Developing A Math Achievement Test for Identifying Primary Students At-risk for Dyscalculia in Hong Kong

by

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Statement of Originality

I, KWAN, Kam Tai, hereby declare that I am the sole author of the thesis and the material presented in this thesis is my original work except those indicated in the acknowledgement. I further declare that I have followed the University's policies and regulations on Academic Honesty, Copyright and Plagiarism in writing the thesis and no material in this thesis has been submitted for a degree in this or other universities.

Abstract

Approximately 6% of the population are estimated to have difficulties with learning about numbers and arithmetic, generally regarded as dyscalculia (or mathematics learning disability). As no standardized procedure for identification of dyscalculia for primary students is currently available in Hong Kong, this study aimed to develop a math achievement test in identifying at-risk Primary students for dyscalculia, as a first step to serve this purpose. A sample of over 1000 Grade 1 to 6 students were invited to complete a comprehensively designed math achievement test, and items were selected based on their functioning. The finalized scales had good reliability and validity. The test provides psychologists and teachers a tool to assess students' math achievement and may potentially serve as a diagnostic tool of at-risk Primary students for dyscalculia in Hong Kong.

Keywords: dyscalculia, math learning disabilities, math achievement scale, vertical scaling, Rasch analysis

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CHAPTER 1

Introduction

Numerical skills are needed in many aspects in our daily lives, such as decision making in purchase, understanding the risks related to medical procedures, or understanding financial risks (Eklund, 2012). However, approximately 6% of the population are estimated to have difficulties with learning about numbers and arithmetic, generally regarded as dyscalculia (or mathematics learning disability) (Butterworth, 2005). A child with such learning disability may exhibit difficulties in recognizing numbers and symbols, counting, memorizing arithmetic facts, understanding abstract concepts (e.g. fractions, place value), basic calculation, measurement, as well as mathematical problem solving.

Children with dyscalculia may find difficulties in non-symbolic and symbolic comparison, manifested by using more time in dot comparison and number comparison tasks (Butterworth, 2011; De Smedt & Gilmore, 2011; Friso-van den Bos, Kroesbergen, &VanLuit, 2014; Kolkman, Kroesbergen, &Leseman, 2013; Xenidou-Dervou, Molenaar, Ansari, van derSchoot, &vanLieshout, 2017). Moreover, they may exhibit low math fluency in simple math facts, which was probably affiliated to poor

arithmetic fact retrieval (Geary, 1993; Mazzocco, Devlin, and McKenney 2008; Price & Ansari, 2013). In practice, they tended to adopt immature strategies, such as finger-counting, in math problem-solving (Butterworth, 2005; Geary, 1993; Jordan & Hanich, 2003).

Methods of measuring math achievement included using curriculum-based measurement (CBM). CBM provided brief tests (usually in several minutes) on a regular basis in order to monitor continuous learning progress (Shapiro, Edward, Gebhardt & Sarah, 2012). There were existing CBM tools, such as AIMSweb (Shapiro, Dennis & Fu, 2015), MBSP Math Computation -Monitoring Basic Skills Progress—Math Computation probes (Fuchs, Hamlett, & Fuchs, 1998), Basic Academic Skills Sample (BASS; Espin & Maruyama, 1985). In dyscalculia literature, standardized math achievement tests are used to examine math achievement of a person suspected with dyscalculia (Shalev, 2004). Examples of tests included Wechsler Individual Achievement Third Edition (WIAT-III) (Wechsler, 2009), Woodcock-Johnson III (Soares &Patel, 2015). For a person with math achievement consistently below a certain percentile (e.g. 10^{th} percentile), he or she would probably be suffering from dyscalculia.

As no standardized procedure for identification of dyscalculia for primary students was currently available in Hong Kong, this study aimed to develop a math achievement scale for identifying at-risk Primary students for dyscalculia (Kolkman et al., 2013) in Hong Kong.

CHAPTER 2

Literature Review

2.1. Definition of dyscalculia

There were various definitions of specific learning disability in numeracy skills. Ladislav Kosc (1970) defined it as dyscalculia (or developmental dyscalculia, DD), which was a structural disorder of mathematical abilities, originated from disorder in parts of brain responsible for mathematical abilities while without other general mental function disorder. To be more specific, dyscalculia was further characterized as a condition affecting a person to acquire arithmetical skills, including difficulty in understanding simple number concepts, facts and procedure. People with dyscalculia may conduct mechanical mathematical procedures without confidence (DfES, 2001). The Diagnostic and Statistical Manual of Mental Disorders – 5 denoted "*Dyscalculia is an alternative term used to refer to a pattern of difficulties characterized by problems processing numerical information, learning arithmetic facts, and performing accurate or fluent calculations."* (DSM-V, 2013, p. 67), which was under the category of specific learning disorder. It described the diagnostic criteria regarding math domain in detail as: "(A) Difficulties learning and using academic skills, as

indicated by the presence of at least one of the following symptoms that have persisted for at least 6 months, despite the provision of interventions that target those difficulties, including difficulties mastering number sense, number facts, or calculation (e.g. has poor understanding of numbers, their magnitude, and relationships; counts on fingers to add single-digit numbers instead of recalling the math fact as peers do; gets lost in the midst of arithmetic computation and may switch procedures), difficulties with mathematical reasoning (e.g. has severe difficulty applying mathematical concepts, facts, or procedures to solve quantitative problems); (B) The affected academic skills are substantially and quantifiably below those expected for the individual's chronological age, and cause significant interference with academic or occupational performance, or with activities of daily living, as confirmed by individually administered standardized achievement measures and comprehensive clinical assessment. For individuals age 17 years and older, a documented history of impairing learning difficulties may be substituted for the standardized assessment; (C)The learning difficulties begin during school-age years but may not become fully manifest until the demands for those affected academic skills exceed the individual's limited capacities (e.g. as in timed tests, reading or writing lengthy complex reports for a tight deadline, excessively heavy academic loads); (D)The learning difficulties are not better accounted for by intellectual

disabilities, uncorrected visual or auditory acuity, other mental or neurological disorders, psychosocial adversity, lack of proficiency in the language of academic instruction, or inadequate educational instruction." (DSM-V, 2013, p. 66-67) To diagnose math-related disorder, specifiers of math impairment including number sense, memorization of arithmetic facts, accurate or fluent calculation, accurate math reasoning, should be reported.

Note that in the current literature in mathematical cognition, some similar terms like 'mathematics difficulties' (Jordan, Kaplan & Hanich, 2002; Jordan, Hanich, & Kaplan, 2003), 'math learning disorder' (Olkun &Denizli, 2015; Soares &Patel, 2015) were used to describe the mentioned phenomenon. One of them was usually directly related to dyscalculia was 'Mathematics Learning Disabilities' (MLD or MD), or learning disabilities in mathematics, to describe population with low mathematic achievement relative to their intelligence and chronological age (Geary &Hoard, 2005; Heine et al., 2010; Lewis, 2014; Wong, Ho, &Tang, 2014). Readers may find it confusing while exploring literatures of both mathematic learning disabilities and dyscalculia as the populations being referred to are rather similar (Butterworth, 2005; Geary &Hoard, 2005; Mazzocco, Feigenson, &Halberda, 2011; Rousselle &Noël, 2007). These studies, although using different names, tend to use a similar research

procedure to identify the relevant population: to define population with low numeracy skills relative to intelligence by first using a low-percentile (e.g. 10th percentile) cut-off on a standardized math achievement test, and then to investigate the differences in cognitive abilities between the target group and the typically-achieving group. Throughout the literature, research procedures and cognitive deficit exploration on both dyscalculia and MLD were nearly the same (e.g. for dyscalculia: Skagerlund &Träff, 2016; Stock, Desoete, &Roeyers, 2009; Szucs, Devine, Soltesz, Nobes, &Gabriel, 2013; for MLD: Geary, Hoard, Nugent, &Bailey, 2012; Toll, VanViersen, Kroesbergen, &VanLuit, 2015; Wong et al., 2014). Therefore, this study employed these two terms interchangeably as the definition discussed above, which were describing an inherent disorder causing difficulty in developing numeracy skills.

2.2. Etiology

Regarding the nature of disorder, current research conceptualized the origin of dyscalculia at the cognitive level (Butterworth, 2005; Geary &Hoard, 2005; Kuhn, 2015; Price &Ansari, 2013). People with dyscalculia are believed to have deficits in certain cognitive abilities (e.g., nonsymbolic magnitude processing, mapping of numerosities and symbolic numbers (known as access skill), symbolic magnitude processing, etc.) that are important for acquiring certain mathematical skills, resulting

in difficulty in learning and applying mathematics. Geary and Hoard (2005) categorized those deficits in three major groups (or subtypes): procedural deficits, regarding inability to develop mature strategies in math problem-solving and relying on immature strategies (e.g. finger counting); semantic memory deficits, regarding an inability to retrieve information from long-term memory and hence affecting arithmetic fact retrieval; spatial deficits, affecting spatial representation of numbers as well as symbol use (e.g. equation writing, number alignment). Karagiannakis, Baccaglini-Frank, and Papadatos (2014) categorized dyscalculia into four subtypes, matched with specific cognitive systems: core number subtype, with internal representation of quantity involved; memory subtype, with various memory systems (e.g. working memory and sematic memory) involved; reasoning subtype, with various executive mechanisms (e.g. entailment, inhibition, updating) involved, and visual-spatial subtype, with visuospatial skills involved.

Moreover, there are some discussions concerning the specific type of deficit that is related to mathematics learning. Though it was suggested that dyscalculia was the result of specific deficits in basic numerical processing, rather than the consequence of deficits in other cognitive abilities (Butterworth, 2003; Landerl, Bevan, &Butterworth, 2004), research evidence showed that dyscalculia might be the result

of general cognitive impairment (Geary &Hoard, 2005; Kucian &vonAster, 2015). Alternatively, the concept of primary dyscalculia and secondary dyscalculia was proposed (Price &Ansari, 2013), distinguishing between an endogenous learning disorder in mathematics and a situation driven by exogenous factors or cognitive deficits not specific to numerical processing, such as deficits in working-memory, visual-spatial processing or attention. Träff, Olsson, Ö stergren, and Skagerlund (2017) conducted a case study by contrasting different cognitive deficit theories and suggested that dyscalculia was a heterogeneous disorder, with multiple possible combination of deficit profiles (such as general magnitude processing [both nonsymbolic and symbolic] only deficit subtype, domain-general [verbal working memory and shifting] only deficit subtype, access deficit and domain-general [visuospatial working memory and shifting] subtype).

2.3. Diagnosis of dyscalculia

There are two types of identification procedures commonly used to identify children with dyscalculia: math achievement-based tests and cognitive-based tests. Regarding math achievement-based approach, standardized mathematic tests are used to examine math achievement of a person suspected with dyscalculia (Shalev, 2004). If a person

with average intelligence performs consistently and significantly lower in the math achievement test compared to his/her same-age peers, it is likely that such person suffer from dyscalculia. Examples of tests mentioned or used are Neuropsychological Test Battery for Number Processing and Calculation in Children (NUCALC) (Shalev, 2004), Wechsler Individual Achievement Third Edition (WIAT-III) (Wechsler, 2009), Woodcock-Johnson III (Soares &Patel, 2015), Wechsler Intelligence Scale for Children Third Edition (WISC-III) (Arithmetic subset) (Santos, DaSilva, Silva Ribeiro, Dellatolas, &vonAster, 2012; Santos et al., 2012). Note that the construction or selection of the tests are mainly curriculum-based (Gliga &Gliga, 2012; Wong, Ho, &Tang, 2017) so that detected low performance would not be affected by unlearned mathematical knowledge. Percentile method (to see whether the low performer is below certain percentile score in a standardized test) is usually used as decision criteria (known as power test) (Butterworth, 2005; Compton, Fuchs, Fuchs, Lambert, &Hamlett, 2012; Desoete, Ceulemans, De Weerdt, 2012; Kaufmann &Aster, 2012 Skagerlund &Träff, 2016; Wong et al., 2017).

Another approach for identifying children with dyscalculia would be cognitive-based tests. The tests are constructed as tests for certain cognitive skills based on different cognitive deficit theories (such as core deficit, access deficit, etc.). Examples include

Dyscalculia Screener (Butterworth, 2003), which includes dot enumeration and number Stroop tasks; and Basic Number Processing Tests (BNPT) (Olkun &Denizli, 2015), which includes random dot enumeration, canonic dot enumeration, symbolic number comparison and mental number line estimation tasks. Yet a consensus has not been reached on the cognitive profiles of children with dyscalculia, leading to validity problem.

2.4. Math achievement test design

With the mentioned validity issues, math achievement-based method becomes a popular method for identification of dyscalculia. Careful consideration is needed to decide which constructs to be included in the achievement test. Math fluency tasks was a common measure among math cognition studies (Mazzocco, Devlin, & McKenney, 2008; Desoete et. al., 2012; Geary, Hoard, Nugent, &Bailey, 2012; Toll, VanViersen, Kroesbergen, &VanLuit, 2015; Vanbinst et. al., 2016). This construct measures how quickly a person could calculate simple arithmetic (such as 2+3=5). According to Geary (2005), student normally learn to do simple addition by counting (both counting-on and counting-all) and eventually memorize the arithmetic facts of different numeric combinations as a result of repeated retrieval from long-term

memory. He suggested that the origin of dyscalculia may lie in sematic memory deficit, which adversely affects the ability to retrieve simple arithmetic fact. Jordan and Hanich (2003) compared achievement profiles of math disability groups and reading disability groups and the results highlighted the uniqueness of math fact retrieval tasks on math disability but not on reading disability. Tempo Tests (e.g. Woodcock–Johnson-III [Fluency subset], Tempo test rekenen) was often adopted to measure the accuracy of simple arithmetic in a short time (e.g. 3 min).

In addition, conceptual and procedural knowledge of numerical computations are required to solve complex arithmetic (Geary, 1994; Geary, 2005, 2010, 2013; Geary & Hoard, 2005). For instance, to solve the arithmetic problem “23 – 17”, one has to recognize the base-10 number system (i.e. meaning of “2” and “1” in “23” and “17”, respectively) that he/she has to borrow “1” from the tenth column to unit column. There were enormous researches focusing on those knowledge in arithmetic domain including whole number (e.g. Fuchs, Fuchs, Prentice, Hamlett, Finelli & Courey, 2004; Burns, 2011), fractions (e.g. Lin, Becker, Byun, Yang, & Huang, 2013; Siegler & Lortie-Forgues, 2015; Siegler, 2016; Bailey, Hansen & Jordan, 2017), decimal (e.g. Durkin & Rittle-Johnson, 2015; Lortie-Forgues & Siegler, 2017) and algebra (e.g. Rittle-Johnso, Star & Durkin, 2009; Schneide, Rittle-Johnso & Star , 2011). While

math fluency targets on arithmetic fact retrieval, arithmetic problems involves uses of both domain-specific and general cognitive skills as their complexity level arises. For example, multi-digit number addition requires symbolic place-value understanding, which was shown to be associated with math achievement (Chan, Au, &Tang, 2014). Lee and Kang (2002) suggested the linkage of phonological loop to multiplication, and visuospitial sketchpad to subtraction, whereas Imbo and LeFevre (2010) suggested both systems were involved in complex multiplication and subtraction, though such relationship might vary across different cultures. After all, arithmetic measurement, though apparently represents math achievement, inherently comprises cognitive development of a student to succeed in math.

Along with the development of one's arithmetic ability, Schoenfeld (1992) briefly explained the goal of math not only stops at skillfully training such abstract, symbolic tool users, but also develops one's mathematical thinking – developing mathematical point of view and making senses with math, to be prepared for an increasingly technological world (Schoenfeld, 1992). This addressed the importance of conceptual and procedural knowledge not only in pure arithmetic, but also in solving arithmetic word problems, as well as problems in other domains (which were also included in math curriculum) such as geometry (e.g. McCool & Holland, 2012; Aydın, 2018) and

area measurement (e.g. Huang, 2017), in order to develop all-rounded math ability for the future challenge. Schoenfeld (1992) further described the central issues in math problem-solving mainly by looking into two aspects: (1) knowledge inventory (what individuals know from memory); (2) how that knowledge was deployed. While solving problems require memory content from corresponding domain knowledge (e.g. the meaning of "cm" for measurement problems), applying suitable procedure such as using geometric tools (e.g. ruler, protractor), mental construction of figures, as well as logical reasoning (Gardner, 1993; Russell, 1919) is included. In the deficit subtype reviewed in Karagiannakis, Baccaglini-Frank, and Papadatos (2014)'s study, mathematical knowledge-based skills such as decoding mathematical terminology (e.g. numerator, denominator, isosceles, etc.) and transcoding verbal rules or orally presented tasks were shown to be related to dyscalculia. Other math procedural skills such as decision making, interpreting graphs, analyzing geometric figures were also reported.

2.5. Research aim

This study was motived from the following issues: firstly, there was no existing standardized tools for identification of students with dyscalculia in Hong Kong.

Although a standardized test, the Learning and Achievement Measurement Kit 3.0 (LAMK 3.0), by Education Bureau in Hong Kong (LAMK 3.0; Hong Kong Education Bureau, 2015), had been developed to assess the math skill mastery of a student with respect to their same-grade peers, only the subtest with respect to student's grade would be used. Therefore, it lacked an ability to examine the position of a student in the whole primary mathematic learning stage, and it could not tell how far the student was lagging behind. Also, development of LAMK did not target on dyscalculia, therefore other specific features of dyscalculia such as poor math fluency and poor math fact retrieval were not accessed. While standardized achievement tests were available in other countries, such standardized tests might not correspond well to syllabus of primary mathematics in Hong Kong and thus failed to reflect the student's math achievement in Hong Kong. For instance, the topic regarding local monetary system (which was introduced Grade 1 and 2 primary students in Hong Kong) was different from countries. And some topics like coordinate system, rate and ratio concept, different statistical measures, etc., which were generally included in the United States (Common Core State Standards for Mathematics, 2017), were not mentioned in Hong Kong primary school math curriculum.

Therefore, this study aimed to develop a math achievement test to measure math

achievement level among primary school students, in order to identify at-risk students in Hong Kong, as a preliminary stage for dyscalculia identification.

CHAPTER 3

Methods

3.1. Participants and procedure

A total of 1410 students (from Grade 1 to Grade 6) from 9 Hong Kong mainstream schools took part in the study. This sample included 561 (60.21%) males and 849 (39.79%) females. Data collection phase started from June 2018 till July 2019. Before participant recruitment, ethics approval from Human Research Ethics Committee of the authors' affiliated institution was sought. Consent forms for school and participants' parents were distributed and received with signature approval. Participants were asked to complete a paper-and-pencil 43-minute comprehensive math achievement test, including four components: Math fluency (3 minutes), Arithmetic (10 minutes), Math knowledge (10 minutes) and Math Problem Solving (20 minutes). The tests were administered in school classrooms by the author and a few undergraduate student helpers.

Table 1 showed the distribution of different Grade students with respect to their school district socio-economic status (SES), which was estimated by their

corresponding district median monthly household income and percentage of post-secondary educational attainment population (Census and Statistics Department, 2018). It suggested the sample included population from all three classes of socio-economic status, although relatively fewer participants came from the middle class districts. Though comparatively fewer Grade 6 students were included (n=81), this study included adequately large sample size (n=1410) for robust data analysis.

Table 1. Distribution of different Grade students with respect to their school district socio-economic status (SES)

District	Grade						
	P1	P2	P3	P4	P5	P6	Total
High SES	233 (74.92%)	113 (27.97%)	92 (30.87%)	30 (18.63%)	64 (41.83%)	7 (8.43%)	539 (38.23%)
Middle SES	40 (12.86%)	32 (7.92%)	29 (9.73%)	25 (15.53%)	12 (7.84%)	7 (8.43%)	145 (10.28%)
Low SES	38 (12.22%)	259 (64.11%)	177 (59.40%)	106 (65.84%)	77 (50.33%)	69 (83.13%)	726 (51.49%)
Total	311 (100.00%)	404 (100.00%)	298 (100.00%)	161 (100.00%)	153 (100.00%)	83 (100.00%)	1410 (100.00%)

Regarding the comorbidity issue of other special education needs (SEN), for a student with other SEN issues (e.g. dyslexia, ADHD, etc.), there was still a possibility that the student was having difficulties in Math. Hence, students with other SEN were included in sampling.

3.2. Math achievement scale development

After literature review, this study conceptualized the measurement in Hong Kong primary math achievement scale for identification of at-risk students as four components: Math Fluency (MF), which included timed simple arithmetic to target on arithmetic fact retrieval; Arithmetic (AR), which included different complexity level (e.g. whole number, fraction, decimal, percentage) of arithmetic (symbolic) problems to measure general arithmetic ability; Math Knowledge (MK), which included problems on math concepts (e.g. odd/even relation, factor/multiple, measurement, L.C.M./H.C.F., area, average) to assess how well students could memorize mathematical facts; Math Problem Solving (PS), which included problems on math application problems (e.g. making purchase, calculating proportion of a set of items, interpreting graphs, percentage change on price) to measure math problem-solving

ability.

3.2.1. Math Fluency (MF)

This component aims at tracking the speed and accuracy of use of simple arithmetic. It was adopted and modified from *Arithmetic Fluency test of the Woodcock Johnson* (Woodcock, McGrew & Mather, 2011). The nature of this component would be timed simple arithmetic. A 1-minute single digit addition (within the sum range from 2 to 18) and a 1-minute simple subtraction (for a two-digit number from 10 to 20, subtracting a single digit number from 1 to 9) were used to access their math fluency. Each operation included 90 items. The large number of items ensured that no ceiling effect would be resulted. Total accuracy will be used to account for their math fluency in each tested operation.

