Disagree	4	10.81	13.51	
	Slightly Disagree	10	27.03	40.54	
	Slightly Agree	18	48.65	89.19	
	Agree	4	10.81	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

Defend Phase

Statement 19: My students are able to explain their findings and conclusions to other students.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.1892**
	Disagree	3	8.11	8.11	
	Slightly Disagree	4	10.81	18.92	
	Slightly Agree	15	40.54	59.46	
	Agree	13	35.14	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 20: My students are able to support their explanations with mathematical knowledge.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.3784**
	Disagree	2	5.41	8.11	
	Slightly Disagree	3	8.11	16.22	
	Slightly Agree	10	27.03	43.24	
	Agree	18	48.65	91.89	
	Strongly Agree	3	8.11	100.00	
	Total	37	100.00	100.00	

Statement 21: My students are able to use data to support their explanations.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	2	5.41	5341	**4.0270**
	Disagree	2	5.41	10.81	
	Slightly Disagree	6	16.22	27.03	
	Slightly Agree	12	32.43	59346	
	Agree	13	35.14	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 22: My students are able to make formal presentations to the class and tell others what they have learnt.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.70	2.70	**4.2432**
	Disagree	2	5.41	8.11	
	Slightly Disagree	5	13.51	21.62	
	Slightly Agree	12	32.43	54.05	
	Agree	13	35.14	89.19	
	Strongly Agree	4	10.81	100.00	
	Total	37	100.00	100.00	

Statement 23: My students are able to critically examine the mathematical explanations of other groups.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	4	10.81	10.81	**3.0270**
	Disagree	5	13.51	24.32	
	Slightly Disagree	16	43.24	67.57	
	Slightly Agree	10	27.03	94.59	
	Agree	2	5.41	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

Overall Arrangement

Statement 24: My students are able to be arranged in seating which facilitate student discussion.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	1	2.7	2.70	**4.5405**
	Disagree	1	2.7	5.41	
	Slightly Disagree	2	5.41	10.81	
	Slightly Agree	10	27.03	37.84	
	Agree	19	51.35	89.19	
	Strongly Agree	4	10.81	100.00	
	Total	37	100.00	100.00	

Statement 25: My students are able to be assigned to work in groups.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.6216**
	Disagree	1	2.70	2.70	
	Slightly Disagree	2	5.41	8.11	
	Slightly Agree	11	29.73	37.84	
	Agree	19	51.35	89.19	
	Strongly Agree	4	10.81	100.00	
	Total	37	100.00	100.00	

Statement 26: My students are able to be given opportunities for student-to-student interaction

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.2972**
	Disagree	1	2.70	2.70	
	Slightly Disagree	5	13.51	16.22	
	Slightly Agree	13	35.14	51.35	
	Agree	18	48.65	100.00	
	Strongly Agree	0	0.00	100.00	
	Total	37	100.00	100.00	

Statement 27: My students are able to be engaged in whole class discussions.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.2162**
	Disagree	1	2.70	2.70	
	Slightly Disagree	8	21.62	24.32	
	Slightly Agree	12	32.43	56.76	
	Agree	14	37.84	94.59	
	Strongly Agree	2	5.41	100.00	
	Total	37	100.00	100.00	

Statement 28: My students are able to engage in small group discussions.

		Frequency	Percent	Cumulative Percent	Mean
Pre-service teachers' response	Strongly Disagree	0	0.00	0.00	**4.4595**
	Disagree	1	2.70	2.70	
	Slightly Disagree	3	8.11	10.81	
	Slightly Agree	15	40.54	51.35	
	Agree	14	37.84	89.19	
	Strongly Agree	4	10.81	100.00	
	Total	37	100.00	100.00	

Appendix 4: Interview questions

1. From your own experience, including your time at the faculty of education, what does inquiry-based learning mean to you?

2. As a student yourself in secondary school, was inquiry-based teaching a part of your learning?

3. What is an example of how you implement inquiry-based learning into teaching mathematics during your teaching practicum?

4. Was this method of inquiry-based teaching successful? Why or why not?

5. What challenges did you faced with in implementing inquiry-based learning in your practicum experience? Consider the following:
   a. Classroom management
   b. Content coverage with limited time
   c. Lack of student base knowledge

6. How do you solve the challenges you faced in your practicum?

7. How can the teacher education training class better prepare you for teaching mathematics through inquiry?

8. Do you have any other comments you feel would be helpful?

Appendix 5: Transcriptions of interviews

Interview transcription 1-PT1

A: Interviewer B: Interviewee

A: From your own experience, including your time at the faculty of education, what does inquiry-based learning (IBL) mean to you?

B: I have attended some workshops about inquiry-based learning. In my opinion, throughout the whole IBL lesson, students are allowed to discover the questions and find the solutions through discussion gradually. When they encounter problems or difficulties which hinder them from progressing, the teacher would provide hints to help them get through.

A: How about the level of guidance provided by the teacher? What is the teacher's role?

B: Teacher must get involved to prevent the students from going in the wrong direction especially during discussion. I remember watching a video about students discussing some questions regarding odd numbers and even numbers. The students could not answer the questions after several attempts. Then the teacher provided another angle for the students. Then the students can answer the questions correctly.

A: Who asked those questions? Students or the teacher? Or those questions were the inquiry questions of the lesson?

B: Those were the assignment questions and some of the students asked how to answer them in the lesson. The teacher also wanted to clarify something regarding those questions. Therefore, the teacher spent the whole lesson to let the students have a discussion to answer those questions.

A: So, you mean the teacher's role should be providing hints to guide the students in the right direction? And students are responsible to discover the question as well as finding the solution(s)?

B: Yes.

A: As a student yourself in secondary school, was inquiry-based learning a part of your learning?

B: No. I did not have any IBL experience in secondary studies.

A: Why do you think the teachers in your secondary school did not use IBL to teach?

B: I think if we spent the whole lesson discussing only one matter or question, our curriculum progress might lack behind. Moreover, for the teachers, they might have a heavier workload. Although their role was the facilitator, they had to think of all the possible questions as well as the corresponding expected answers and solutions when they prepare for the lesson. There might be so many possibilities and they had to spend a rather great amount of time for preparation. Hence, they might not be able to do the preparation since they were very busy. Furthermore, the intended learning outcome for

one IBL lesson can only be one or two. Therefore, in order to make efficient use of time to cover more subject content, they might not consider IBL as their teaching method.

A: What is an example of how you implement inquiry-based learning into teaching mathematics during your teaching practicum? Like what topic did you teach? Why did you use IBL to teach that topic? And how did you implement IBL in your lesson?

B: In my first teaching practicum, I found out that my P.3 students were quite smart after having few lessons with them. At the time, I had taught them about all different kinds of triangle. Then I had to teach them how to form a closed triangle with three edges. The rule they had to know is "the length of any side of a triangle is shorter than the sum of the other two sides". I did not want to teach this rule through direct teaching therefore I designed an inquiry activity for them. My students were divided into groups of four. Each group was given six sticks with different lengths. My question for them was "how to use three of the tools (those sticks) to form a closed triangle. I played a very happy and hyper song for timing keeping. When the song finished, they would have to stop. I think I did not explain my question very clearly and the song I played created a very exciting atmosphere, none of the groups could finish the task of forming a closed triangle. All the triangles formed were not closed. Since the sticks provided were having holes on them, so that other sticks could be inserted into the holes to make two sticks together. Then the students would just place the sticks wrongly. When I walked around to see how the groups were doing, I found out that all the groups were going in the wrong direction. Since the song almost came to the end, therefore I stopped the song and asked the groups to present their products. Then I realized that I did not explain my question very clearly and hence I clarified the questions and told the students what I wanted. The students all shouted that I did not explain well. After my clarification and suggestion, two groups understood the task requirement and could almost make a closed triangle. Due to the limited time, the two groups could only make an almost finished triangle.

A: When you walked around and found out that your students were all going in the wrong direction, and therefore you clarified the question and restate the task requirement? After your clarification, some groups understood and followed your direction. They could almost make a closed triangle. However, with the limited time, they could not finish it, right?

B: Yes. There were six groups, two groups could understand and try to make a closed triangle. Since the sticks were a little bit hard to make them stick together, therefore they could not make a closed triangle within the given time. For the other four groups, they still could not think of a solution. Therefore, they asked for my help. After I listened to their idea, I provided them some hints on making their plan work. Moreover, I would check on them to make sure they were on the right direction.