3.2.2. Arithmetic (AR), Math knowledge (MK) and Math Problem Solving (PS)

The three components were adopted and modified questions with reference to the selected local mainstream textbooks. A textbook analysis was conducted before the construction of the remaining three subscales. Local textbooks from 3 publishers with reference to ‘suggested textbook by Education Bureau’ (Hong Kong Education

Bureau, 2017) were selected. Questions from ‘Chapter Exercise’ and ‘Revision test’ were coded by the author and an expert in math according to the learning outcomes set by the Education Bureau, which divided the whole primary mathematics curriculum into five learning dimensions: *Number, Shape and Space, Measures, Data Handling* and *Algebra*. Moreover, by inspecting the whole curriculum, the dimensions *Shape and Space, Measures, Data Handling* mainly involved problems targeted on both pure mathematical concepts and problem-solving techniques (such as recognizing shapes, calculating area, volume, etc.), while *Number* and *Algebra* included numerical computation skills, together with related-mathematical concepts and application problems (e.g. place-value recognition, constructing equations, decision on making a purchase, etc.). And thus, in calculation, their contents were assumed to be more or less evenly distributed among the three components. For AR component, relevant items were selected to include a variety of combinations among number types, operations and complexity. For each PS and MK component, theoretical item proportion of each learning outcomes were estimated using suggested learning time of each learning outcome set by Education Bureau (Hong Kong Education Bureau, 2000). Relevant items from all learning outcomes were selected to construct both PS and MK scale. Empirical item proportion of each learning outcome in both scales was calculated. The number of items in both PS and MK scale in each

grade were modified, so that the empirical item proportion average of both PS and MK scales did not deviate from the theoretical proportion for more than 5%. Moreover, content proportion were controlled such that empirical dimension proportion average of both scales did not deviate from the theoretical dimension proportion for more than 5%. Appendix A summarized the item distribution of MK and PS with respect to learning outcome categorization.

Constructed items of the four components were presented in Appendix B. There were 14 AR items, and 20 items in each of the PS and MK components for each grade. While it was not realistic for one student completing all the items, in the phase of data collection, tests for each Grade were designed so that each grade would include items of the grade, together with a small number of linking items below 2 grades and above 1 grade. The selection of linking items were based on discussion with relevant experts, with the aim of covering all the five primary learning dimensions at different learning stages. For AR, the number of linking items in each grade would be 3, and for MK and PS, it would be 4, such that the proportion of linkage in each grade is appropriate (≥20%) (Kolen & Brennan, 2004). In addition, three junior secondary school items were included in each component to possibly measure students with exceptionally high achievement. Moreover, two different sets of linking items were

selected to create two testlets (set A and B) for data collection.

For the final scale, the number of items in each AR, MK and PS component were constrained by final test length. While piloting the whole test, for each minute, students were generally able to finish 1.4 AR item, 2 MK items and 1 PS item. As a 20-minute comprehensive test would be practical for clinical use, 4 –minute sections for AR and MK, and a 9-minute section for PS, combined with a 3-minute MF section would be appropriate. Moreover, data showed that students generally could complete around 1 higher level item. As a result, for each Grade, the number of items for AR, MK, PS component in the final scale would be designed as (4*1.4)-1≈5, (4*2)-1=7 and (9*1)-1=8 respectively. Such design would expect a student to finish all the items of the Grade and a higher level item within the mentioned duration.

3.3. Data analysis

3.3.1. Analysis on Math Fluency (MF)

For MF, Cronbach's α of the two items would be used to investigate internal consistency as a measure of reliability. A measure above .7 suggested good consistency. Validity were examined by correlating average raw scores of the two items against their school report scores. A pooled correlation above .3 suggested moderate relationship of the component to math achievement measured from school, and a correlation above .5 suggested strong relationship.

3.3.2. Analysis on Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

For AR, MK and PS, Rasch analysis was conducted on each component using Winstep v4.4.8 to calibrate item difficulties and person abilities. Extreme items (items with minimum or maximum responses) were excluded and the first item selection was performed according to the following criteria: (1) outfit statistic were considered to detect item misfit. It is computed as standardized residual average and is sensitive to

detect unexpected responses (e.g. lucky guessing and careless mistakes) (Linacre, 1994). For an individual item, outfit mean square statistic (MNSQ) should be consistently within the range of 0.5 and 1.5, which was shown by a series of Rasch analyses after item-wise deletion a suspected misfit item; (2) each component should retain a minimum number of items (i.e. 5 for AR, 7 for MK and 8 for PS) as designed for the final scale; (3) upon deleting linking items, the proportion of linkage should be at least 20% for good estimation in empirical analysis. Analysis were performed iteratively by deleting misfit items, until no more items should be deleted to retain a set of eligible items.

For constructing the final scale, a second item selection was further performed considered to shortlist items as a recommended short version: (4) one's item difficulty should not be too deviated from the general difficulty of its Grade. An item with median absolute deviation (MAD) greater than 2 would be considered as 'not representative' of its Grade; (5) item difficulties in each Grade should have an even spread, so as to measure a person ability in greater detail. This was done by ranking item difficulties in each Grade and selecting items for each difficulty separation; (6) as a curriculum-based assessment, items from the five primary learning dimensions should be included whenever possible; (7) the easiest item in each component would

be inspected if it was easy enough to measure achievement at the lower end. Raw correct rates of the easiest item were scrutinized to ensure that more than 85% of students of the Grade can score that item. Otherwise, items which item difficulty levels lower than that would be backwardly included in the final scale despite item misfit. Finally, Rasch analysis of the selected items, with anchored item difficulties from first item selection would be conducted to evaluate the performance of the short version.

Reliability of AR, MK and PS component were checked by item and person reliabilities, as well as item and person separation index in Winsteps. Reliability index measures the degree of reproducing relative measure locations, that student with high math achievement would be possibly estimated as having high person ability, and difficult items would be estimated as having high item difficulty. Separation index suggests the number of strata of person or item that could be separated by measurement. High person separation suggests that the scale can distinguish different level of performers into more groups, while high item separation indicates more different difficulty clusters of items can be identified, which indicates item hierarchy. For item, same criteria suggested good item difficulty hierarchy to measure high and low performers. For person index, a reliability above .8 and a separation index above

2 suggested good sensitivity to distinguish between high and low performers (Linacre, 1994). Validity were examined by pooled correlation between person abilities and their math achievement scores in school. A pooled correlation above .3 suggested moderate relationship of the component to math achievement measured from school, and a correlation above .5 suggested strong relationship.

Unidimensionality of each AR, MK and PS component were checked by Principal Component Analysis (PCA) of the residuals. This was to ensure the items only measuring their corresponding component but not including other factors. Correlation matrix of the residuals was decomposed into components (or contrasts). Linacre (1994) suggested an eigenvalue of the first contrast less than 3 (i.e. the second dimension had the strength less than 3 items) would be indicate unidimensionality.

3.3.3. Confirmatory Factor Analysis on Math Fluency (MF), Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

After item selection, confirmatory factor analysis (CFA) was used to confirm the four-factor structure of MF, AR, MK and PS. In each grade, responses from sorted items after item selection were used and separate CFA with diagonally weighted least squares (WLSMV) were conducted by *Mplus*. Due to the fact of small sample sizes in

Grade 4 to 6, CFA were selectively conducted in Grade 1 to 3. Regarding The following criteria were used to evaluate CFA models: Root Mean Square Error of Approximation (RMSEA) < .08, and Comparative Fit Index (CFI) > .90. These criteria were suggested by Hooper, Coughlan & Mullen (2007). Standardized factor loadings as well as factor correlation were reported.

CHAPTER 4

Results

4.1. Results in Math Fluency (MF)

For MF, the Cronbach's α was 0.87, indicating good internal consistency of the two items (simple addition and subtraction). Moreover, average raw scores of the two items were significantly correlated with their school report scores. A pooled correlation of .49 ($p<.001$) showed that the whole MF component was valid as a measure to primary math achievement.

4.2. Results in Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

Appendix C displayed summaries of Rasch analysis of AR, MK and PS component after first item selection. After first item selection, there were 61, 100 and 92 items retained in AR, MK and PS components respectively. All three components had item difficulty measures (logit) mean of 0, with ranges from -6.26 to 6.51 for AR, from -5.13 to 6.37 for MK, from -5.65 to 6.42 for PS. They all achieved a low mean

standard error of item estimates (item s.e. mean =.18). The most difficult items were 5AR11 (6.51), 6MK4 (6.37) and 6PS9 (6.42), while the easiest items were 1AR5 (-6.26), 1MK1 (-5.13), 1PS9 (-5.65). Note that there were misfit items retained (10 items for AR, 12 for MK, 14 for PS), mainly due to criteria (2) and (3).

Second item selection were summarized in Appendix D. There were 33, 42 and 48 items retained in AR, MK and PS components. The mean item difficulty measures (mean standard error of item) of AR, MK and PS were -.84 (.19), .60 (.20) and .73 (.20) respectively. Item difficulty ranges changed from -8.62 to 4.83 for AR, while MK and PS remained unchanged. This was due to the exclusion of the most difficult item (5AR11) by criteria (4), and the inclusion of easy items (1AR1, 1AR2, 1AR4) by criteria (7).

Figure 1,2,3 and 4,5,6 showed the item-person map (Wright map) of AR, MK and PS component after first and second item selections respectively. Distributions of person abilities and item difficulties were compared to check item alignment, i.e. the scale provide adequate information to measure math abilities in the whole primary stage. From the figures, AR and MK component showed matched distribution of person and item, suggesting a reasonable item span to measure primary math achievement,

whereas PS component included more difficult items than person starting from Grade 4.

Figure 1. Item-person map (Wright map) of Arithmetic (AR) after first item selection

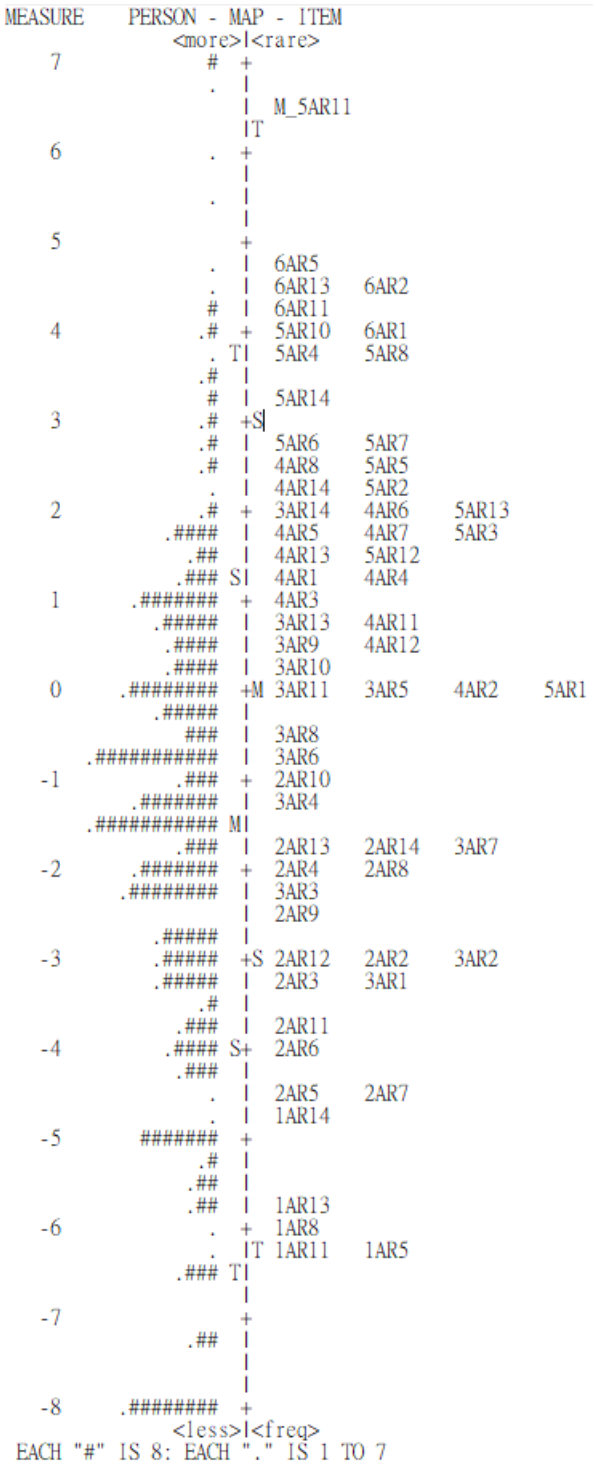


Figure 2. Item-person map (Wright map) of Math Knowledge (MK) after first item selection

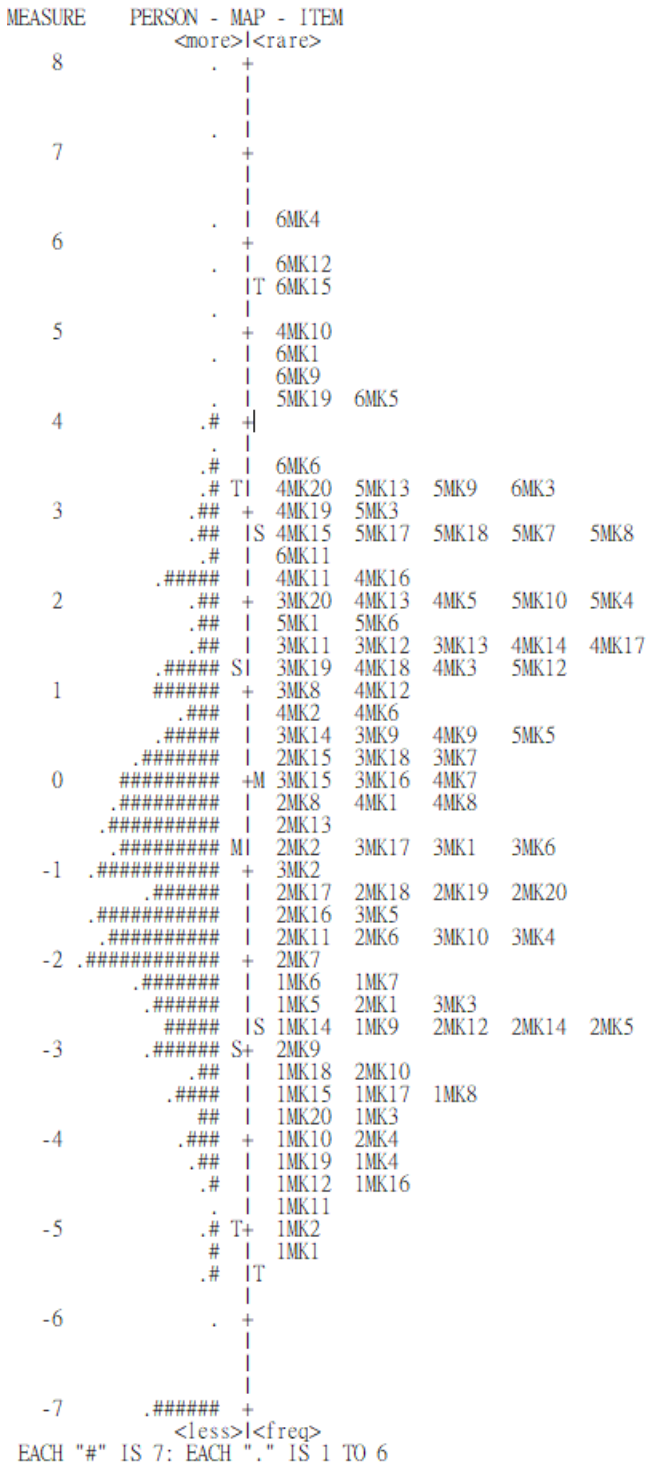


Figure 3. Item-person map (Wright map) of Math Problem Solving (PS) after first item selection

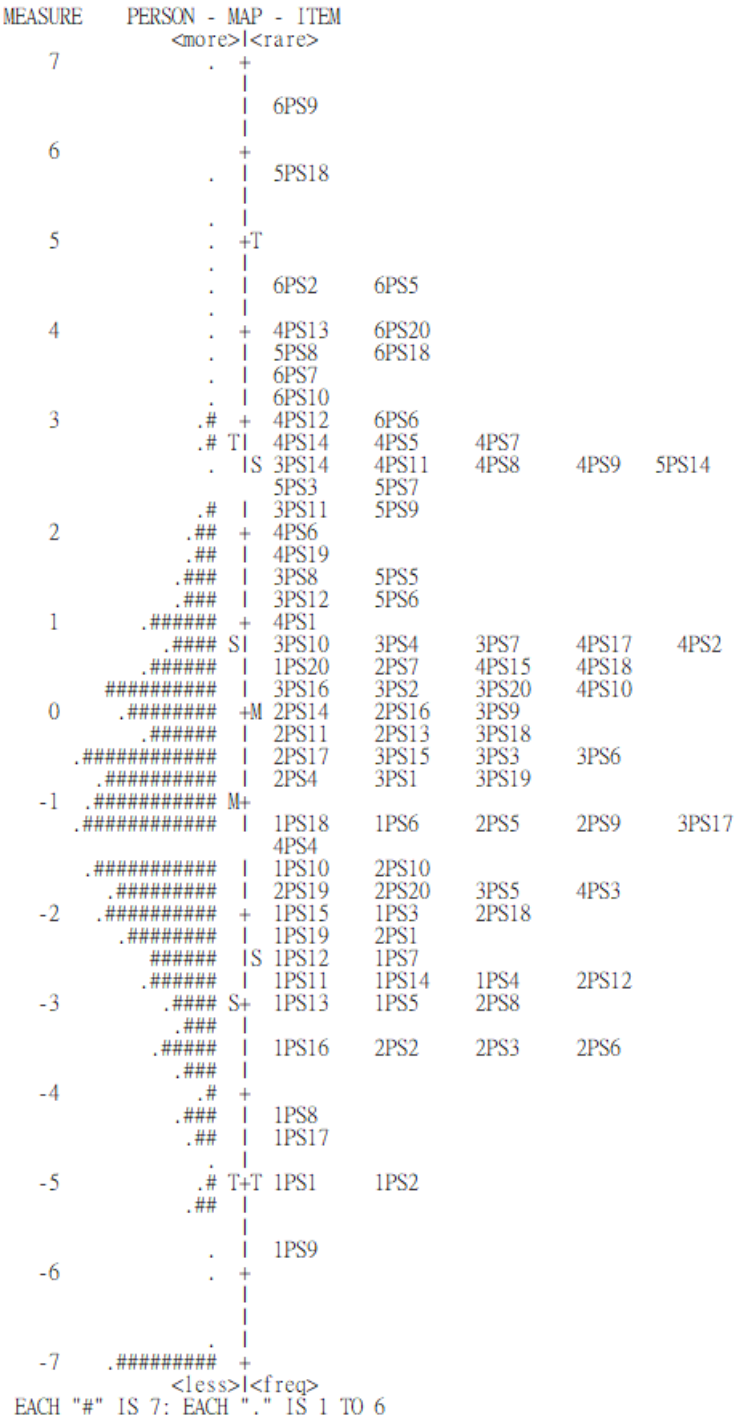


Figure 4. Item-person map (Wright map) of Arithmetic (AR) after second item selection

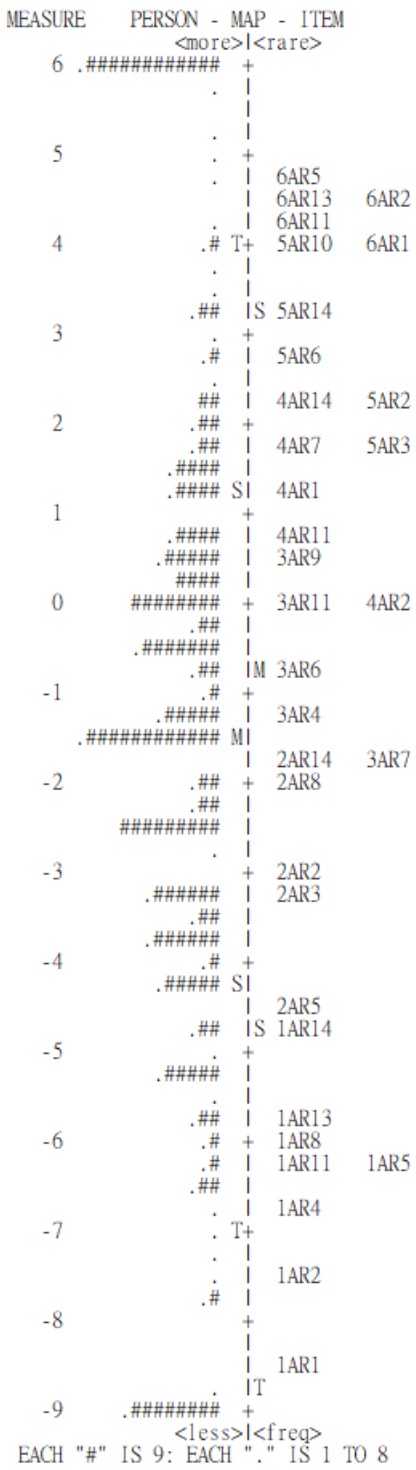


Figure 5. Item-person map (Wright map) of Math Knowledge (MK) after second item selection