A: Why do you think the four groups could not think of a way to form a closed triangle?
B: First of all, they might not have the ability. Secondly, the sticks were not very suitable for the activity. I spent a long time finding the more appropriate materials for the activities. However, I could find the sticks that I wanted. As the lengths of the stick have a huge difference, the difficulty of the task would increase and it affects the effectiveness of my lesson. I had thought of giving them four sticks. However, with less tools, they might find the task very easy. If I gave them more sticks, I would have to spend a much longer time staying in the teaching material room to find the needed sticks since there were not many of the sticks in the room. Therefore, I thought six sticks would be good. However, after the lesson, I think maybe I can distribute the number of sticks to the group according to their abilities.
A: So, you think there can be differentiated inquiry? Like the task difficulty level would correspond to their students' abilities?
B: Yes.
A: So, you think the teaching materials (i.e. the sticks) were a problem in your IBL lesson?
B: Yes.
A: Was this method of inquiry-based learning/ teaching successful? Why or why not? Could your intended learning outcomes of the lesson be achieved?
B: I think my IBL lesson was not successful. I did not know whether the students would consider the IBL activity as a learning or just for fun. Since the learning mode of the IBL lesson was very different from the usual lessons. In usual lessons, students would remember everything the teacher said. However, in the IBL lesson, the conclusion was made by the students. Hence, they would think that the IBL lesson was a lesson designed for them to have fun rather than learning. The content learnt in the IBL lesson might not be as important as the other content learnt in the usual lessons. That could be reflected by their homework. They might remember the game they played in the IBL lesson but they did not realize they have to apply the knowledge learnt in the IBL lesson when they finish their homework.
A: So you mean that your students think the IBL lesson was just a lesson for fun?
B: Yes. They thought that lesson was a break to let them relax.
A: You mentioned that your students made the conclusion in the IBL lesson. Was the made conclusion your expected conclusion or the intended learning outcome of the IBL lesson?
B: Yes, it was. However, only few students could come to that conclusion. Only the two groups who understood my clarification could make the correct conclusion. For the other four groups, they might not fully understand the conclusion since they could not

make a closed triangle. Therefore, they might not understand the intended learning outcome as clearly as the two groups.

A: Since they think that lesson was for fun and they could not finish the task, therefore you think only a small group of students gained the intended learning outcomes?

B: Yes.

A: Also, you think the learning mode of the IBL lesson is different from the usual lesson?

B: Yes. They were very different. As in usual lessons, the conclusion (or the intended learning outcome) was given to the students at the beginning. Then, throughout the whole lesson, the learning would be done based on the intended learning outcome. It was like 'brainwashing' and the students would remember the intended learning outcome very well. However, in the IBL lesson, the conclusion was made in the late five minutes (after all the discussion and sharing). Hence the students lacked the time to apply the knowledge learnt to deepen the learning.

A: So you think the application of the knowledge would deepen the learning?

B: Yes. I think practice is very important. Maybe one lesson is not enough.

A: What challenges did you faced with implementing inquiry-based learning in your teaching practicum experience? Consider the following: a) classroom management, b) content coverage with limited time, c) lack of student base knowledge.

B: Firstly, classroom management. I mentioned before that I used a very happy song to do the time keeping. I guess maybe the song is very hyper and exciting which cause the students to be a bit too hyper and some even left their seats.

A: Was it more difficult to maintain the discipline?

B: Yes. And among the sticks, there were only three that could be used to form a closed triangle. However, there were four students in a group, so one student might be left out. Most importantly, if the students could not think of a solution, all of them would just sit there and do nothing. Their involvement would be discouraged. Maybe groups of two would be a better option to increase student involvement.

A: So, you think the grouping and the difficulty of the task affected the student involvement?

B: Yes. As well as the song. The students were crazy about the song!

A: Was it your first time to use a song for time keeping?

B: Yes. Secondly, content coverage with limited time. I think I could cover all the content I expected in the lesson. However, I do not think the students have a very deep learning since some students could not finish the task requirement.

A: So, you mean their learning is not deep because they cannot execute it?

B: Yes. Lastly, student base knowledge. I think they have enough student base knowledge. They all know different kinds of triangles. However, I might use inappropriate wordings to explain the question so that most of them did not get it. Perhaps a given example would help to visualize and explain the question better. I could also show a counterexample.

A: You think they have enough prior knowledge but you can provide more information when explaining the question?

B: Yes.

A: Did you experience another challenge?

B: Yes. As the students made the conclusion at the end of the lesson, they did not have the time to apply the knowledge and practice it. I think it is crucial for them to practice. I think the questions written in the textbook are not suitable for practice.

A: Then how should the students practice? Use the sticks to actually make a closed triangle?

B: Yes. If they can make one, then they can gain the intended learning outcome.

A: Was you IBL lesson a single lesson or a double lesson?

B: Single lesson.

A: You think a single lesson could not provide enough time for the students to practice after making the conclusion? It would be better for students to practice after making the conclusion?

B: Yes. Since some of the students might not even understand the question while the others already reached the conclusion, giving them time to practice can offer hands-on experience to the students.

A: How do you solve the challenges you faced in your practicum?

B: Since it involves explanation of rules and the activity, a double lesson would be a better choice. In a double lesson, there is sufficient time for students to trial-and-error, make their conclusion, practice and then apply the knowledge in finishing the questions in the textbook. Their learning can be deepened.

A: How about the classroom management aspect?

B: Maybe dividing them into groups of two can allow me to manage the class easier.

A: How about in that lesson? What did you do to manage your students?

B: I talked to every group and asked them to sit properly.

A: How about the students' base knowledge aspect? You mentioned you did not explain the question clearly, but you have clarified the question, right?

B: Yes. I clarified and explained what is a closed triangle to every group.

A: Why didn't you clarify and explain the question in front of the whole class?

B: I did not want to stop the whole activity since I assume some groups might have finished the task. But turned out none of them had finished.

A: How can teacher education training class better prepare you for teaching mathematics through inquiry?
B: We have micro-teaching in our courses. Most of us would just present all the prepared teaching materials instead of teaching by inquiry. We just carry out direct teaching. We do not have any experience in IBL in micro-teaching.
A: SO you suggest that we should implement IBL in our micro-teaching?
B: Maybe. Also, in our teaching practicum, there would be a lesson supervision for the FE department. I think the lesson could be used to require us to teach by inquiry to let us experience IBL.
A: You mean to make it compulsory for students to implement IBL in that lesson observation?
B: Yes. Then every pre-service teacher can experience IBL in their teaching practicum.
A: Did you learn about IBL in any of our education courses?
B: No.
A: Do you think the University should make IBL be included in our courses and teach us how to implement IBL? Or do you think this pedagogy is needed to be included?
B: I think we learnt very little about the pedagogies. I cannot say which pedagogy is the best. At least we should learn about the most popular pedagogies. Maybe we should get into school to try out different pedagogies when we learn them instead of learning them from lecture notes.
A: Do you have any other comments you feel would be helpful?
B: No.
A: Thank you for your time!

Interview transcription 2-PT2
A: Interviewer B: Interviewee

A: From your own experience, including your time at the faculty of education, what does inquiry-based learning (IBL) mean to you?
B: I think it is the opposite of the traditional teaching mode. It is not teacher-centered. Instead, teachers are as facilitators to ask questions or to help students to summarize their learning outcomes. It is a process of student-centered learning.
A: So you think the teachers' role is to guide the students along the learning. How and when do you think the teachers should guide the students in IBL?
B: For instance, when learning some new concepts, teachers can use questioning to ask questions about their daily surroundings which can be related to the concepts to be learnt. Also, teachers can use questioning to bring out how to use, when to use and why the concepts are like that. Through questioning, discussion among students can be initiated and students can share their points of view afterwards. Then, the teacher would help to summarize their ideas and make the conclusion for them.
A: You mean the conclusion should be made by the teacher?
B: Yes. The teacher can help to organize and summarize the main concepts after the students have done their learning through discussion. Moreover, I think the students might not be able to conclude and summarize all the ideas by themselves. Hence, that should be the responsibility of the teacher.
A: As a student yourself in secondary school, was inquiry-based learning a part of your learning?
B: Almost none. It might be because of the tight teaching schedules and they have to cover a lot of the curriculum contents. Most of the teaching was direct teaching rather than letting us learn through inquiry. Moreover, I think they would rather spend the time to help us practice the past exam paper. They focused more on the execution part rather than helping us to understand the theories or concepts. Not much time was spent letting us do inquiry.
A: What is an example of how you implement inquiry-based learning into teaching mathematics during your teaching practicum? Like what topic did you teach? Why did you use IBL to teach that topic? And how did you implement IBL in your lesson?
B: I remember I was teaching the unit of volume in P.3. My students only have very basic knowledge about volume like its definition as it was the beginning of learning that unit. Before that lesson, I asked them to bring a bottle/container for observation in the IBL lesson. Then in that lesson, they observed the bottles or containers brought to realize that there were different units for volume (i.e. L and ml). Then I asked them

what L and ml stand for. After that, they would observe how much is 100ml, 1000ml and 1L and their relationships.