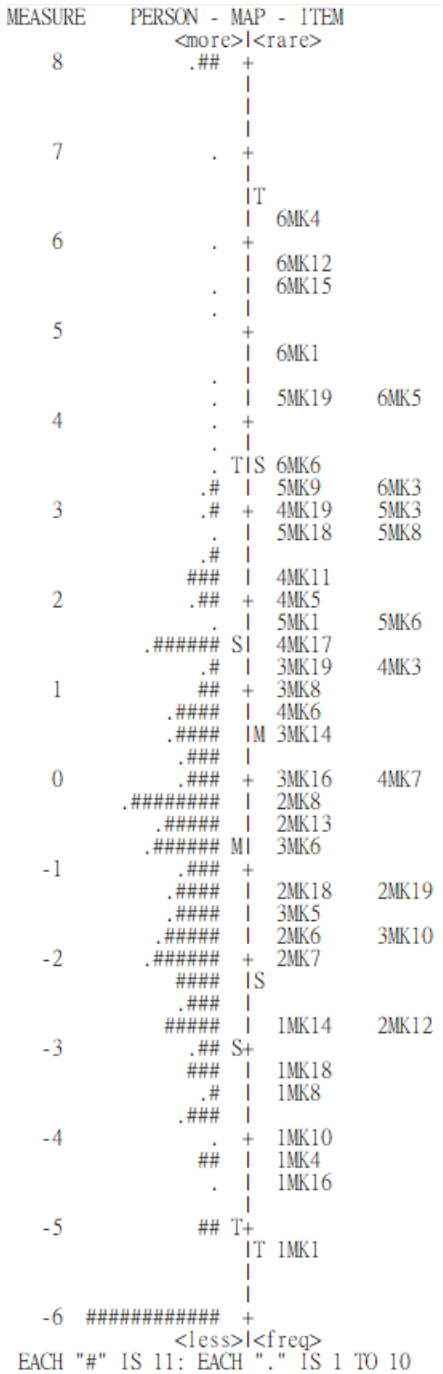
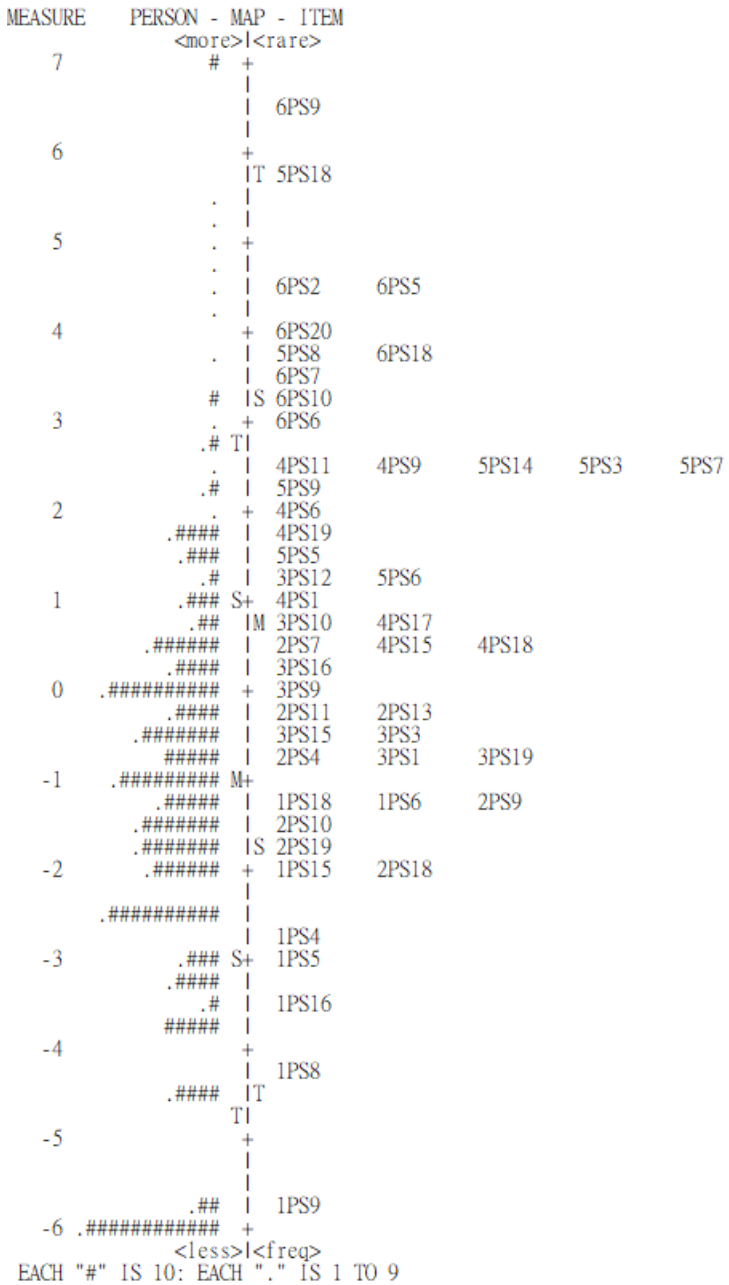


Figure 6. Item-person map (Wright map) of Math Problem Solving (PS) after second item selection



Reliability and separation index were summarized as below. After first item selection, item reliability indexes of AR, MK and PS component were 1.00, .99, .99; and their item separation indexes were 15.92, 13.56, 12.38. These values implied good item hierarchy. Person reliability indexes (for non-extreme person) of AR, MK and PS component were .90, .92, .99; and their person separation indexes were 2.95, 3.31, 2.95. These suggested the scales had good differentiation on person into around three strata. Regarding the short version after second item selection, the corresponding item reliability indexes did not change, while item separation indices increased to 18.90 for AR, decreased to 12.46 and 10.88 for MK and PS respectively, which still implied that the scales were reliable and included a variety of items from different difficulty level. For person reliability indexes, the corresponding values of AR, MK, PS dropped to .83, .83, .80. Person separation indexes were also decreased to 2.18, 2.19, 2.03, which was expected as the number of items after second item selections were about halved. Yet the scale still showed good distinguishing power of person into high and low ability strata. For validity, pooled correlation between person ability estimates for AR, MK, PS after first item selection and math achievement scores in school were .64 ($p<.001$), .59 ($p<.001$), .69 ($p<.001$) respectively, and .59 ($p<.001$), .53 ($p<.001$), .64 ($p<.001$) respectively after second item selection. These implied the scale were valid to measure primary math achievement.

Principal Component Analysis (PCA) of the residuals after first and second item selections for AR, MK and PS respectively. After the first item selection, the corresponding observed variance explained by the measures were 48.7%, 44.2% and 42.8%, and the eigenvalues in the first contrast were 2.65, 2.67, 2.26. And after the second item selection, the corresponding observed variance explained by the measures were 50.1%, 44.3%, 39.6%, and the eigenvalues in the first contrast were 1.77, 1.89, 2.26. All of the eigenvalues were less than 3, which implied the 'secondary dimension' (if possible) had a strength of item less than 3. These findings underpinned the unidimensionality of each component.

4.3. Confirmatory Factor Analysis on Math Fluency (MF), Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

The model fit results of confirmatory factor analysis in Grade 1 to 3 were summarized in Table 2. In the case of Grade 1 and 3, the model attained adequate model fit (Grade 1: RMSEA=.056, CFI=.938; Grade 3: RMSEA=.055, CFI=.939), while for Grade 2, the model was slightly below the mark (RMSEA=.075, CFI=.817).

Table 2. Model fit results of confirmatory factor analysis (Grade 1 to 3)

		Chi-squared statistics			RMSEA			
Grade	n	χ^2	df	p-value	RMSEA	90% CI	CFI	TLI
P.1	311	531.275	269	.000	.056	[.049,.063]	.938	.930
P.2	404	658.718	203	.000	.075	[.068, .081]	.817	.792
P.3	298	387.938	203	.000	.055	[.047, .064]	.939	.930

Standardized factor loadings and factor correlation were reported in Appendix E and Table 3,4,5 respectively. All factor loadings were significant (Grade 1: range = [.513,.965], median = .717; Grade 2: range =[.150,.875], median = .601; Grade 3: range = [.448,.908], median = .704). Among Grade 1 to 3, all factor correlations were significant (p<.000) and positive (Grade 1: range = [.522,.890], median = .647; Grade 2: range = [.501, .721], median = .670; Grade 3: range = [.668, .908], median = .738). The highest factor correlation in Grade 1, 2 and 3 were PS with MK, PS with MF, and MK with AR respectively. These results supported the claim of a four-factor structure of the scale.

Table 3. Factor correlation in Grade 1

Factor correlation (Grade 1)				
	MF	AR	MK	PS
Math Fluency (MF)	-			
Arithmetic (AR)	.566	-		
Math Knowledge (MK)	.816	.522	-	
Math Problem Solving (PS)	.727	.522	.890	-

Table 4. Factor correlation in Grade 2

Factor correlation (Grade 2)				
	MF	AR	MK	PS
Math Fluency (MF)	-			
Arithmetic (AR)	0.643	-		
Math Knowledge (MK)	0.57	0.501	-	
Math Problem Solving (PS)	0.721	0.706	0.697	-

Table 5. Factor correlation in Grade 3

Factor correlation (Grade 3)				
	MF	AR	MK	PS
Math Fluency (MF)	-			
Arithmetic (AR)	0.668	-		
Math Knowledge (MK)	0.689	0.908	-	
Math Problem Solving (PS)	0.699	0.777	0.868	-

4.4. Scale finalization

Developmental score tables and percentile tables of each MF, AR, MK, PS component as well as composite score were presented in Appendix F. Distribution of standardized scores of each component were centered at 500 with standard deviation of 30. In practice, average raw scores of MF, total raw scores of AR, MK and PS component could be matched on corresponding developmental score table, which would locate corresponding ability of student in the whole primary ability continuum. Grade percentile tables displayed information of ability location with respect to Grade, which would be useful to inspect his/her developmental process of corresponding

component compared to peers. Finally, Grade percentile table of Composite score tables provided location and guidelines for diagnosis. As suggested by Geary et. al. (2007), a person with performance below the 10th percentile would be at-risk to have math learning disability, and those between the 10th and 25th percentiles would be low achieving, students with composite scores consistently below the 10th percentile score with respect to their grades in the whole math achievement scale, yet with normal intelligence level (from Raven's Standard Progressive Matrices guideline) are having very high risk to be classified as dyscalculia, which may require further diagnostic heuristics.

Finally, the items selected for the final scale were sorted in ascending order of item difficulty. Basal and stopping rule of AR, MK and PS component were further defined. For basal rule, the subject should start from the first (easiest) item of his/her Grade, with the assumption that all items before were correct. Appendix G summarized the calculation of probabilities of correct with respect to each AR, MK and PS component. While the mean of consecutive item difficulty differences for the three component were 0.54, 0.38, and 0.33, the probability of getting a score would be equal to or below .05 when the subject answering items with difficulty 6 item units higher in AR, 8 units in MK, and 9 units in PS. Thus, if the subject answered

consecutively wrong with the mentioned number, the test could be stopped. Moreover, if the subject could not score from the beginning item, he/she should work backward until getting consecutive correct of the mentioned number. After that the process continued after the beginning item.

CHAPTER 5

Discussion

This study aimed to develop a math achievement test for identifying at-risk Primary student for dyscalculia in Hong Kong, as a preliminary stage for dyscalculia identification. Math achievement-based method was adopted and scale conceptualization was inspired by existing literature describing behavior of dyscalculic children. This study included a large population (n=1410) of Hong Kong primary students to establish representativeness of the scale. Rasch analysis for selecting AR, MK, PS items were conducted with respect to a number of selection criteria. Reliability and validity analysis were conducted among MF, AR, MK and PS components. Finally, a finalized math achievement scale, as well as norm tables and administrating rules, was yielded as a practical diagnostic tool of at-risk students for dyscalculia in assessment.

The scale as a whole exhibited good psychometric properties in reliability and validity analyses. It furnished high confidence in measuring primary math achievement in a vertical manner. After all, this study managed to develop a clinical diagnostic tool which could also be used for research purpose. In practice, the expected time for

completing the final scale with suggested rules would be 20 minutes, which was practically feasible in the clinical and educational settings. A primary student math achievement could be effectively located with reference to development score tables, so as to inform practitioners how far a student is lagging behind. Further intervention at corresponding math level could be reasonably proposed to achieve higher effectiveness.

This study introduced a four-component (MF, AR, MK and PS) factor structure. This enriched conceptualization in math domain. An exploratory factor analysis across all grades would be a promising evidence, while such approach would be difficult to be conducted in current research design. Firstly, there were designed missings on item responses in AR, MK and PS. Secondly, there were different sample sizes in each grade. The confirmatory factor analysis in Grade 1 to 3 tended to provide supportive evidence on the factor structure. Further investigation in other Grades could provide justification more completely. In addition, further including more standardized math achievement measures (e.g. LAMK) could help improve scale validity, as well as examine the relationship between the four components and other math achievement measurements. After all, though the factor analysis seemed to be incomplete, unidimensionality from Rasch analysis suggested rationality of these four components

conceptualization. Furthermore, most tests did not differentiate math knowledge and problem solving. For example, Wechsler Individual Achievement Third Edition (WIAT-III) (Wechsler, 2009) only included the subset “Math Problem Solving” as a mix of them, which lacked an ability to tell whether a student’s difficulty lied on memory-based math fact, or application skills specific to problem-solving. Current study highlighted such difference to explore manifested math difficulties in a more precise way.

Rasch analysis provided good guidance in item selection, as well as calibrating more accurate measurement in standardization procedure. This study included comprehensive item selection criteria, which kept a balance between statistical estimation, practical issues and curriculum. Yet still there were misfit items included in the final scale. These cases mainly occurred when the misfit item was a linking item. As Kolen & Brennan (2004) reported that too few linking items may result in inadequate equating problems, the criterion (2) ensured stability in statistical estimation of the item parameters. Although this study already included two sets of linking items, the current situation suggested further extension of the linking item set to tolerate the removal of misfit linking items. To the least extent, this study supplied a pool of calibrated fit items which could be selected as linking items in future scale

calibration.

The choice of using composite score increases the robustness of the sample at scale developmental stage. Since the weighting of each subscale are not yet calibrated during the process of scale development, it would be subjective to define any criteria using subscale scores (e.g. poor performance in at least 2 subscales). Moreover, poor result in terms of composite score suggests general poor math achievement with respect to the primary math curriculum. Indeed currently, the weighting of each component for the composite score was arbitrary. While the nature of MF apparently differed from the other three components, weighting by item-response model would be perplexing. Future exploration on weighting methods, such as constructing correlation profiles between components and well-known math achievement measures (e.g. LAMK), or constructing a more promising confirmatory factor analysis model, could be considered.

This study indeed had limitations in sampling. For example, there were fewer senior Grade students (P.4 to P.6) and middle-SES students in the current sample, and further sampling on those area could increase the representativeness of the sample. In fact, for more concise sample, stratified sampling with respect to school banding would

imply a greater representativeness of a Hong Kong primary math scale. Yet such method would require collaboration with relevant official education organization in Hong Kong.

This study adopted math achievement-based test approach to identify at-risk students as a first step of identifying dyscalculia, which at the end required a percentile criterion for identification. Some researcher argued if such an arbitrarily set cut-off (Wong et al., 2014) may imply that any a priori criterion would simply define a particular proportion of the population as dyscalculia if the criterial dimension is normally distributed (Butterworth, 2005; Geary &Hoard, 2005). Wong et al. (2014) proposed latent class growth curve analysis as an alternative to identify dyscalculia, by comparing the development of arithmetic skills over three years, a group of children whose development fell significantly below the other groups were identified, and this group may be the potential dyscalculia group. Such modelling technique enables growth tracking in nonequidistant measurement occasions in longitudinal studies (Jordan, Hanich, &Kaplan, 2003), yet laborious to apply in practical setting. Moreover, low-percentile cut-off evidently identify really poor achievers who are possibly succumbed to dyscalculia. In addition, dyscalculic population defined by such cut-off showed distinct profiles against typical achiever (Geary et al., 2007),

which coherently suffices the use of the cut-off. As a result, this study adopted the common approach to use percentile method to identify a student with dyscalculia in similar researches (e.g. Compton et. al. 2012; Desoete et al. 2012; Kaufmann &Aster, 2012 Skagerlund &Träff, 2016; Wong et al., 2017)), with the choice of percentile summarized by Butterworth (2005).

Moreover, by addressing the approach of using cognitive-based method to identify dyscalculia, several hypothesis, such as core deficit theory (Butterworth, 2005) and access deficit hypothesis (Rousselle &Noël, 2007; Skagerlund &Träff, 2016) were proposed to explain this developmental problem. Other domain-general cognitive skills, such as visuospatial skills, working memory and executive function, are found to be related to dyscalculia (Kucian &vonAster, 2015), which may contribute to comorbidity of other learning disabilities such as dyslexia, ADHD, etc. (Butterworth, 2005; Kaufmann &Aster, 2012; Moll, Göbel, &Snowling, 2015). With the existing literature, however, not much consensus has been reached regarding the cognitive profiles of children with dyscalculia, leading to validity problem. After all, math achievement screening is sound to screen out students in need, yet specific intervention could only be applied with what they are lacking identified. Further exploration on cognitive profiles of dyscalculia by contrasting well-established

cognitive tasks should be included as the latter stage for enriching the whole dyscalculia identification procedure. Specifically, relationships between different cognitive skills and the four math components of current scale would provide more information on math performance and characteristics with different cognitive deficits.

To sum up, this study had the following limitation: Firstly, this study did not include complete analysis on the factor structure of the scale. Secondly, misfit linking items were still included. Thirdly, weighting of the four components to calculate composite score was arbitrary. Fourthly, fewer senior Grades and middle-SES students were included in the study. These could be solved by: (1) further study with larger sample size (especially in senior Grades) to complete the factor analysis; (2) using existing fit items as linked items for future math achievement scale development; (3) using factor analysis method to conduct weighting procedure; (4) recruiting more participants from mentioned sector. In addition, future research could put more resources on identifying existing students with dyscalculia by other methods, such as long-term tracking clinical diagnosis using DSM-V by an educational psychologist. This could further establish criterion validity of the scale.

In conclusion, this study developed a math achievement scale for identifying at-risk

Primary students for dyscalculia in Hong Kong, as a first step for dyscalculia identification. Definition of dyscalculia, identification method, as well as four important constructs in math domain were discussed. Scale construction and item selection by Rasch analysis were done to produce a refined final version. Reliability and validity analysis revealed its good psychometric properties. Furthermore, developmental score tables as well as administrative rules of scale were further constructed for practical usage.

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Appendix A. Item distribution of Math Knowledge (MK) and Math Problem Solving (PS) with respect to Education Bureau learning outcome categorization

P.1									
Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
1N1	17	5.67	6.80%	0	1	0.00%	5.00%	2.50%	-4.30%
1N2	14	4.67	5.60%	1	2	5.00%	10.00%	7.50%	1.90%
1N3	23	7.67	9.20%	1	4	5.00%	20.00%	12.50%	3.30%
1S1	10	10.00	12.00%	2	2	10.00%	10.00%	10.00%	-2.00%
1S2	3	3.00	3.60%	1	1	5.00%	5.00%	5.00%	1.40%
1M1	6	6.00	7.20%	3	1	15.00%	5.00%	10.00%	2.80%
1N4	10	3.33	4.00%	3	0	15.00%	0.00%	7.50%	3.50%
1N5	18	6.00	7.20%	2	1	10.00%	5.00%	7.50%	0.30%
1S3	12	12.00	14.40%	2	2	10.00%	10.00%	10.00%	-4.40%
1M2	10	10.00	12.00%	2	2	10.00%	10.00%	10.00%	-2.00%
1M3	7	7.00	8.40%	0	2	0.00%	10.00%	5.00%	-3.40%
1M4	8	8.00	9.60%	3	2	15.00%	10.00%	12.50%	2.90%
Total	138	83.33	100.00%	20	20	100.00%	100.00%	100.00%	0.00%
Number			32.80%					37.50%	4.70%
Shape and Space			30.00%					25.00%	-5.00%
Measures			37.20%					37.50%	0.30%
Total			100.00%					100.00%	

P.2

Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
2N1	6	2.00	2.22%	0	1	0.00%	5.00%	2.50%	0.28%
2N2	12	4.00	4.44%	0	1	0.00%	5.00%	2.50%	-1.94%
2N3	20	6.67	7.41%	2	2	10.00%	10.00%	10.00%	2.59%
2S1	8	8.00	8.89%	2	1	10.00%	5.00%	7.50%	-1.39%
2S2	4	4.00	4.44%	2	0	10.00%	0.00%	5.00%	0.56%
2M1	8	8.00	8.89%	2	1	10.00%	5.00%	7.50%	-1.39%
2M2	9	9.00	10.00%	2	2	10.00%	10.00%	10.00%	0.00%
2N4	4	1.33	1.48%	0	1	0.00%	5.00%	2.50%	1.02%
2N5	13	4.33	4.81%	0	1	0.00%	5.00%	2.50%	-2.31%
2N6	20	6.67	7.41%	2	2	10.00%	10.00%	10.00%	2.59%
2S3	4	4.00	4.44%	1	1	5.00%	5.00%	5.00%	0.56%
2S4	9	9.00	10.00%	2	1	10.00%	5.00%	7.50%	-2.50%
2M3	9	9.00	10.00%	1	2	5.00%	10.00%	7.50%	-2.50%
2M4	8	8.00	8.89%	2	2	10.00%	10.00%	10.00%	1.11%
2D1	6	6.00	6.67%	2	2	10.00%	10.00%	10.00%	3.33%
Total	140	90	100.00%	20	20	100.00%	100.00%	100.00%	0.00%
Number			27.78%						2.22%
Shape and Space			27.78%						-2.78%
Measures			37.78%						-2.78%
Data Handling			6.67%						3.33%
Total			100.00%						0.00%

P.3

Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
3N1	4	1.33	1.60%	1	1	5.00%	5.00%	5.00%	3.40%
3N2	9	3.00	3.60%	0	1	0.00%	5.00%	2.50%	-1.10%
3N3	10	3.33	4.00%	1	1	5.00%	5.00%	5.00%	1.00%
3N4	15	5.00	6.00%	1	1	5.00%	5.00%	5.00%	-1.00%
3S1	3	3.00	3.60%	1	1	5.00%	5.00%	5.00%	1.40%
3S2	6	6.00	7.20%	2	1	10.00%	5.00%	7.50%	0.30%
3M1	10	10.00	12.00%	3	1	15.00%	5.00%	10.00%	-2.00%
3M2	7	7.00	8.40%	2	1	10.00%	5.00%	7.50%	-0.90%
3N5	16	5.33	6.40%	1	2	5.00%	10.00%	7.50%	1.10%
3N6	10	3.33	4.00%	1	1	5.00%	5.00%	5.00%	1.00%
3S3	5	5.00	6.00%	1	1	5.00%	5.00%	5.00%	-1.00%
3S4	10	10.00	12.00%	2	2	10.00%	10.00%	10.00%	-2.00%
3M3	10	10.00	12.00%	3	3	15.00%	15.00%	15.00%	3.00%
3M4	5	5.00	6.00%	1	1	5.00%	5.00%	5.00%	-1.00%
3D1	6	6.00	7.20%	0	2	0.00%	10.00%	5.00%	-2.20%
Total	126	83.33	100.00%	20	20	100.00%	100.00%	100.00%	0.00%