A: Why did you want to use IBL to teach this topic? Why didn't you show them what 1000ml is directly?

B: Since the topic is very daily life and students might have seen it in their daily life. Since they already had some knowledge or experience about volume, they might be able to learn through IBL. Therefore, I tried to teach this topic by IBL.

A: So you think IBL should start from something the students are familiar with?

B: Yes.

A: After observing the containers, what else did they do?

B: There were two learning outcomes for the IBL lesson: "1000ml = 1L" and "same volume can be shown as different shapes which depend on the containers themselves". I divided the students into groups of 4. Each group was given one 100ml bottle, a 1L cube and a 1L cylinder. Before they carried out their experiment, I showed them the 1L cube and the 1L cylinder and asked them to guess which container was of a greater volume. All of them had raised their hands to show their opinions. After that, they would use the tools to find out which container was larger. Actually, they first fill either one of the 1L containers with water and then pour the water contained into the other 1L container. Then, they would discover that both containers were of the same volume but with different shapes. They were very surprised about that which I think would have a great impact. For that little experiment, they learnt the same volume can be shown as different shapes which depend on the containers themselves. For the "1000ml = 1L" part, I asked them to try to fill either one of the 1L containers with water by using the 100ml bottle. Then after 10 times, the 1L container was full and hence they would understand 1000ml = 1L.

A: Was this method of inquiry-based learning/ teaching successful? Why or why not?

B: I think the IBL lesson was quite successful. All students can gain the two learning outcomes.

A: What challenges did you faced with implementing inquiry-based learning in your teaching practicum experience? Consider the following: a) classroom management, b) content coverage with limited time, c) lack of student base knowledge.

B: I think the time was the greatest challenge. Since each group needed to send a representative to go to the tap and get water and that took up quite a long time. With the use of water, it was a bit messy since the students were very excited and they spilt water somewhere. It was a bit difficult to manage the class. It was the challenge for the classroom management aspect.

A: How did they assign the representative to get the materials (such as water, container)? You assigned or they decided themselves?

B: There was a leader in each group and the leader was responsible for getting the tools and materials. The group leader would go to get the materials according to their group number.

A: So, you mean the challenge for the classroom management aspect was to assign the group leaders to get the needed materials accordingly and carefully, and the challenge for the content coverage with limited time aspect would be students spent quite a long period of time getting the water as well as some other tools, right?

B: Yes.

A: Was it a single lesson or double lesson?

B: It was a double lesson. I think a double lesson was enough but it would be better to have more time.

A: Did you cover all the knowledge planned for that lesson?

B: Yes. After the whole activity, I did the conclusion part with them and stated out what they had done and what they learnt throughout the activity. Actually, I concluded the lesson by asking them questions and letting the students answer the questions for me. Then I would summarize their answers to make the final conclusion.

A: How about the student base knowledge aspect? Did you think your students have enough prior knowledge for your activity?

B: I think so. They all knew about the definition of volume.

A: So far, the major challenge for your IBL lesson was the matter of time. Did you do anything to cope with this challenge? If so, how do you solve the challenge you faced in your practicum?

B: Actually I predicted that the time would be a problem, and therefore I used a powerpoint to list all the rules and procedures of the activity to explain to the students. Then the students were very clear about what to do and how to do the activity and they can refer to the powerpoint whenever they had problems with the rules or procedures. I think this could save a little bit of time as the class was still under control.

A: So you did not have to rush to make the conclusion, right?

B: Yes. If there was time left, I think I would let them do some exercise to deepen their learning. I think this would be better.

A: So your lesson ended after the conclusion part without any consolidating activity?

B: Yes. I think it is better to have a consolidating activity.

A: How can teacher education training class better prepare you for teaching mathematics through inquiry?

B: As I remember, we only learnt about the definition or some example of IBL in our education courses. I think it was more like a very brief introduction.

A: Do you think IBL should be included in our education course and be taught in detail? Or do you think there isn't such a need?

B: I think this pedagogy can be taught in detail. For example, topics suitable for and can be taught through IBL can be introduced. Or lecturers can teach us how to choose a topic when we want to teach by inquiry. Moreover, we can also learn how to implement IBL in our courses since we do not have much experience with IBL.

A: So, you suggest it is better for the pre-service to have experience with IBL? If so, how do you think the university can do to let pre-service teachers experience teaching by inquiry?

B: Yes. For the courses that require pre-service to do micro-teaching, the lecturer can assign certain topics which are suitable to be taught through IBL for the pre-service teachers to carry out their micro-teaching by inquiry. This can at least provide pre-service teachers with one experience of teaching by inquiry. Although the target of the micro-teaching would be other pre-service teachers instead of students, pre-service teachers would experience what IBL is like. Maybe the other way round. The micro-teaching assessment requires pre-service teachers to teach through IBL but they have to decide their own topic. Then they would do their research to determine which mathematics topics are suitable to be taught through IBL.

A: Do you have any other comments you feel would be helpful?

B: No.

A: Thank you for your time!

Interview transcription 3-PT3
A: Interviewer B: Interviewee

A: From your own experience, including your time at the faculty of education, what does inquiry-based learning (IBL) mean to you?
B: To me, IBL is a way of active learning for students. It is a way that we can learn through different means. Normally, the teacher would provide some information to initiate some questions for the students to explore either individually or in groups and hence gain some knowledge.
A: You mentioned it is a way of active learning for students. What do you think is the role of the teacher in IBL?
B: I think the role of the teacher is a guide along the way. The teacher would provide the direction to guide the students to gain the knowledge instead of giving all the information and knowledge to them directly. At the same time, students should have the ability to do the exploration and inquiry to do the learning.
A: As a student yourself in secondary school, was inquiry-based learning a part of your learning?
B: I guess most of the students in recent year all have experienced IBL through doing their IES. It was a kind of individual IBL when we needed to set the inquiry question and search for information in order to answer the inquiry question. Throughout the whole process, the supervising teacher would guide us by giving advice and direction.
A: Apart from IES, did you experience IBL in lessons?
B: I did. They would announce the inquiry question and let us do the inquiry. When we encountered any problems in our investigation or in the presentation, we can seek help from our teachers. They would provide some hints or tell us the correct direction. However, sometimes my teachers would give us the answers, tell us the correct methods or teach us the theories directly when they found out that we might not be able to finish the whole inquiry in that lesson or we were not capable of exploring and finding out the answer on our own. I think it was a pity that we could not find the whole inquiry and gain the knowledge on our own.
A: With your experience of IBL, what do you think about this learning mode? Do you prefer the usual way of learning?
B: In my opinion, it was a little bit of a waste of time. I think IBL is for exploring and discovering some rather interesting knowledge or theories. With the tight teaching schedule in secondary school, spending one or two lessons to explore only one or two pieces of findings or knowledge might make us lack behind the teaching schedule. The whole planned teaching progress would be affected. However, take my classmates as an example, they would prefer this kind of teaching and learning as it boosts their

learning motivation and incentive. IBL can elevate their interest towards the subjects they were not fond of.

A: Did you implement inquiry-based learning into teaching mathematics during your teaching practicum?

B: I did not but I had a lesson observation on IBL to observe how the in-service implement IBL.

A: Would you show the details of that lesson? Like what was the topic and was the method of inquiry-based learning/ teaching successful? Why or why not?

B: It was a lesson for P.4 and the topic for that lesson was area. Even with the knowledge of perimeter, area was still a quite abstract topic for the students. For them, area means an enclosed area which is either big or small. Since the lesson is open for public observation, hence it took place at the school hall. Students were allowed to walk around the whole hall. At the beginning of the lesson, the teacher introduced briefly about the definition of area and shared some examples of area. The inquiry question for them was "What is the best way to measure an area and to compare the areas?" Students could only use the tools they have on their own, such as a school bag, their legs or fingers. The major focus of the lesson was to use appropriate tools to measure an area and appropriate units to describe the area.

A: Did you think the students achieved the learning outcome(s) of the IBL lesson?

B: I did think the students were able to gain the knowledge. Since the teacher used an E-platform to test the students' understanding towards the concepts of area and other related knowledge, most of the students were able to answer those questions correctly.

A: With your observation, did you think that IBL lesson was successful?

B: From the students' perspective, that lesson came out quite well. They had strong learning motivation and I think they enjoyed the process of active learning. When being tested, they could answer the questions with the expected answers. From a teacher's perspective, the lesson was well executed since the learning outcomes of the lesson could be achieved. Moreover, the lesson was managed well as the activity took place smoothly.

A: As you mentioned you didn't implement IBL into teaching mathematics during your teaching practicum. Why didn't you use IBL?