Number	25.60%	4.40%
Shape and Space	28.80%	-1.30%
Measures	38.40%	-0.90%
Data Handling	7.20%	-2.20%
Total	100.00%	0.00%

P.4

Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
4N1	10	10.00	3.24%	1	1	5.00%	5.00%	5.00%	1.76%
4N2	10	10.00	3.24%	0	1	0.00%	5.00%	2.50%	-0.74%
4N3	5	5.00	1.62%	0	0	0.00%	0.00%	0.00%	-1.62%
4N4	10	3.33	9.71%	3	0	15.00%	0.00%	7.50%	-2.21%
4N5	9	3.00	8.74%	2	1	10.00%	5.00%	7.50%	-1.24%
4S1	10	10.00	9.71%	2	2	10.00%	10.00%	10.00%	0.29%
4M1	11	11.00	10.68%	2	2	10.00%	10.00%	10.00%	-0.68%
4N6	10	3.33	3.24%	0	2	0.00%	10.00%	5.00%	1.76%
4N7	16	5.33	5.18%	3	0	15.00%	0.00%	7.50%	2.32%
4N8	5	5.00	4.85%	2	1	10.00%	5.00%	7.50%	2.65%
4S2	9	9.00	8.74%	0	2	0.00%	10.00%	5.00%	-3.74%
4S3	8	8.00	7.77%	2	1	10.00%	5.00%	7.50%	-0.27%
4M2	12	12.00	11.65%	3	3	15.00%	15.00%	15.00%	3.35%
4D1	12	12.00	11.65%	0	4	0.00%	20.00%	10.00%	-1.65%
Total	137	107.00	100.00%	20	20	100.00%	100.00 %	100.00%	0.00%
Number			39.81%						2.69%
Shape and Space			26.21%						-3.71%
Measures			22.33%						2.67%
Data Handling			11.65%						-1.65%
Total			100.00%						0.00%

P.5

Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
5N1	5	5.00	5.38%	2	0	10.00%	0.00%	5.00%	-0.38%
5N2	15	5.00	5.38%	0	1	0.00%	5.00%	2.50%	-2.88%
5N3	14	4.67	5.02%	0	1	0.00%	5.00%	2.50%	-2.52%
5S1	6	6.00	6.45%	1	1	5.00%	5.00%	5.00%	-1.45%
5M1	14	14.00	15.05%	3	4	15.00%	20.00%	17.50%	2.45%
5D1	7	7.00	7.53%	1	1	5.00%	5.00%	5.00%	-2.53%
5A1	10	10.00	10.75%	2	2	10.00%	10.00%	10.00%	-0.75%
5N4	8	2.67	2.87%	1	1	5.00%	5.00%	5.00%	2.13%
5N5	10	3.33	3.58%	1	0	5.00%	0.00%	2.50%	-1.08%
5N6	11	3.67	3.94%	1	1	5.00%	5.00%	5.00%	1.06%
5S2	10	10.00	10.75%	3	1	15.00%	5.00%	10.00%	-0.75%
5M2	11	11.00	11.83%	2	3	10.00%	15.00%	12.50%	0.67%
5D2	8	8.00	8.60%	2	2	10.00%	10.00%	10.00%	1.40%
5A2	8	2.67	2.87%	1	2	5.00%	10.00%	7.50%	4.63%
Total	137	93.00	100.00%	20	20	100.00%	100.00%	100.00%	0.00%
Number			26.16%						-3.66%
Shape and Space			17.20%						-2.20%
Measures			26.88%						3.12%
Data Handling			16.13%						-1.13%
Algebra			13.62%						3.88%
Total			100.00%						0.00%

P.6

Education Bureau learning outcome	Suggested learning time	Corrected learning time	Theoretical proportion of item	Empirical number of item		Empirical proportion		Average	Deviation
				MK	PS	MK	PS		
6N1	14	4.67	5.00%	0	1	0.00%	5.00%	2.50%	-2.50%
6N2	8	2.67	2.86%	1	0	5.00%	0.00%	2.50%	-0.36%
6N3	12	4.00	4.29%	2	1	10.00%	5.00%	7.50%	3.21%
6S1	14	14.00	15.00%	2	3	10.00%	15.00%	12.50%	-2.50%
6M1	14	14.00	15.00%	3	2	15.00%	10.00%	12.50%	-2.50%
6D1	5	5.00	5.36%	1	1	5.00%	5.00%	5.00%	-0.36%
6D2	6	6.00	6.43%	1	1	5.00%	5.00%	5.00%	-1.43%
6N4	24	8.00	8.57%	2	2	10.00%	10.00%	10.00%	1.43%
6S2	6	6.00	6.43%	1	1	5.00%	5.00%	5.00%	-1.43%
6M2	6	6.00	6.43%	2	2	10.00%	10.00%	10.00%	3.57%
6M3	10	10.00	10.71%	2	2	10.00%	10.00%	10.00%	-0.71%
6D3	8	8.00	8.57%	2	2	10.00%	10.00%	10.00%	1.43%
6A1	15	5.00	5.36%	1	2	5.00%	10.00%	7.50%	2.14%
Total	142	93.33	100.00%	20	20	100.00%	100.00 %	100.00%	0.00%
Number			20.71%						1.79%
Shape and Space			21.43%						-3.93%
Measures			32.14%						0.36%
Data Handling			20.36%						-0.36%
Algebra			5.36%						2.14%
Total			100.00%						0.00%

Appendix B. Constructed items of Math fluency (MF), Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

Math Fluency (MF)

. 第一部分：數學流暢(Math fluency)

1a. 加法 – 時限 1 分鐘　　姓名：_____

請由**左邊直行**開始，由**上**至**下**，完成每道題目。　　班別：____（　　）

5+8=	9+1=	2+1=	1+3=	1+8=
6+4=	2+7=	9+2=	4+1=	4+6=
8+9=	1+8=	1+9=	2+8=	2+2=
2+1=	3+2=	8+6=	1+9=	5+8=
2+5=	4+3=	3+9=	2+7=	3+2=
7+9=	5+3=	8+5=	3+8=	3+8=
2+1=	2+6=	9+1=	2+8=	8+1=
6+5=	2+2=	6+5=	3+6=	8+5=
1+1=	7+8=	2+5=	9+6=	2+6=
1+3=	7+7=	2+1=	5+7=	5+9=
9+7=	8+4=	8+1=	4+3=	2+2=
7+3=	9+3=	7+9=	8+3=	5+9=
4+6=	2+7=	3+4=	2+8=	6+7=
5+9=	7+1=	3+5=	8+3=	3+8=
8+4=	9+5=	9+7=	9+2=	8+1=
5+8=	5+5=	5+6=	3+1=	3+9=
3+7=	9+8=	8+5=	8+8=	8+1=
8+9=	3+2=	9+3=	4+7=	9+4=

第一部分：數學流暢(Math fluency)

1b. 減法 – 時限 1 分鐘　　姓名：_____

請由**左邊直行**開始，由**上**至**下**，完成每道題目。　　班別：____（　　）

19-2=	19-3=	13-2=	12-10=	19-6=
15-9=	14-5=	10-3=	10-1=	18-7=
15-4=	11-10=	16-7=	12-3=	16-5=
17-2=	14-3=	18-10=	14-9=	12-4=
18-8=	13-3=	10-8=	15-3=	16-3=
12-3=	16-7=	10-6=	20-2=	20-10=
18-7=	11-9=	15-6=	12-3=	13-1=
10-2=	15-2=	19-7=	19-7=	11-2=
10-6=	12-7=	12-1=	19-6=	13-4=
16-3=	13-5=	20-2=	15-2=	18-1=
13-4=	13-9=	20-8=	10-9=	15-6=
18-8=	13-8=	15-3=	12-4=	19-4=
18-5=	17-8=	20-2=	13-2=	11-5=
14-1=	13-3=	18-10=	12-2=	12-10=
13-10=	14-2=	20-6=	12-6=	18-3=
19-1=	20-9=	14-3=	20-2=	14-8=
14-9=	16-5=	18-1=	10-3=	19-4=
19-9=	17-1=	15-9=	16-2=	14-9=

Arithmetic (AR)

Item code	Question	Answer	Education Bureau learning outcome	Linking set
1AR1	5+3=	8	1N3	A
1AR2	9-7=	2	1N3	B
1AR3	0+0=	0	1N3	
1AR4	17-3=	14	1N3	
1AR5	12-8=	4	1N3	
1AR6	8+7=	15	1N5	B
1AR7	62+14=	76	1N5	
1AR8	57+35=	92	1N5	
1AR9	64+26=	90	1N5	A
1AR10	69+13+27=	109	1N5	
1AR11	77-54=	23	1N5	
1AR12	48-36=	12	1N5	
1AR13	39-31=	8	1N5	A
1AR14	96-11-25=	60	1N5	B
2AR1	574+360=	934	2N2	
2AR2	90+351+115=	556	2N2	
2AR3	84-17=	67	2N2	

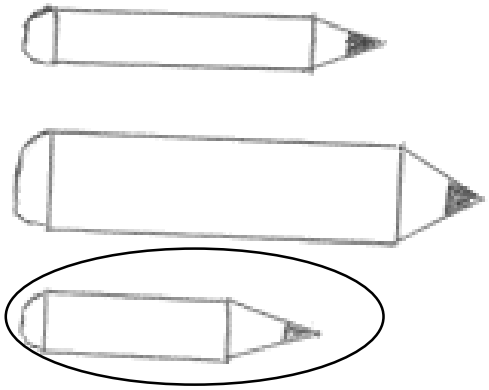
2AR4	66-19-38=	9	2N2	
2AR5	3×8=	24	2N3	A
2AR6	7×4=	28	2N3	B
2AR7	6×9=	54	2N3	
2AR8	528+134-420=	242	2N5	
2AR9	487-152-20=	315	2N5	A
2AR10	273-320+194=	147	2N5	B
2AR11	24÷3=	8	2N6	
2AR12	72÷9=	8	2N6	B
2AR13	20÷3=___...___	6…2	2N6	A
2AR14	97÷9=___...___	10…7	2N6	
3AR1	3262+151=	3413	3N2	
3AR2	7541-3550=	3991	3N2	
3AR3	1291+52+107=	1450	3N2	
3AR4	5325-164-2050=	3111	3N2	
3AR5	1027-2234+1639=	432	3N2	A
3AR6	9×36=	324	3N3	B
3AR7	325×4=	1300	3N3	

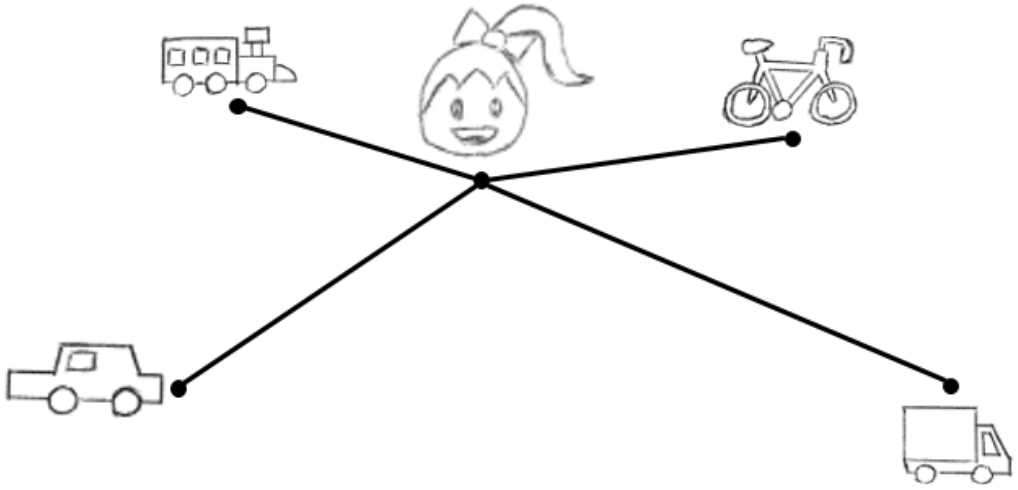
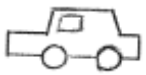
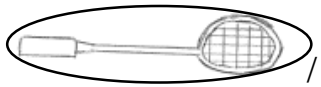
3AR8	67×5×7=	2345	3N3	
3AR9	462÷3=	154	3N4	A
3AR10	648÷8=	81	3N4	B
3AR11	86-(36-18)=	68	3N5	
3AR12	460-53×7=	89	3N5	
3AR13	4×(15+175)=	760	3N5	A
3AR14	421×6-238×7=	860	3N5	B
4AR1	13×12×15=	2340	4N1	
4AR2	85÷28=___...__	3…1	4N2	B
4AR3	326÷61=___...___	5…21	4N2	A
4AR4	544÷17÷8=	4	4N2	
4AR5	64-51÷3-27=	20	4N6	
4AR6	44+8×12÷6=	60	4N6	
4AR7	132×(20-13)+253=	1177	4N6	A
4AR8	168÷(32÷4) ×4+307=	391	4N6	B
4AR9	75-(80+7×3)+262=	236	4N6	
4AR10	108÷36×48÷24=	6	4N6	
4AR11	$\frac{5}{9}+\frac{2}{9}=$	$\frac{7}{9}$	4N7	

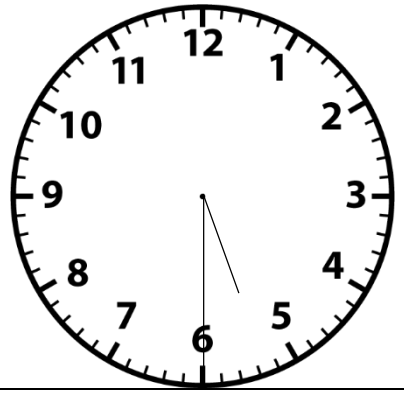
4AR12	$4\frac{8}{13}+\frac{4}{13}=$	$4\frac{12}{13}$	4N7	B
4AR13	$3\frac{9}{14}+1\frac{3}{14}=$	$4\frac{6}{7}$	4N7	A
4AR14	$\frac{5}{28}+1+2\frac{15}{28}=$	$3\frac{5}{7}$	4N7	
5AR1	$\frac{11}{12}+\frac{3}{4}=$	$1\frac{2}{3}$	5N2	
5AR2	$14-3\frac{7}{12}+5\frac{2}{3}=$	$16\frac{1}{12}$	5N2	B
5AR3	$14\times\frac{13}{28}=$	$6\frac{1}{2}$	5N3	A
5AR4	$4\frac{2}{7}\times3\frac{1}{3}\times1\frac{1}{20}=$	15	5N3	
5AR5	7.13+8.5+24=	39.63	5N4	A
5AR6	62.1-22.05-3.4=	36.65	5N4	
5AR7	13×5.06=	65.78	5N5	
5AR8	0.15×8.03=	1.2045	5N5	B
5AR9	$8\div1\frac{7}{9}=$	$4\frac{1}{2}$	5N6	
5AR10	$9\frac{3}{7}\div\frac{22}{35}\div2\frac{6}{7}=$	$5\frac{1}{4}$	5N6	
5AR11	$4\frac{2}{5}\div1\frac{5}{6}\times1\frac{2}{3}=$	4	5N6	A
5AR12	a+6=15，a=___	9	5A2	
5AR13	b ÷7=20，b=___	140	5A2	
5AR14	9-z=1.5，z=___	7.5	5A2	B
6AR1	0.76÷0.8=	0.95	6N1	

6AR2	6.25÷1.25×8.4=	42	6N1	B
6AR3	47.8 – 64.22÷3.8=	30.9	6N1	A
6AR4	$\frac{13}{50}$ + 0.72×9=	6.74	6N2	
6AR5	75% - 0.243=	0.507	6N3	A
6AR6	30.05×20%	6.01	6N3	B
6AR7	$4\frac{1}{15} \times 60\%$=	2.44	6N3	
6AR8	$10\frac{1}{3} \div 62\% \times \frac{3}{4}$=	12.5	6N3	
6AR9	(3.42 – 172%)× $5\frac{4}{5}$=	9.86	6N3	A
6AR10	$35\% \times (5.3 + \frac{2}{10})$=	1.925	6N3	B
6AR11	10f – 3=17，f =___	2	6A1	
6AR12	16 – p ×25% = 8，p=___	32	6A1	
6AR13	5×(y – 13)÷8 = 10，y=___	29	6A1	
6AR14	$\frac{82 - x}{8} = 5x, x =$ __	2	6A1	
JSC1	(-5) + (-14) – (+3)=	-22	-	
JSC2	20÷(-4) + (-2×2)=	-9	-	
JSC3	$2^3 \times 3 \div 3^2 + 5^0$=	$3\frac{2}{3}$	-	

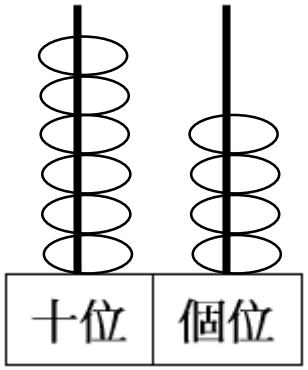
Math Knowledge (MK)

Item Code	Question and Model Answer	Education Bureau learning outcome	Linking set
1MK1	 圈出最短的筆。	1M1	

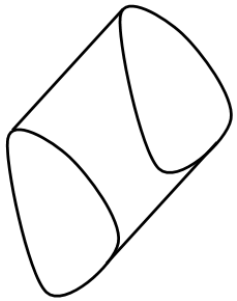
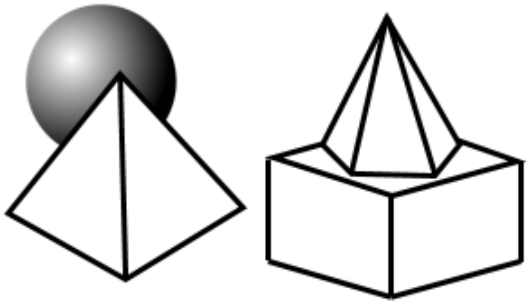
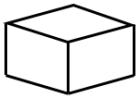
1MK2	 圈出答案。 ( /  /  / ) 距離  最近。	1M1	B
1MK3	圈出答案。 量度客廳之間的距離可以用( /  / ) 。	1M1	A

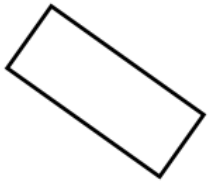
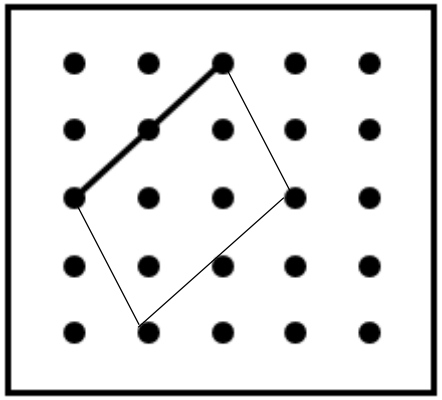
1MK4	填上數值，並圈出正確的量詞。  這個是 5 (元 / 毫)。	1M2	
1MK5	 可以換得 6 個。	1M2	
1MK6	在鐘面上劃上 5 時半。 	1M4	B
1MK7	如果今天是星期六，明天是星期 日/天 ，後天是星期 一 。	1M4	A

<table>
<tr>
<td>1MK8</td>
<td>
<table>
<tr><td colspan="7">2017年 12月</td></tr>
<tr><td>日</td><td>一</td><td>二</td><td>三</td><td>四</td><td>五</td><td>六</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>1</td><td>2</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr>
<tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td></tr>
<tr><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
小文在月曆上圈出了鋼琴比賽的日子。小文會在 2017 年 12 月 19 日 (星期二) 參加鋼琴比賽。
</td>
<td>1M4</td>
<td></td>
</tr>
<tr>
<td>1MK9</td>
<td>左  右
圈出從右邊數起的第 6 朵花。</td>
<td>1N2</td>
<td></td>
</tr>
<tr>
<td>1MK10</td>
<td>請在橫線上填寫答案。</td>
<td>1N3</td>
<td></td>
</tr>
</table>

	□ + ○ = 12 □ ○ + □ = <u>12</u>		
1MK11	 在上面畫上串珠，表示出 64 這個數字。	1N4	A
1MK12	請在橫線上填寫適當的數字。 26，28，<u>30</u>，32，<u>34</u>，<u>36</u>，38	1N4	B

1MK13	把 85，76，80，91 由大至小排出來。 91，85，80，76 (最大) (最小)	1N4	
1MK14	請在橫線上填寫答案。 21 + 32 + 39 大約等於 (90 -–95) 。	1N5	
1MK15	請在橫線上填寫答案。 84 + 11 = ☆ ☆ - 84 = 11	1N5	
1MK16	圈出答案。	1S1	