B: I thought the teaching schedule for P.1 and P.5 was very tight. Moreover, most of the topics were about Number (i.e. addition and subtraction for P.1 and fractions for P.5). I needed to spend most of the time helping students understand some specific wordings and teach them the calculation techniques. Hence, if I need to implement IBL into my teaching, I might need to spend two lessons to teach one concept. With the consideration of the coming tests, I thought implementing IBL into my teaching was not a wise use of time. Moreover, I think IBL should be used to teach topics which are

more concrete which was not the case for the units of Number. I think the units of Number are quite abstract in nature. Therefore, I did not consider IBL as my pedagogy for teaching topics about Number. Therefore, due to the schedule and the nature of the topic (i.e. numbers), I thought using IBL was not quite feasible in my case. I think IBL can be implemented to teach topics about Shape and Space or Measures as they are more concrete and not much prior knowledge is needed when I start to teach these topics.

A: Other than time and the nature of the topics, what other elements would you consider when you decide what pedagogy to be implemented?

B: I would consider the students' discipline. If my students are very active and they may run around the class when they are excited, I might not consider using IBL as I could predict that they may be challenging to manage. This may get hurt or they may not gain the learning outcomes successfully. In this case, I probably would not consider implementing IBL into my teaching. Moreover, I think which grade they are in also matters. For example, for junior primary students, they may not be capable of doing inquiry. They may not be able to understand the inquiry question and need to rely much on the teacher. If the teacher interferes very often, this would affect the whole process of IBL and they may not be doing the learning on their own.

A: How can teacher education training class better prepare you for teaching mathematics through inquiry?

B: I think we did learn about IBL in our courses but very briefly. I think the teacher's role in IBL is quite challenging as the teacher is not long the centre of the learning. We have to be very clear about our role starting from we set the inquiry question, guiding students to do the discussion and result presentation. Most importantly, it is challenging for pre-service teachers to respond according to students' different responses. Furthermore, teachers have to spend longer time preparing how to respond to all possible answers from students in order to ensure the inquiry is feasible. On the other hand, the preparation needed for ordinary teaching mode would be less. To equip us better to teach mathematics through inquiry, I think we need more experience on inquiry. We can make good use of the micro-teaching, everyone needs to teach through inquiry. This can ensure we have at least experience teaching mathematics through inquiry once. Maybe this can be part of the assessment of the pedagogic course or even we can do it in every lesson.

A: Do you have any other comments you feel would be helpful?

B: No.

A: Thank you for your time!

Interview transcription 4-PT4
A: Interviewer B: Interviewee

A: From your own experience, including your time at the faculty of education, what does inquiry-based learning (IBL) mean to you?
B: I think I do not learn much about my education course even if my minor is teaching and learning studies. For me, IBL is a rather students-based learning method which students would discover knowledge on their own.
A: How about the teacher's role in IBL? What do you think about it?
B: In IBL, it is a kind of self-directed learning while the teacher is responsible for designing lessons which students can experience inquiry. During the IBL lesson, the teacher needs to guide the students step by step to explore the contents to be learnt and achieve the learning objectives. Moreover, the focus of the IBL lesson should be on students' hands-on experimental experience instead of the direct teaching given by the teacher.
A: Therefore, you mean the role of the teacher should be more likely to be a guide who assists and leads the operation of the lesson and the activities which students can experience the learning?
B: Yes.
A: As a student yourself in secondary school, was inquiry-based learning a part of your learning?
B: Yes. I did experience IBL in my secondary mathematics lessons. We had IBL lessons once or twice a month. The IBL lessons were conducted with the Jigsaw teaching method which provided differentiated instruction. The teaching content and mode of the IBL lessons would be very different from our usual lessons. They were not focused on the knowledge. Instead, the IBL lessons involved a story-based mathematical inquiry which required us to solve some challenges. I think this kind of lessons deepened our learning.
A: So you mean the IBL lessons were an extension of which you have learnt instead of letting you explore or discover some new knowledge?
B: Yes. For example, the IBL lessons conducted in October would be focused on the topics taught in September. For the students with higher abilities, they may discover the knowledge they learnt could be used or applied in various ways to solve the problems practically. Hence, students could learn from doing experiments rather than absorbing knowledge from the book.
A: With your experience of IBL in secondary school, what do you think about this learning mode? You may consider the effectiveness of this learning mode.