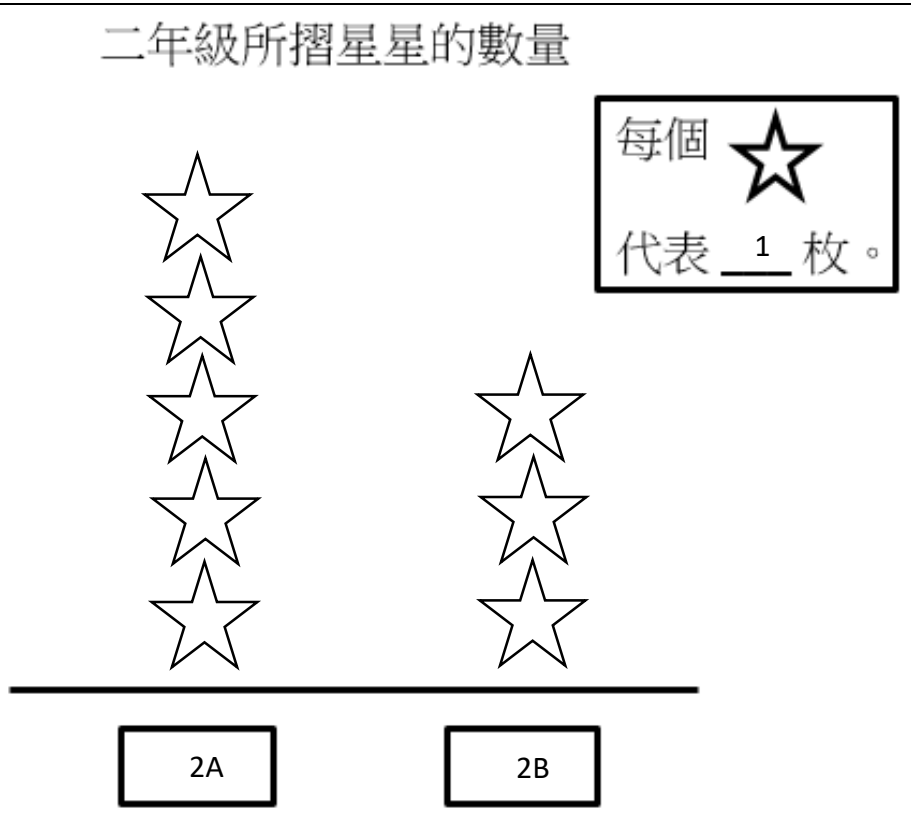
	 這個是（球體 / 柱體 / 錐體）。		
1MK17	 圈出答案。  在  的（上 / 下 / 左 / 右 / 前 / 後）方。	1S1	A
1MK18	把下面的字母分類：	1S2	B

	MONDAY 只由直線組成的字: M,N,A,Y		
1MK19	填上適當的英文字母，代表圖形。 A 形 B. 三角形 C. 四邊形 D. 五邊形 86.  這個是 C 。	1S3	
1MK20		1S3	

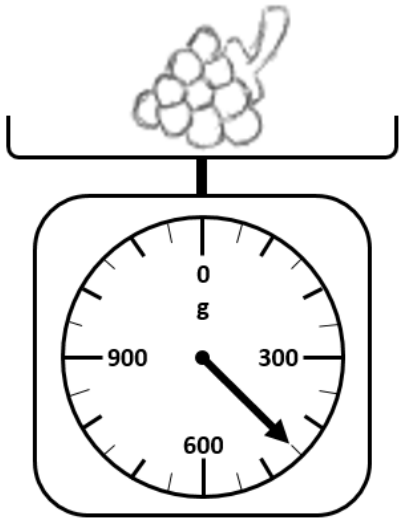
	加上直線，使在釘板上的線能圍成一個四邊形。		
2MK1	 小軒有貨車 4 架。	2D1	A
2MK2	以下是二年級所摺星星的數量。	2D1	B

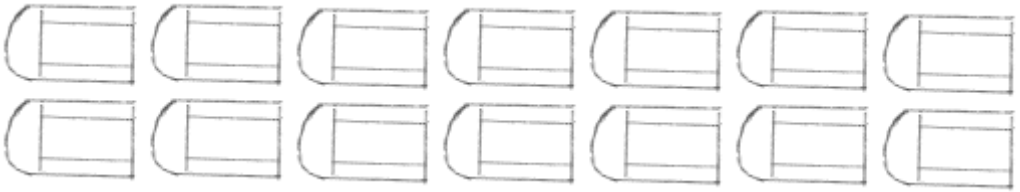
班別	2A	2B
記錄	正	下

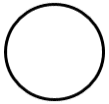
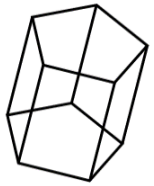
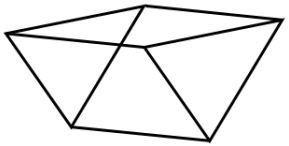
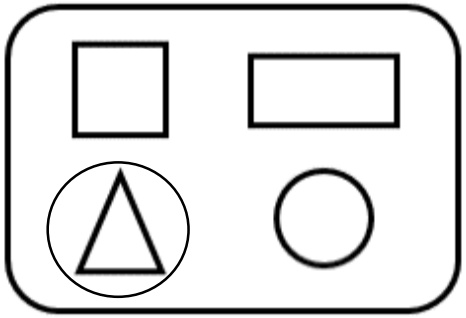
利用上表的資料製作象形圖。

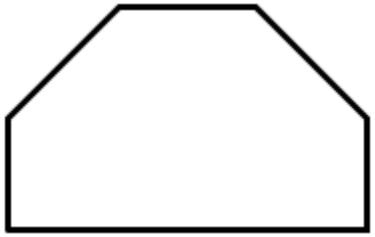
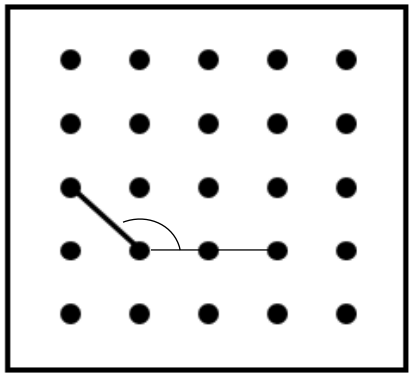
			
2MK3	圈出答案。 一隻水杯約高 11 (米 / 厘米) 。	2M1	

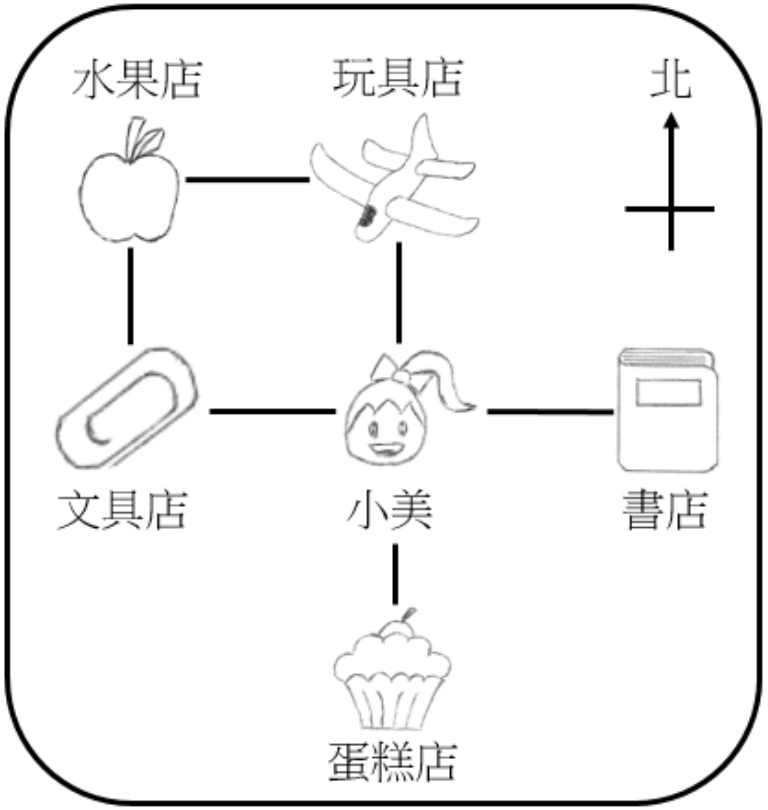
2MK4	6 米 70 厘米 = <u>670</u> 厘米	2M1	
2MK5	 這個鐘面顯示的是 <u>11</u> 時 <u>48</u> 分。	2M2	
2MK6	閏年的 2 月有 <u>29</u> 天，全年有 <u>366</u> 天。	2M2	
2MK7	請在橫線上填寫答案。 1 張  可換成 1 張  和 <u>5</u> 張  。	2M3	A

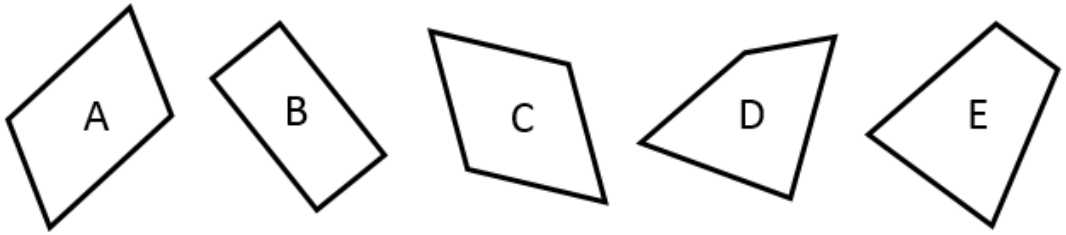
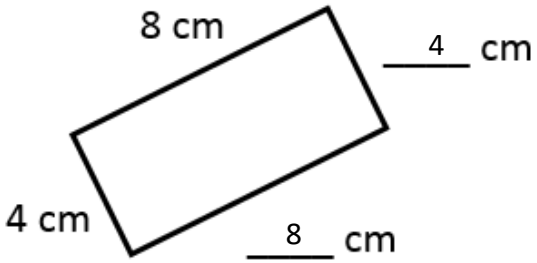
2MK8	  的重量是 <u>450g</u> 。(答案需寫上單位。)	2M4	B
2MK9	請在橫線上填寫答案。 6 公斤 150 克 = <u>6150</u> 克。	2M4	
2MK10	請在橫線上填寫答案。	2N3	

	$4 + 4 + 4 + 4 + 4 + 4 = \underline{4 \times 6 / 6 \times 4}$		
2MK11	請在橫線上填寫答案。 ☆ × <u>7</u> = 7 × ☆	2N3	A
2MK12	 上面有 14 塊橡皮刷，每 3 塊分成一盒，可分成 <u>4</u> 盒，餘下 <u>2</u> 塊橡皮刷。	2N6	B
2MK13	請在橫線上填寫答案。 ○ × ☆ = 54 54 ÷ ○ = ☆	2N6	

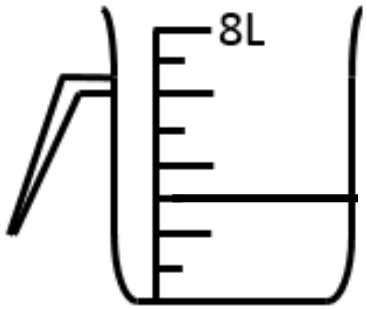
	54 ÷  = 		
2MK14	填上適當的英文字母，代表圖形。 A 角錐體 B. 五角柱體 C. 圓柱體 D. 圓錐體 86.  這個是 <u>B</u> 。	2S1	
2MK15	圈出圖形，代表答案。  的底是  。	2S1	

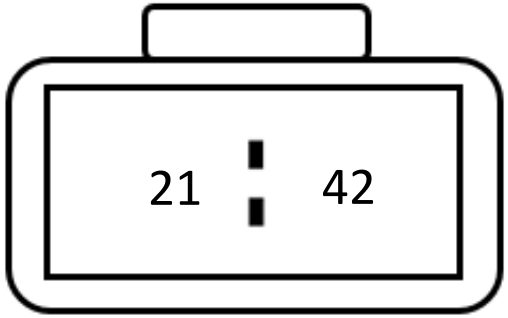
2MK16	 這個圖形內有 6 個角，其中 2 個是直角。	2S2	
2MK17	 在釘板上加上直線，使在釘板上的線能形成一個比直角大的角，並用  標示。	2S2	

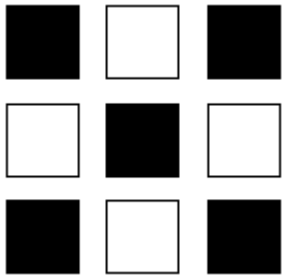
2MK18	 圈出答案。 蛋糕店在玩具店的（東／南／西／北）方。	2S3	B

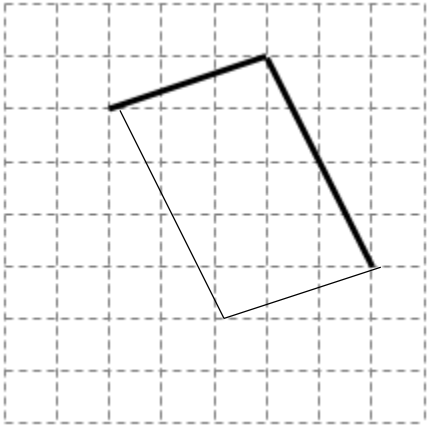
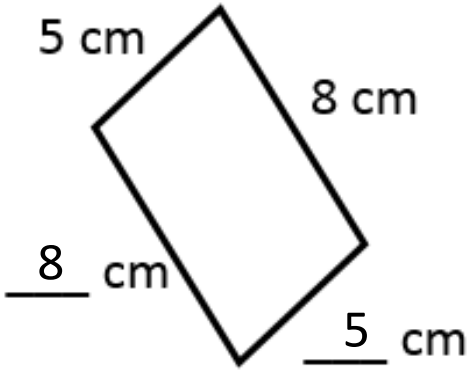
2MK19	請在橫線上填上英文字母。  梯形是 E 。	2S4	A
2MK20	請在橫線上填寫答案。  	2S4	
3MK1	圈出答案。 我的食指指甲約長 (⑨/ 90) mm。	3M1	A

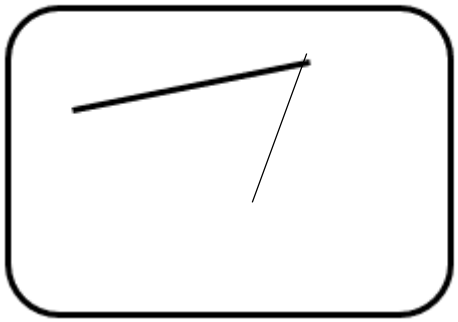
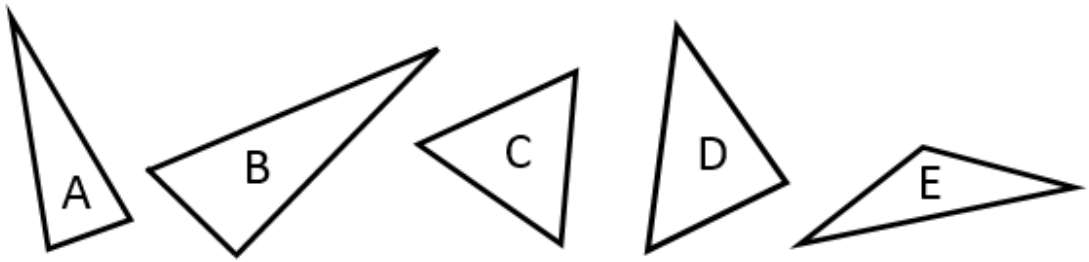
3MK2	12 cm 8 mm = <u>128</u> mm	3M1	B
3MK3	選擇最適合的量度工具來量度以下長度，填寫代表答案的英文字母。 A 輪 B. 捲尺 C. 直尺 D. 軟尺 86. 足球場的長度應該用 <u>A</u> 來量度。	3M1	
3MK4	秒錶的指針走 1 圈，時間過了 <u>60</u> 秒，即 <u>1</u> 分鐘。	3M2	
3MK5	2 分鐘 = <u>120</u> 秒	3M2	A
3MK6	以下哪些容器的容量少於 1 升? 請圈出答案。 (答案可多於一個。)	3M3	

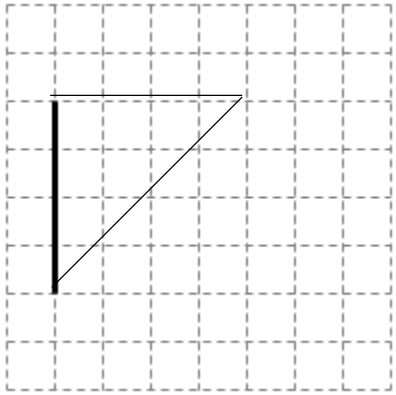
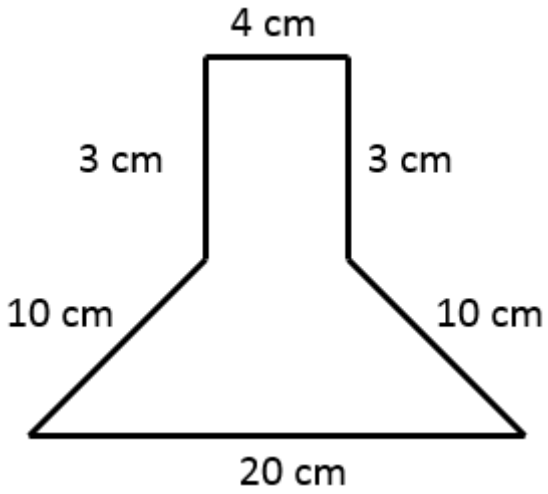
			
3MK7	在量杯上畫上橫線，表示容量。  容量：3 升	3M3	
3MK8	9 L 6 mL = <u>9006</u> mL	3M3	B

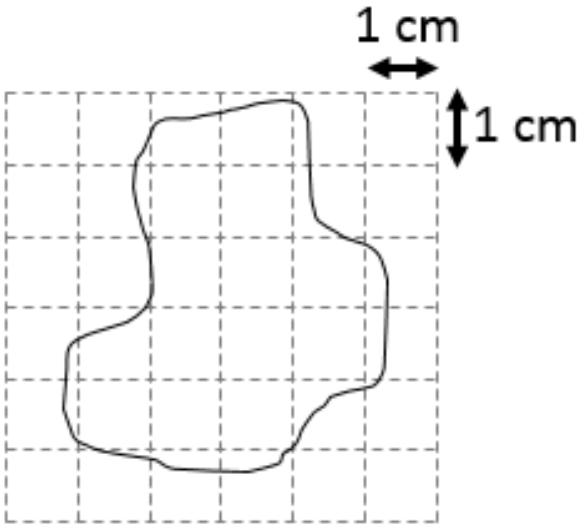
3MK9	用 24 小時報時制在電子鐘上表示下午 9 時 42 分。  	3M4	
3MK10	在每條橫線上填上一個數字，使算式成立。 $25394 = 20000 + \underline{5}000 + \underline{3}00 + \underline{9}0 + 4$	3N1	
3MK11	請在橫線上填寫答案。 297 × 6 大約是 <u>300</u> × 6 = <u>1800</u>。	3N3	A
3MK12	請在橫線上填寫答案。 596 ÷ 3 大約是 <u>600</u> ÷ 3 = <u>200</u> 。	3N4	

3MK13	A，B，C 分別是三個不同的數字。以下有兩道算式，請圈出結果較大的那一道。 $A-(B-C)$　$A-(B+C)$	3N5	
3MK14	 黑色部分佔全個圖形的 $\frac{5}{9}$。(請在方格內填寫數字。)	3N6	B
3MK15	MATH 有平行線的字母： M,H	3S1	
3MK16	在方格紙上加上直線，作成一個平行四邊形。	3S2	

			
3MK17	請在橫線上填寫答案。 	3S2	

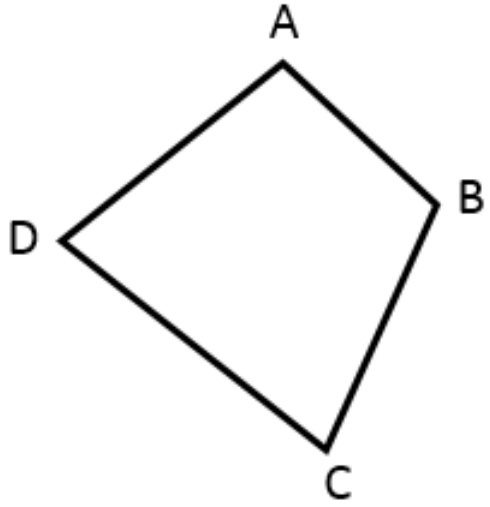
3MK18	加上直線，作成指定的角度。 銳角 	3S3	B
3MK19	填上英文字母，把以下三角形分類。  直角三角形： B	3S4	A

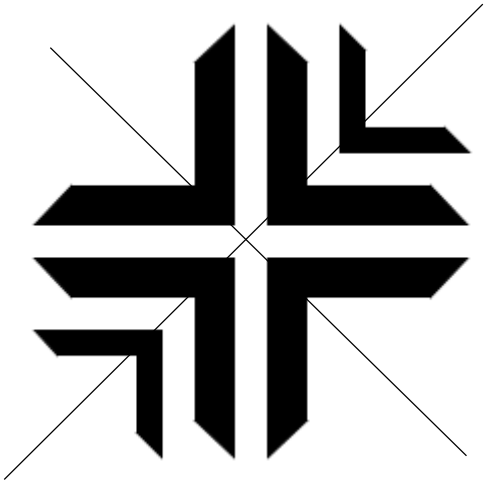
3MK20	加上直線，作成一個等腰直角三角形。 	3S4	
4MK1	 	4M1	

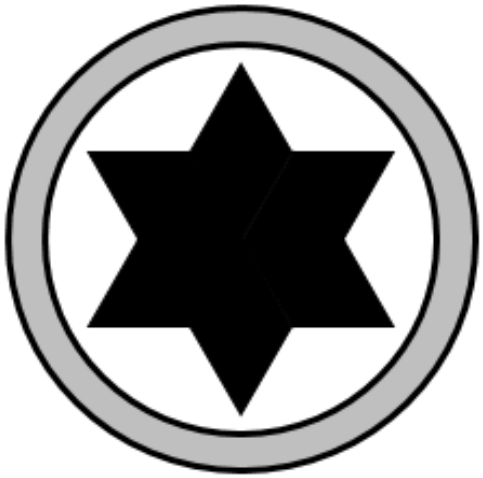
	這個圖形的周界為 <u>50</u> cm。		
4MK2	一個邊長為 12 cm 的正方形，它的周界為 <u>48</u> cm。	4M1	B
4MK3	 這個圖形的面積約為 (13-14) cm^2。	4M2	
4MK4	圈出適當的單位。 一張書簽的面積約為 45 (平方厘米 / 平方米)。	4M2	