B: I think this way of learning is more interesting than the usual lessons. As in the usual lessons, we would probably sit there and absorb knowledge from the teacher's direct teaching. For the effectiveness aspect, I think IBL is probably as effective as the usual teaching mode. Maybe for IBL, the learning objectives in each lesson would be one or two. However, I think students can consolidate their learning in this learning mode. Their learning would be deeper as they can better remember what they have learnt with their hands-on experiences.

A: So in this case, the quantity in learning decreases but the quality increases?

B: Exactly.

A: What is an example of how you implement inquiry-based learning into teaching mathematics during your teaching practicum? Like what topic did you teach? Why did you use IBL to teach that topic? And how did you implement IBL in your lesson?

B: I think I have implemented some inquiry elements in my teaching but I am not very sure. However, I did not implement IBL in the form of a lesson. I was teaching P.2 about pictograms. I wanted to show and let my students know the advantage of using pictograms. Hence, I incorporated some inquiry elements in my introduction of pictograms. I showed my students a picture which was a pictogram but which all elements scattered all over the picture for 5 seconds. Then I asked them what information they could get from the picture. They all could not get any useful information from that picture. After that, I showed them an organized pictogram for 5 seconds and asked them the same question. Although they did not have enough time to count the numbers of objects on the pictogram, they at least could answer what fruit were shown and which one has the greatest number. It was easier to get useful information from a pictogram. They understood that there was a meaning behind how the icons for each fruit were placed.

A: Did you have any other activities after the introduction?

B: After I taught them about all the elements of a pictogram, I organized some activities for them throughout the whole unit of pictograms instead of one lesson. For example, I gave them a story-line and asked them to construct a pictogram according to the story. Hence the students would learn how to make a pictogram in that activity. In activity, my students were divided into four groups. The background story was about all of them working in a department store and each group worked on one floor. They would have to answer some questions such as what and how many products they sold by reading the given pictogram. Then, they had to present their answers to the whole class with the use of the given pictogram. Then each group needed to mark down the best sales of each floor. After that, each group needed to make another pictogram to show the best sales of each floor.

A: Was this method of inquiry-based learning/ teaching throughout the whole unit of pictograms successful? Why or why not?
B: Not very successful as they sometimes lose their focus in lessons.
A: What challenges did you faced with implementing inquiry-based learning in your teaching practicum experience? Consider the following: a) classroom management, b) content coverage with limited time, c) lack of student base knowledge.
B: For the classroom management part, I think it was a bit hard to manage the class. Firstly, they were P.2 students which meant they were very young and they often lose their focus in lessons. As in that pictogram activity, the use of grouping might increase the chance for them to lose focus. I had thought about letting them do the activity individually. However, I thought it was quite hard as they would have a lot of things to do and it would take so much time to do the presentation one by one. However, in a group manner, some students might not get involved. It was hard to decide how many students in one group to ensure everyone was attentive and involved throughout the whole activity. For the time matter, I think this kind of activity takes up much time. Since the teaching schedule is very tight, spending so much time doing this kind of activity would make the situation worse. Moreover, I doubt the effectiveness of this kind of activity. Maybe I use questioning to bring out the topic would save much time but have similar effects. Therefore, I might cover less content but spend more time by conducting this kind of activity. For the student base knowledge part, I think they had adequate base knowledge for the topic of pictogram since it was not a very difficult topic. It was not very hard for them to handle the topic of pictogram.
A: Did you achieve your learning objectives?
B: I think I could achieve the objectives. The reason I am not sure whether my lessons were IBL or not was I thought my students learned about the important elements of pictogram through my questioning, explanations and demonstration. And in my activity, they deepened what they had learnt instead of discovering some knowledge.
A: How do you solve the challenge you faced in your practicum?
B: I think I could not solve the challenges I faced at the moment. However, I reflected and realized that maybe I preferred teaching directly which can save time and I could cover more contents. Maybe I could implement IBL better after I gain more experience on that. For classroom management, I think the problem is quite minor as the students were not very attentive. However, I think students losing their focus would decrease the effectiveness of that lesson.
A: How can teacher education training class better prepare you for teaching mathematics through inquiry?
B: I think it is important for pre-service teachers to know how to design the IBL lessons. If pre-service teachers are better equipped with the knowledge and abilities to

implement IBL into their teaching, this offers another pedagogy option for them. As not all our lecturers would think IBL is effective, hence I think we need an IBL specialist to teach us how to implement IBL into our teaching. Maybe the university can offer an independent course for IBL which allows us to explore and learn about IBL thoroughly.

A: Do you have any other comments you feel would be helpful?

B: No.

A: Thank you for your time!