4MK5	請在橫線上填寫答案。 一個邊長為 6 m 的正方形，它的面積為 <u>36</u> m^2。	4M2	A
4MK6	請在橫線上填寫答案。 $\underline{5} \times 101 \times 37 = \underline{37} \times (101 \times 5)$	4N1	
4MK7	3 的首 5 個倍數是： <u>3,6,9,12,15</u>	4N4	
4MK8	列出 12 的所有因數： <u>1,2,3,4,6,12</u>	4N4	
4MK9	圈出答案。 72 是 6 和 12 的 (因數 / 倍數)。 6 和 12 是 72 的 (因數 / 倍數)。	4N4	A
4MK10	8，12 和 14 的最小公倍數是 <u>168</u> 。	4N5	B
4MK11	6，15 和 24 的最大公因數是 <u>3</u>。	4N5	

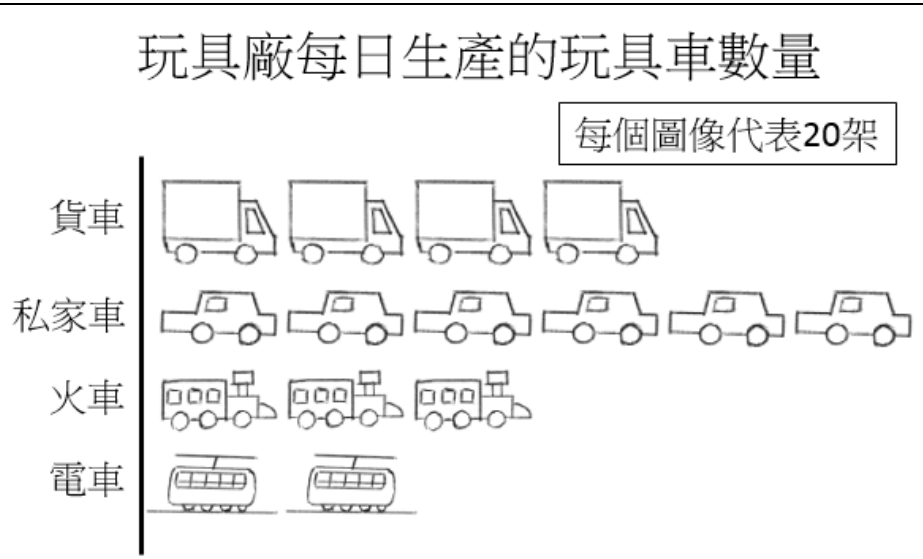
4MK12	請把以下分數分類。 $\frac{9}{10}$ $1\frac{25}{27}$ $\frac{7}{6}$ $\frac{3}{4}$ $8\frac{1}{2}$ $\frac{12}{7}$ 帶分數： $1\frac{25}{27}, 8\frac{1}{2}$ _____	4N7	
4MK13	將 $2\frac{5}{6}$ 化為假分數： $\frac{17}{6}$ ____	4N7	
4MK14	將 $2\frac{4}{15}$ ，$3\frac{1}{2}$，$2\frac{11}{15}$ 由小至大排列。 $2\frac{4}{15}$ ， $2\frac{11}{15}$ ， $3\frac{1}{2}$ (最小) (最大)	4N7	
4MK15	用小數表示$50\frac{4}{100}$： 50.04	4N8	A
4MK16	6 升 732 毫升 = 6.732 升	4N8	B
4MK17	根據資料，圈出資料所代表的四邊形。	4S1	

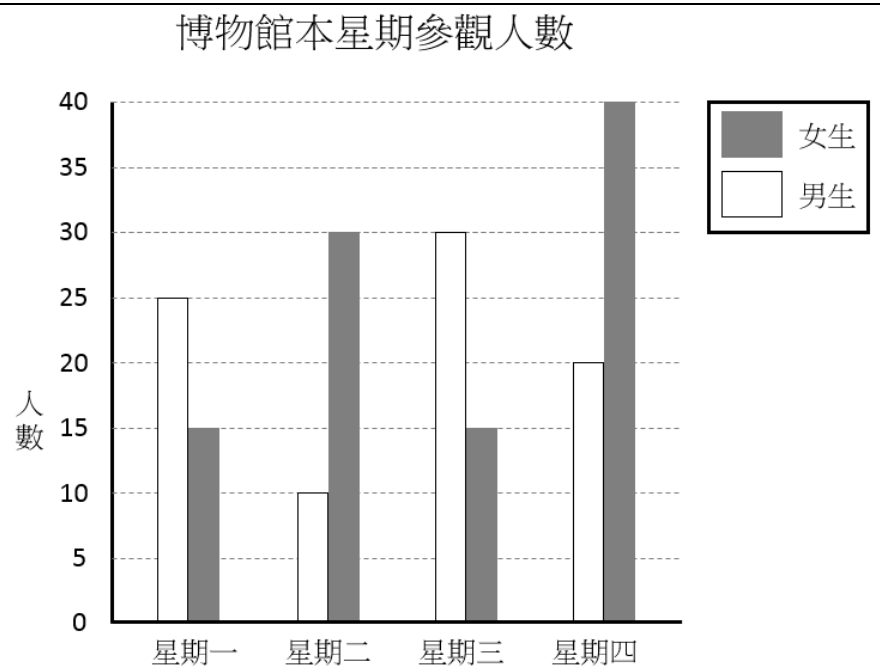
	<table><tr><th>四邊形的資料</th><th>這個四邊形是</th></tr><tr><td>• 鄰邊不相等 • 兩組對邊平行 • 沒有直角</td><td>正方形　長方形　(平行四邊形) 梯形　菱形</td></tr></table>		
4MK18	圈出答案。  BC 和 AD 是一組 (對邊 / 鄰邊 / 平行線)。	4S1	B

4MK19	用直線畫出以下圖形的所有對稱軸。 	4S3	A

4MK20	 這個圖形有 6 條對稱軸。	4S3	
5MK1	下面哪些是代數式? 請圈出來。 (答案可多於一個。) $d + 3$ $5w$ $50 - 3$ $\frac{200}{y}$ $5 - 4$ $b \div (20 + 7)$ $30 - 6 \times 5$ $20 - 5t$	5A1	

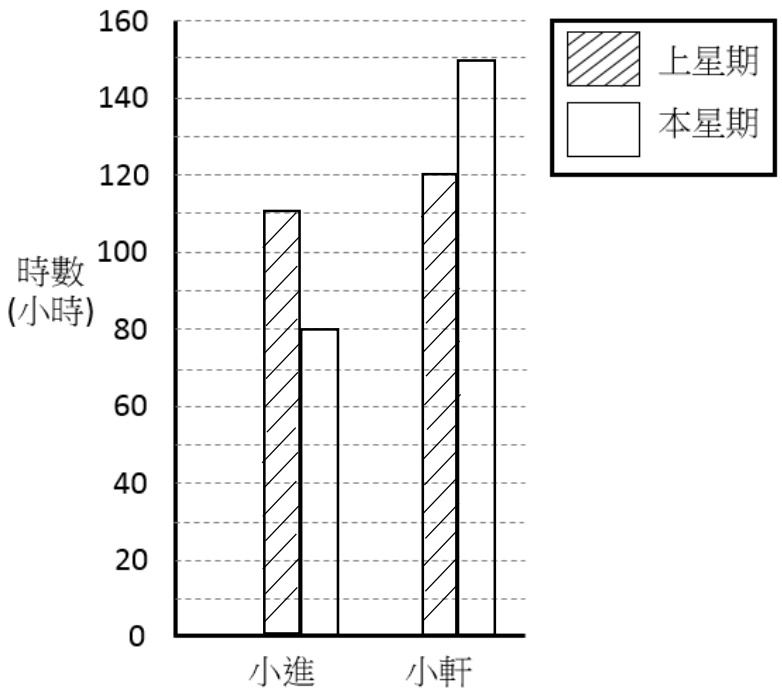
5MK2	用代數式表示以下句子。 H　減去 3，再除以 7：$\frac{H-3}{7}$ _____	5A1	B
5MK3	下面哪些是方程式? 請圈出來。 (答案可多於一個。) $d-72=5$　$4\times3-5=7$　$k+3>21$ $\frac{y}{32}=5$　$50-u<2$　$9p=45+2$　$t\div3=5$	5A2	A
5MK4	以下是玩具廠每日生產的玩具車數量。	5D1	

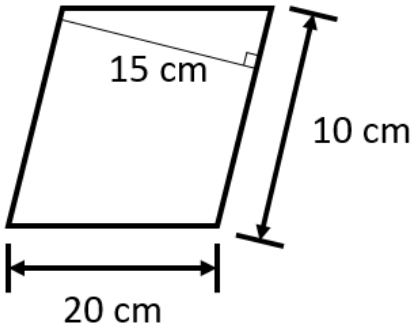
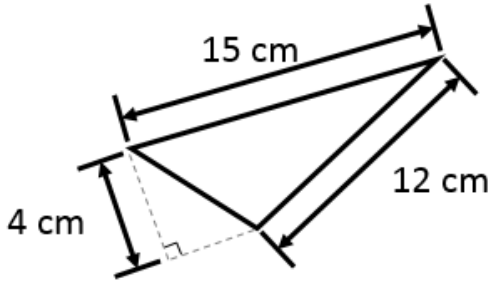
	 每日生產的貨車數量，比電車 (多 / 少) 40 架。		
5MK5	以下是博物館本星期的參觀人數。	5D2	

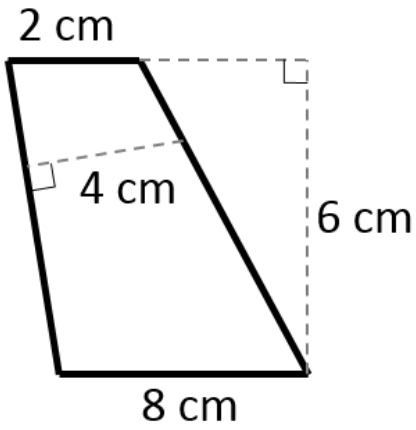
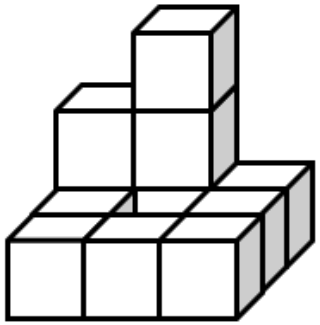
	 星期三參觀的男生比女生（(多)／少） 15 人。		
5MK6	以下是小進和小軒二人於兩星期的工作時數。	5D2	

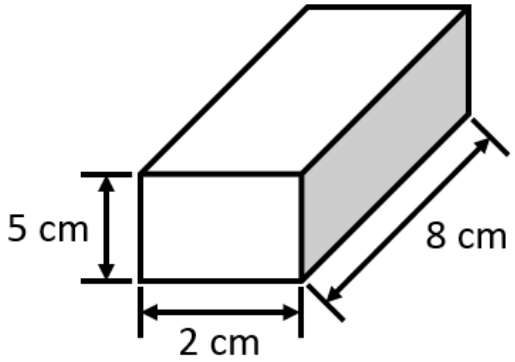
	小進	小軒
上星期 (小時)	110	120
本星期 (小時)	80	150

根據上表的資料，製作出一個並排的複合棒形圖。

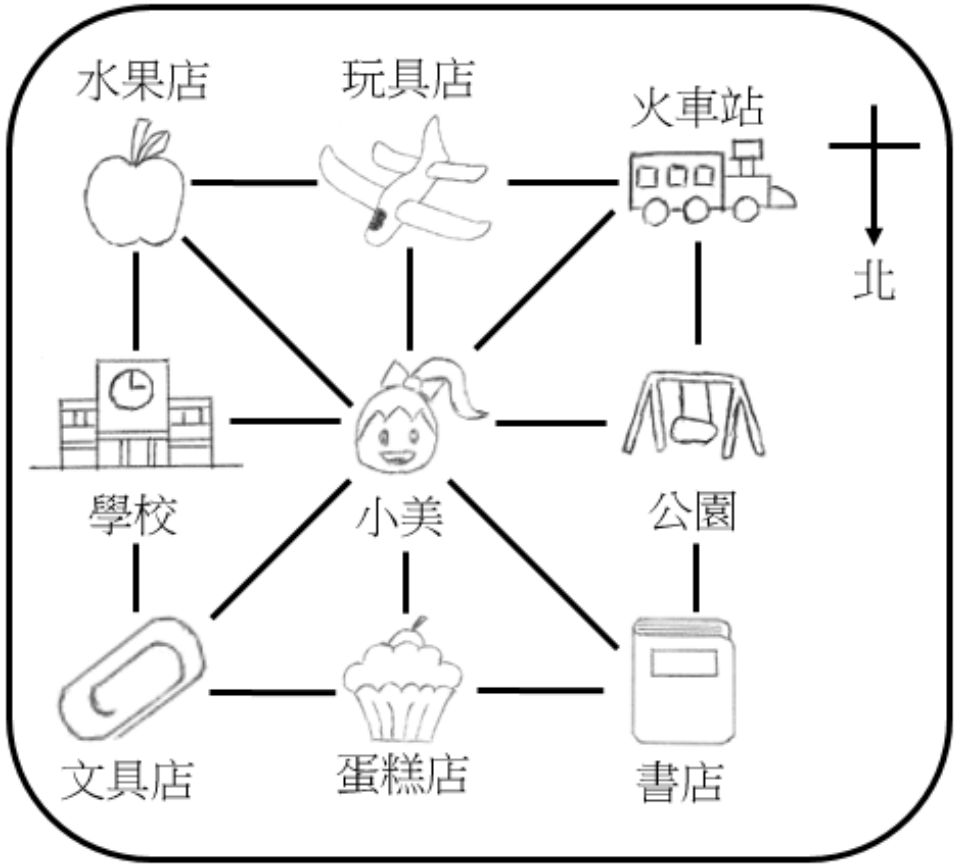


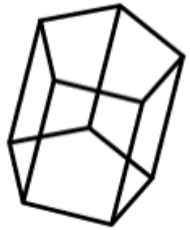
5MK7	 這個圖形的面積是 <u>150</u> cm^2。	5M1	
5MK8	 這個圖形的面積是 <u>30</u> cm^2。	5M1	

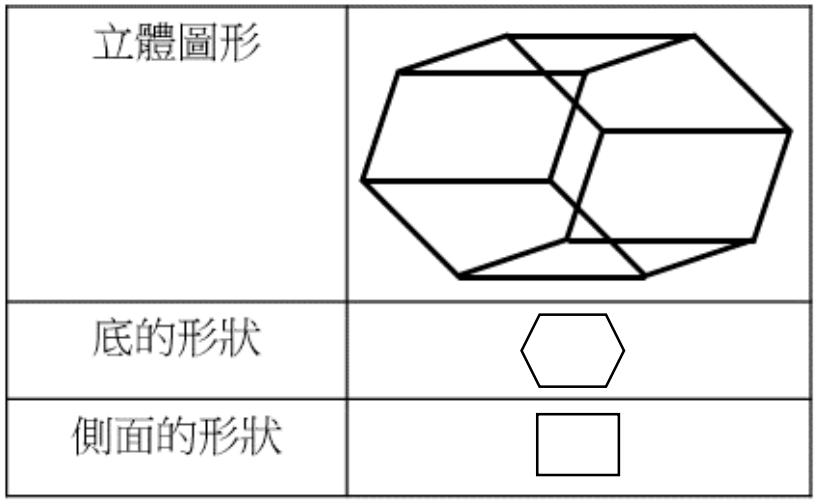
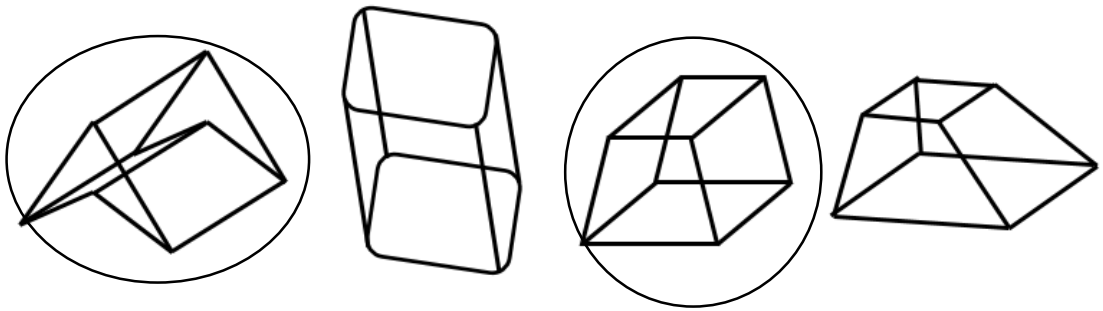
5MK9	 這個圖形的面積是 <u>30</u> cm^2。	5M1	A
5MK10	 每個  都是 1 立方米，上面立體的體積為 <u>11</u> m^3。	5M2	B

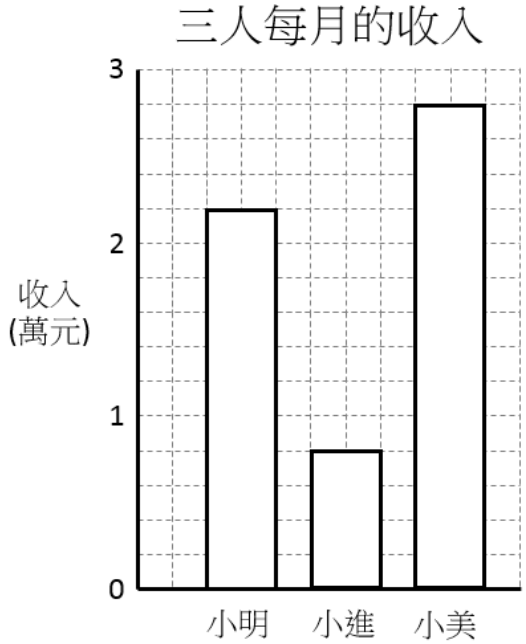
5MK11	 這個立體的體積是 80 cm^2。	5M2	
5MK12	用中國數字寫出以下數字： (可參考以下中文字：億，萬，零) 100 034 005： 一億零三萬四千零五	5N1	B
5MK13	24 534 937 (近似值取至萬位)： 24 530 000 139 469 602 (近似值取至千萬位)： 140 000 000	5N1	A
5MK14	30.203 + 1.89 -–19.02 大約等於 30 + 2 -–20 = 12	5N4	
5MK15	請在橫線上填寫答案。	5N5	

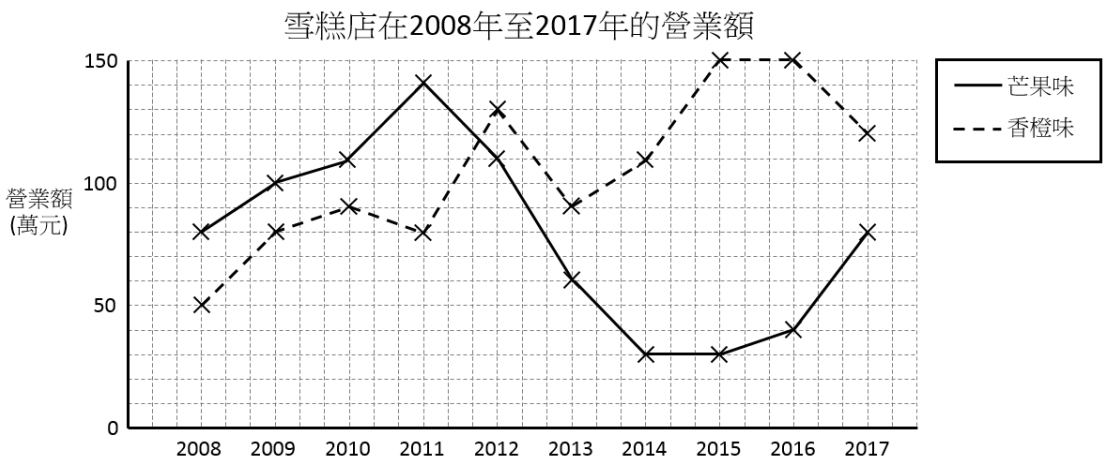
	25.37 × 4.13 大約等於 $\underline{25 \times 4 = 100}$		
5MK16	$84\frac{1}{9} \div 3\frac{9}{11}$ 大約等於 $\underline{84 \div 4 = 21}$	5N6	

5MK17	 圈出答案。 火車站在文具店的（東南 / 東北 / 西南 / 西北）方。	5S1	A

<table>
<tr>
<td>5MK18</td>
<td>請寫出對應的數量。
<table>
<tr><td>立體圖形</td><td>  </td></tr>
<tr><td>底的數量</td><td>2</td></tr>
<tr><td>側面的數量</td><td>5</td></tr>
</table>
</td>
<td>5S2</td>
<td>B</td>
</tr>
<tr>
<td>5MK19</td>
<td>請畫出對應的圖形。</td>
<td>5S2</td>
<td></td>
</tr>
</table>

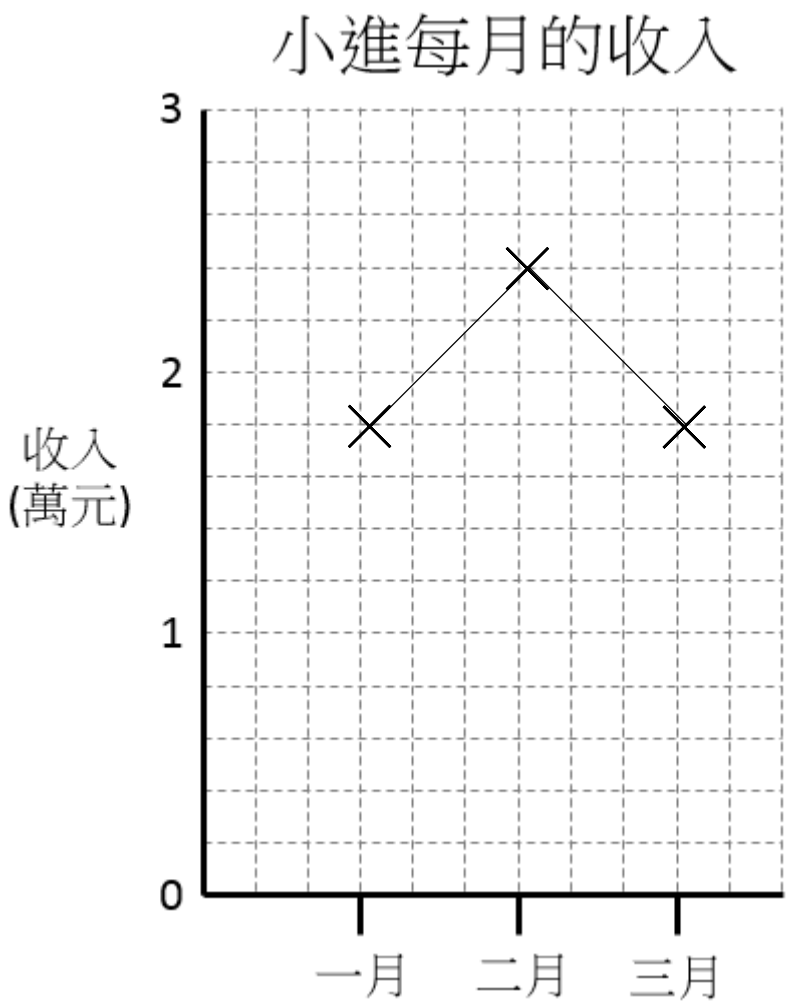
			
5MK20	圈出所有四角柱體。(答案可多於一個。) 	5S2	
6MK1	請以方程式表示以下句子：	6A1	A

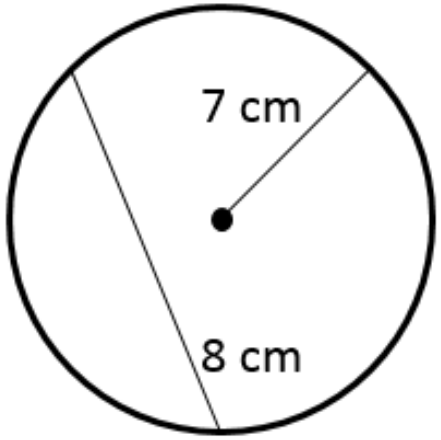
	u 減去 2.8 的結果，再除以 5，是 4.1。 $\frac{u-2.8}{5}=4.1$		
6MK2	計算 37，0， 13，10，12，30 的平均數。<u>17</u>	6D1	
6MK3	 這個棒形圖，每一小格代表 <u>2000</u> 元。	6D2	

<table>
<tr>
<td>6MK4</td>
<td> 

芒果味雪糕在 2011 年所賺得的營業額最多，有 1 400 000 元。</td>
<td>6D3</td>
<td>B</td>
</tr>
<tr>
<td>6MK5</td>
<td>以下是小進每月的收入。
<table>
<tr><td>月份</td><td>一月</td><td>二月</td><td>三月</td></tr>
<tr><td>收入 (元)</td><td>18 425</td><td>23 962</td><td>17 850</td></tr>
<tr><td>湊整至千位</td><td>18 000</td><td>24 000</td><td>18 000</td></tr>
</table></td>
<td>6D3</td>
<td></td>
</tr>
</table>

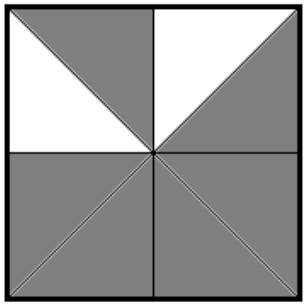
	請完成上面的表格，並根據資料，完成下面的折線圖。		

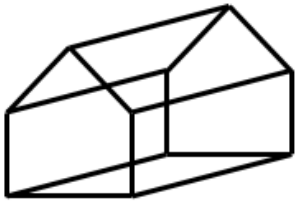


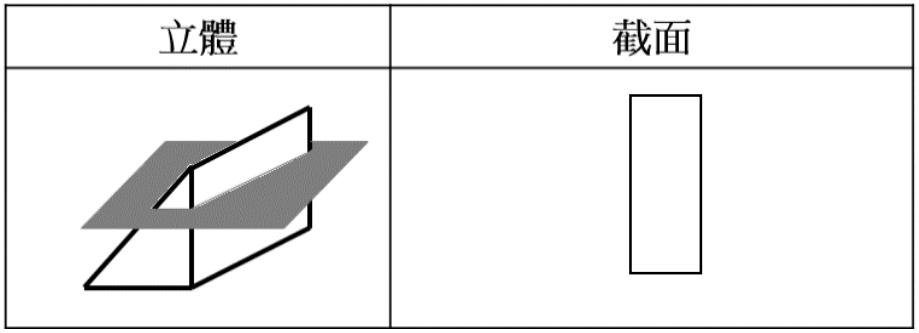
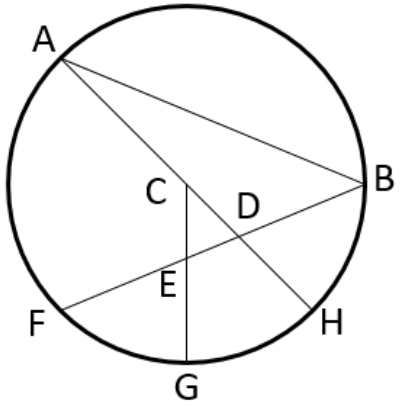
6MK6	5 升 = 5000 立方厘米	6M1	
6MK7	圈出答案。 一瓶大枝裝汽水約 1.25 (L / cm^3 / m^3)。	6M1	
6MK8	長方體水箱長 50 厘米，闊 10 厘米，高 20 厘米。它的容量是 10 升。	6M1	
6MK9	 這個圓形的圓周是 44 cm。 (取 $\pi = \frac{22}{7}$ 。)	6M2	A
6MK10	完成表格。 (取 $\pi = \frac{22}{7}$ 。)	6M2	

<table>
<tr><td></td><td>
<table>
<tr><th>圓形</th><th>資料</th><th>圓周</th></tr>
<tr><td>A</td><td>直徑 42 cm</td><td>$\underline{132}$ cm</td></tr>
</table>
</td><td></td><td></td></tr>
<tr><td>6MK11</td><td>圈出答案。

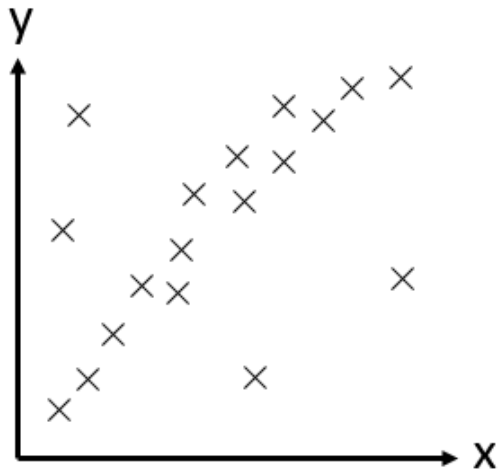
民航客機的平均速率為 900 (m/s / km/h)。</td><td>6M3</td><td></td></tr>
<tr><td>6MK12</td><td>完成表格。
<table>
<tr><td>路程</td><td>時間</td><td>速率</td></tr>
<tr><td>5 米</td><td>20 秒</td><td>0.25 米每秒</td></tr>
</table>
</td><td>6M3</td><td>B</td></tr>
<tr><td>6MK13</td><td>將 $2\frac{1}{4}$ 化為小數： 2.25</td><td>6N2</td><td></td></tr>
</table>

<table>
<tr>
<td>6MK14</td>
<td> 
陰影部份佔整個正方形的 <u>75</u> %。</td>
<td>6N3</td>
<td>B</td>
</tr>
<tr>
<td>6MK15</td>
<td>將 0.207% 化為小數： <u>0.00207</u></td>
<td>6N3</td>
<td></td>
</tr>
<tr>
<td>6MK16</td>
<td>224 的 25% 是 <u>56</u> 。</td>
<td>6N4</td>
<td></td>
</tr>
<tr>
<td>6MK17</td>
<td>完成表格。
<table>
<tr><td>原價</td><td>售價</td><td>折扣</td></tr>
<tr><td>120 元</td><td><u>72</u> 元</td><td>六折</td></tr>
</table>
</td>
<td>6N4</td>
<td>A</td>
</tr>
</table>

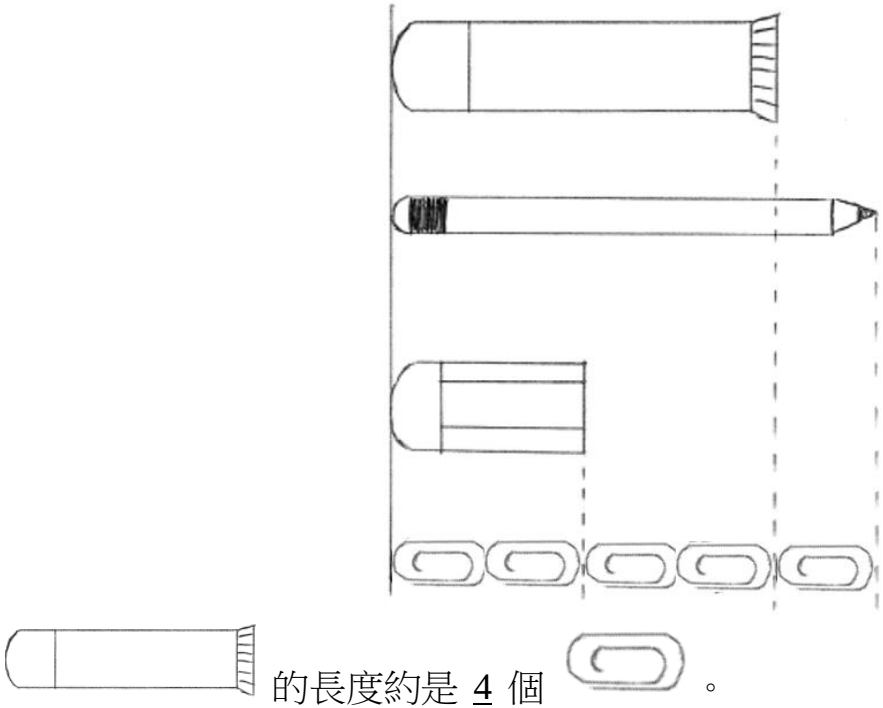
<table>
<tr><td>6MK18</td><td>完成表格，填寫立體圖形的資料。
<table><tr><td>立體</td><td>  </td></tr><tr><td>面的數量</td><td>7</td></tr><tr><td>頂的數量</td><td>10</td></tr><tr><td>棱的數量</td><td>15</td></tr></table></td><td>6S1</td><td></td></tr>
<tr><td>6MK19</td><td>如果將下面的立體，沿灰色的平面分開，所得的截面將會是怎樣? 請在表格中把截面的形狀畫出來。</td><td>6S1</td><td>A</td></tr>
</table>

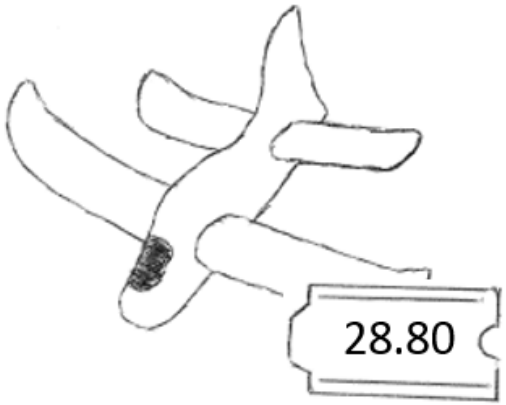
			
6MK20	 請在橫線上填寫答案。 在這個圓形中，AH 是直徑。	6S2	B

JSCMK1	以下哪些算式的答案是負數?請圈出答案。 (-2) + (-3) –(-6)　(-5) + (-5)×(+4) (-2)÷(+4)×(+5)　(+8)×(+2)-(-9)	JSC	
JSCMK2	以下哪些是多項式?請圈出答案。 $x^2 + 3x - 4$　$\frac{1}{y}$ $z - 1$　$w^2 + \sqrt{w}$	JSC	
JSCMK3	用一把準確度為 1cm 的間尺，量度出一個橡皮刷的長度是 5cm。這個橡皮刷的真實長度界乎於 _4.5_ cm 至 _5.5_ cm。	JSC	

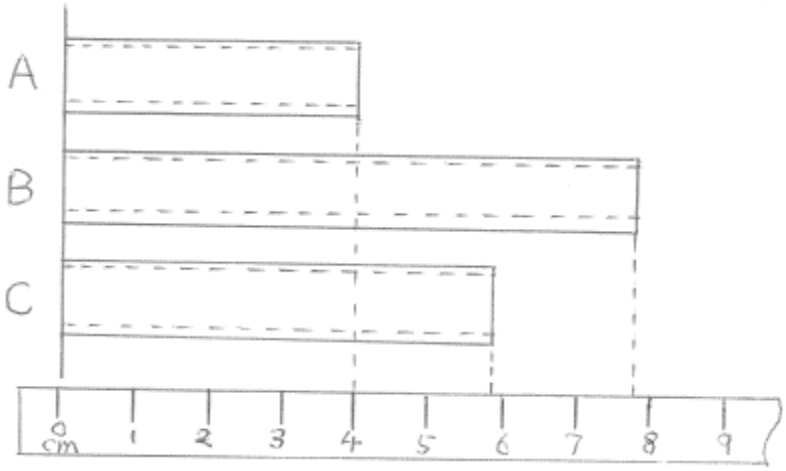
JSCMK4	 這個散點圖顯示 x 與 y 的關係為 (正相關 / 負相關 / 無關係)。	JSC	

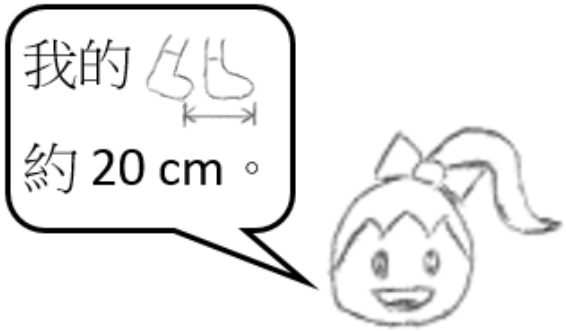
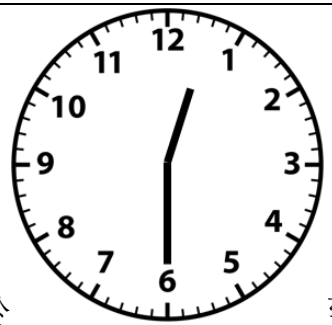
Math Problem Solving (PS)

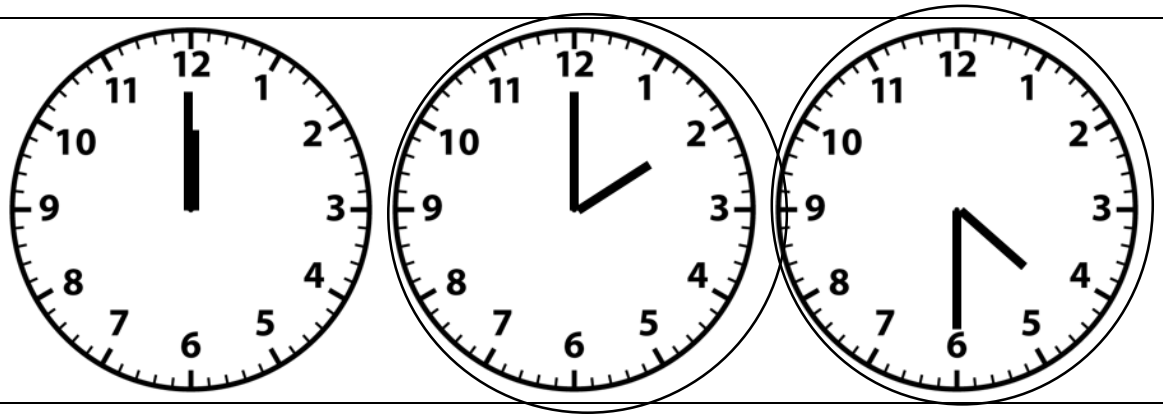
Item Code	Question and Model Answer	Education Bureau learning outcome	Linking set
1PS1	 的長度約是 4 個 。	1M1	
1PS2	小明共有	1M2	B

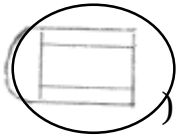


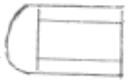
小明 (能夠 / 不能夠) 購買  。(圈出答案。)

1PS3	  黃小姐有 10 元，她買了一個  後，剩下 7 元 6 角。	1M2	A
1PS4		1M3	

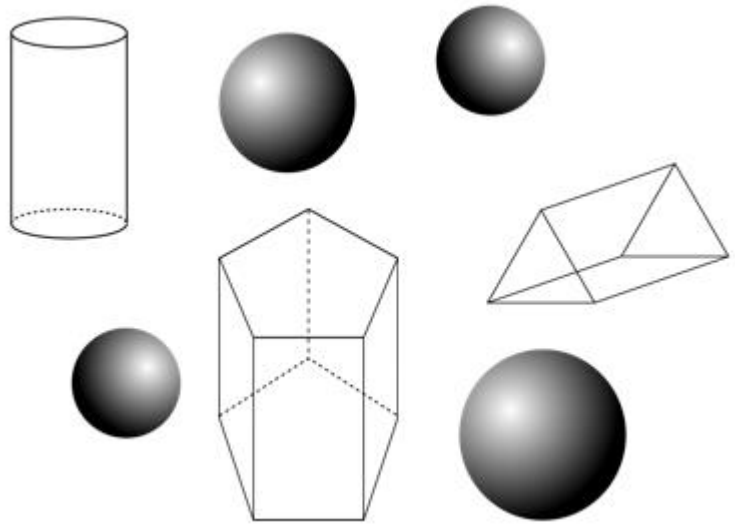
	A 和 C 相差 2 厘米。		
1PS5	 睡房的門和睡床的距離是 4 個  ，睡房的門和睡床相距約 80 cm 。	1M3	
1PS6	 小明與家人於 到達公園。 小明與家人可能在哪些時間離開公園? 請圈出答案。 (答案可多於一個)	1M4	B

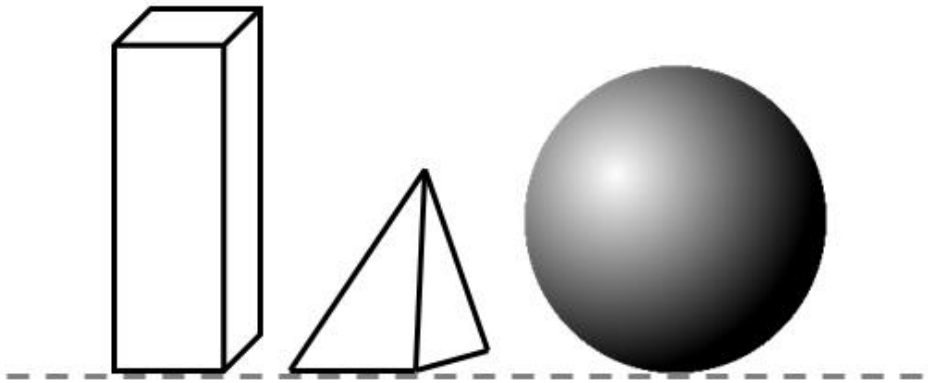
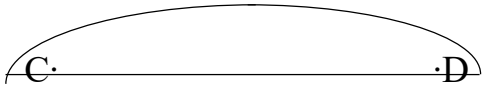
<table>
<tr><td></td><td>  </td><td></td><td></td></tr>
<tr><td>1PS7</td><td>
<table>
<tr><td colspan="7">2017年 12月</td></tr>
<tr><td>日</td><td>一</td><td>二</td><td>三</td><td>四</td><td>五</td><td>六</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>1</td><td>2</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td></tr>
<tr><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td></tr>
<tr><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
鋼琴比賽結果將於第 3 個星期四公佈，即 12 月 21 日。</td><td>1M4</td><td>A</td></tr>
<tr><td>1PS8</td><td>圈出答案。</td><td>1N1</td><td></td></tr>
</table>

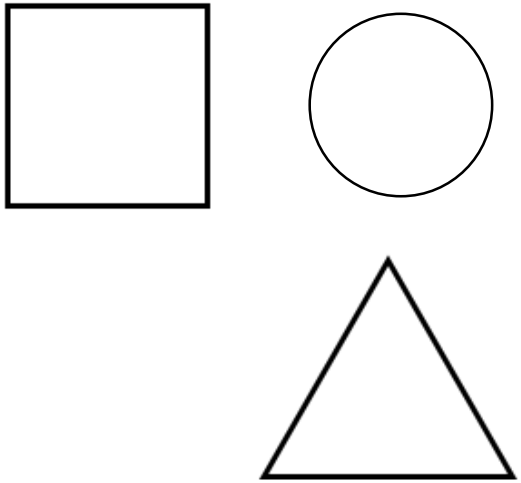
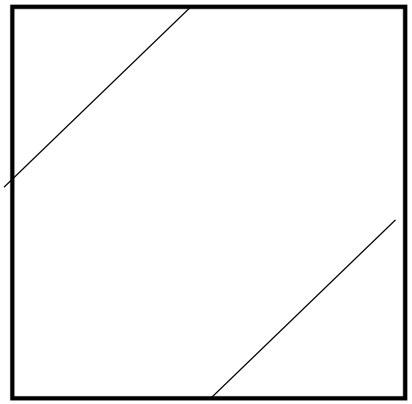
	☆是一個雙數，10+☆是一個 (單數 / 雙數)。		
1PS9	 ( / ) 較多。(圈出答案。)	1N2	
1PS10	把 14 塊  分成 2 份，其中一份有 5 塊  ，另一份有 9 塊  。	1N2	

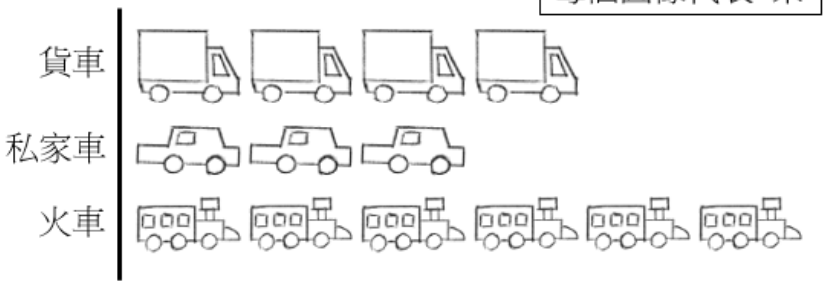
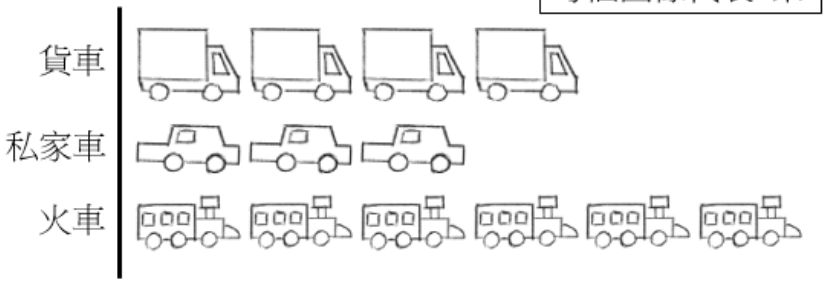
1PS11	小明吃了  4 個，小星吃了  7 個，兩人共吃了  多少個? 列式： 4+7 答案： 11	1N3	
1PS12	家俊畫了 17 枝鉛筆  和 4 個橡皮刷  ，兩種文具相差多少? 列式： 17-4 答案： 13	1N3	
1PS13	哥哥早上做了 8 個  ，下午再多做了 9 個。哥哥共做了多少個  ? 列式： 8+9 答案： 17	1N3	

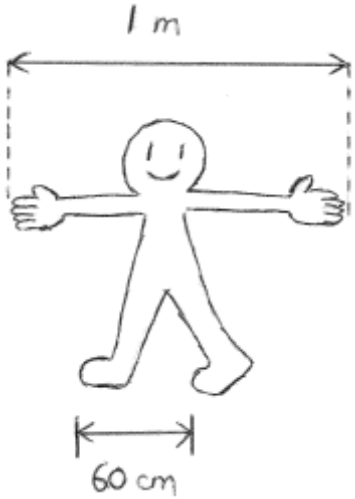
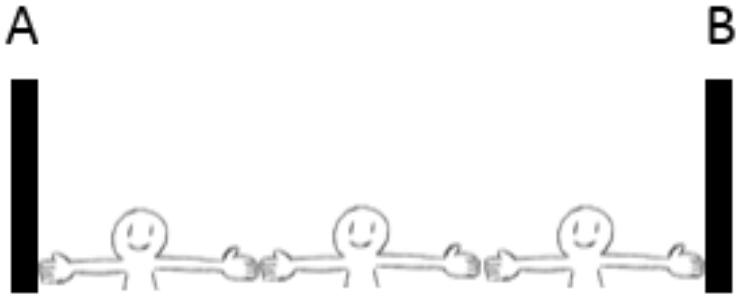
1PS14	小明有  18 個，吃了 12 個後，小明還有  多少個? 列式： 18-12 答案： 6	1N3	B
1PS15	小美，小進和一心三人一同摺  。小美摺了 13 顆，一心摺了 12 顆，小進摺了 15 顆。三人合共摺了  多少顆?。 列式： 13+12+15 答案： 40	1N5	A

1PS16	 這裡有 4 個球體和 3 個柱體。	1S1	

1PS17	 最高的立體圖形是（球體／柱體／錐體）。(圈出答案。)	1S1	
1PS18	 請分別地用一條直線和一條曲線，把 C，D 兩點連起來。	1S2	

1PS19	 從你的方向看，在正方形的右面及三角形上面畫出一個圓形。	1S3	A
1PS20		1S3	B

	用線將上面的圖形，分成一個六邊形和兩個三角形。		
2PS1	小軒的玩具車數量 每個圖像代表1架  小軒最多的玩具車比最少的多 3 架。	2D1	A
2PS2	小軒的玩具車數量 每個圖像代表1架  	2D1	B

	小軒總共有車 13 架。		
2PS3	  A，B 相距約 3 m。	2M1	

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>2PS4</td><td>媽媽在 10:15pm 開始看電視，看了 45 分鐘後把電視關掉，當時是 下 午 11

時 0 分。</td><td>2M2</td><td></td></tr>
<tr><td>2PS5</td><td>
<table>
<tr><td colspan="7">2月</td></tr>
<tr><td>日</td><td>一</td><td>二</td><td>三</td><td>四</td><td>五</td><td>六</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td></tr>
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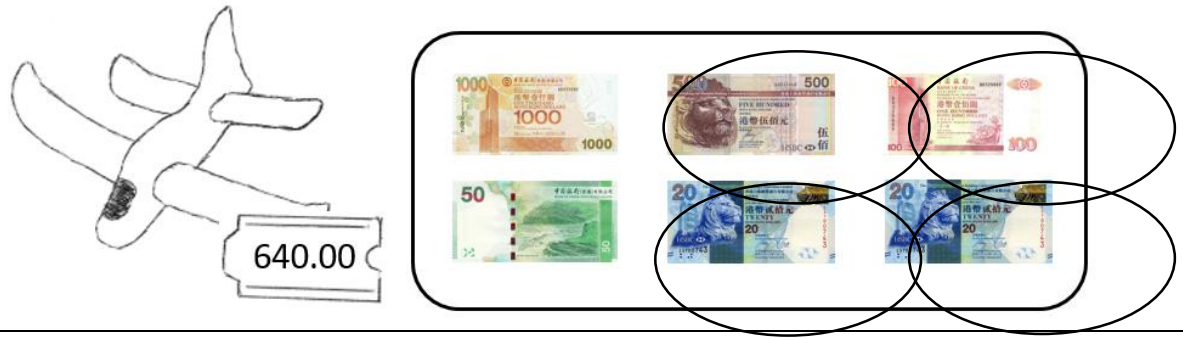
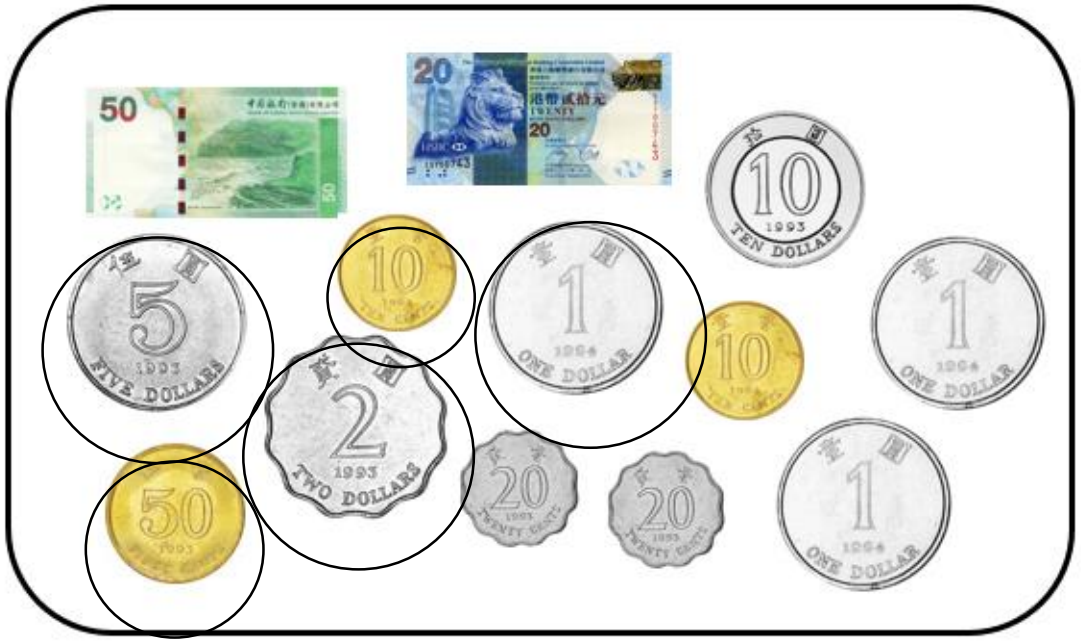
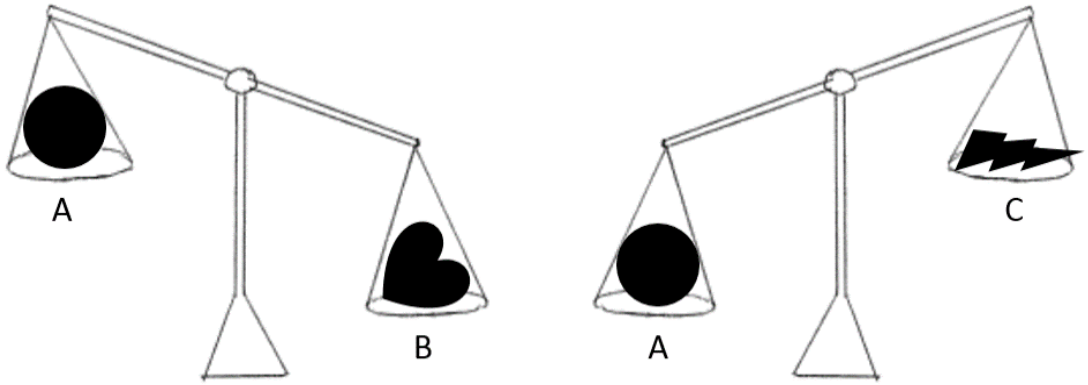
<tr><td>27</td><td>28</td><td>29</td><td></td><td></td><td></td><td></td></tr>
</table>

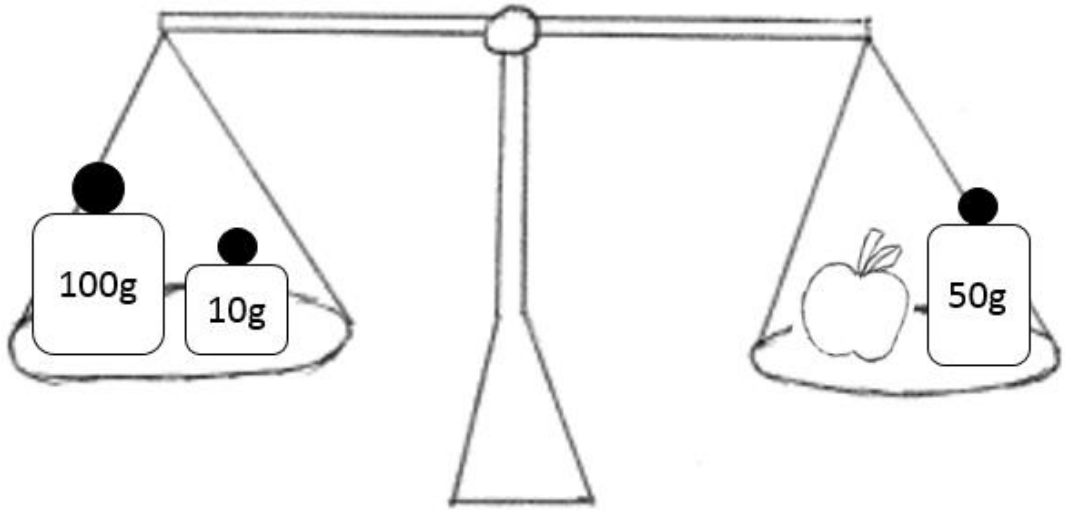
爸爸在這個月和下一個月都在外地工作，他這次在外地工作共 60 天。</td><td>2M2</td><td></td></tr>
<tr><td>2PS6</td><td>圈出剛好要付的款項。</td><td>2M3</td><td></td></tr>
</table>

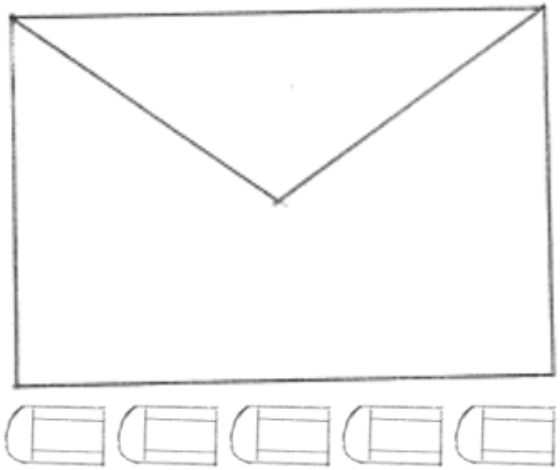
			
2PS7	小新買了羽毛球拍和乒乓球拍各一塊，付了 80 元，應找回多少? 請圈出適當的貨幣。	2M3	

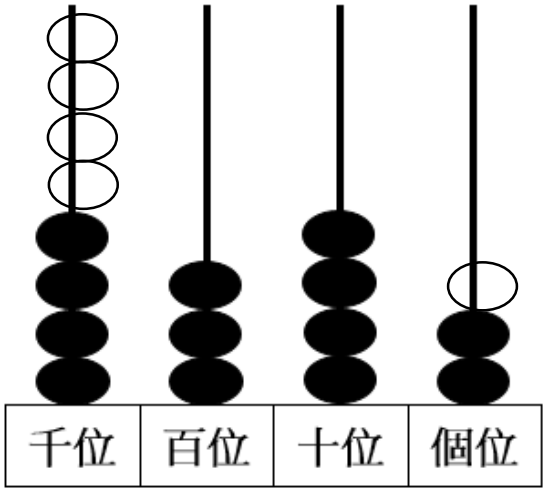


(8 元 6 角)

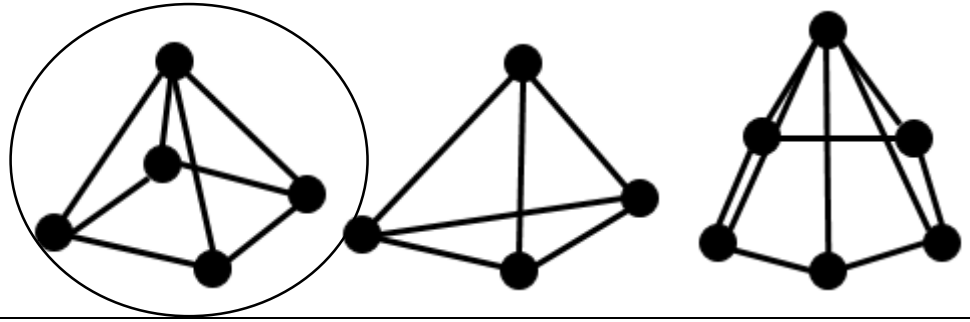
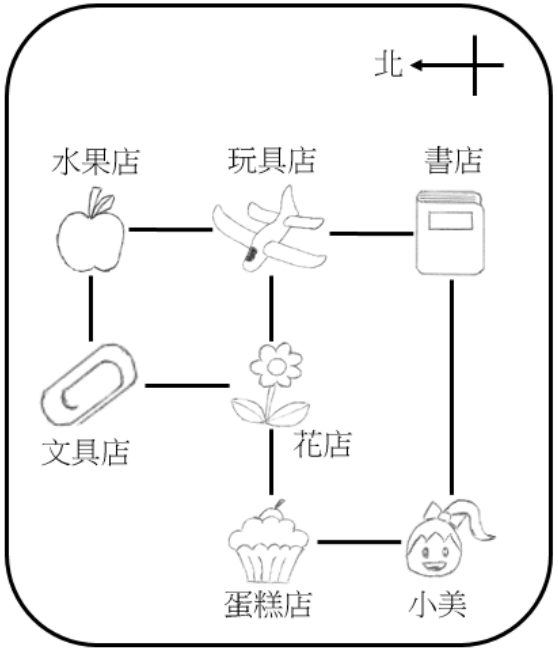
2PS8	 將 A，B，C 由重輕排列。 B ， A ， C (最重) (最輕)	2M4	B

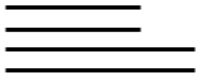
2PS9	  重 <u>60</u> 克。	2M4	A
2PS10	把 2，0，9 排列成最小的三位雙數：<u>290</u>	2N1	
2PS11	小賣部有 92 盒蘋果汁，蘋果汁比牛奶少 118 盒，牛奶比檸檬茶少 165 盒。小賣部有檸檬茶多少盒?	2N2	

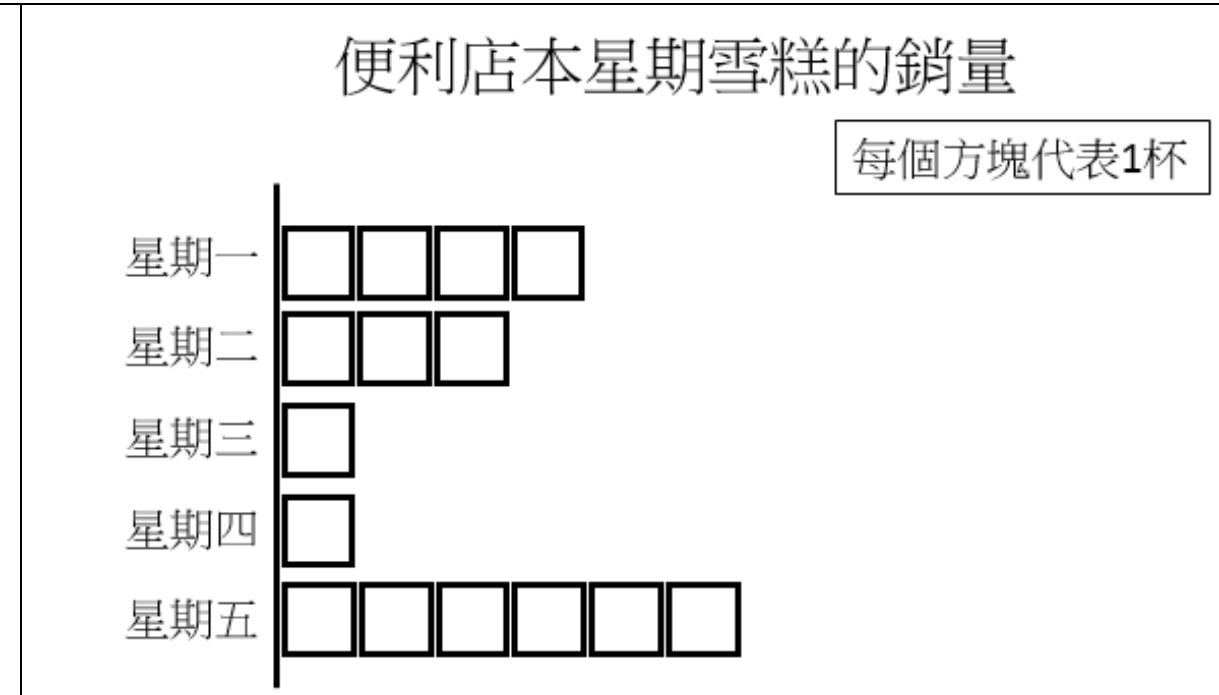
	列式：92+118+165 答案：375		
2PS12	 每個橡皮刷長 4 厘米，這個信封約長 20 厘米。	2N3	
2PS13	兩個一位數相乘的積是 14。這兩個一位數是 2 和 7 。	2N3	A
2PS14	在算柱上加畫 5 粒算珠，組成最大的四位單數。	2N4	B

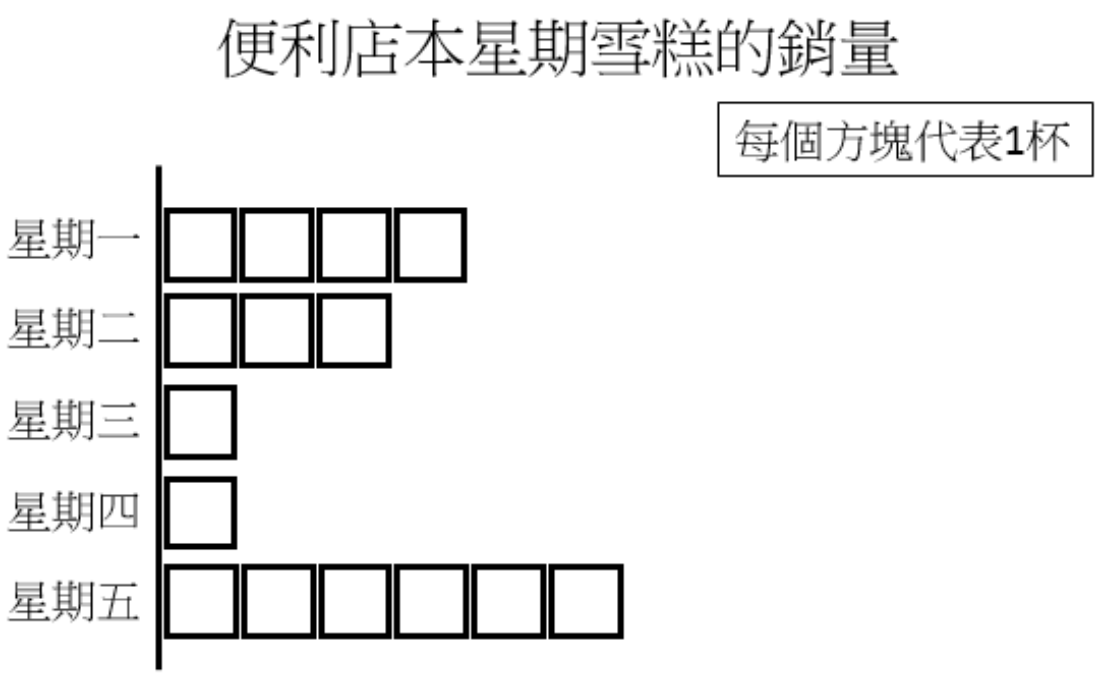
			
2PS15	水果店買入 279 個蘋果後，售出 66 個。如果水果店現有蘋果 310 個，原有蘋果多少個? 列式： 310+66-279 答案： 97	2N5	
2PS16	哥哥付 42 元買 7 包朱古力，每包朱古力有 8 粒。每包朱古力售多少元?	2N6	

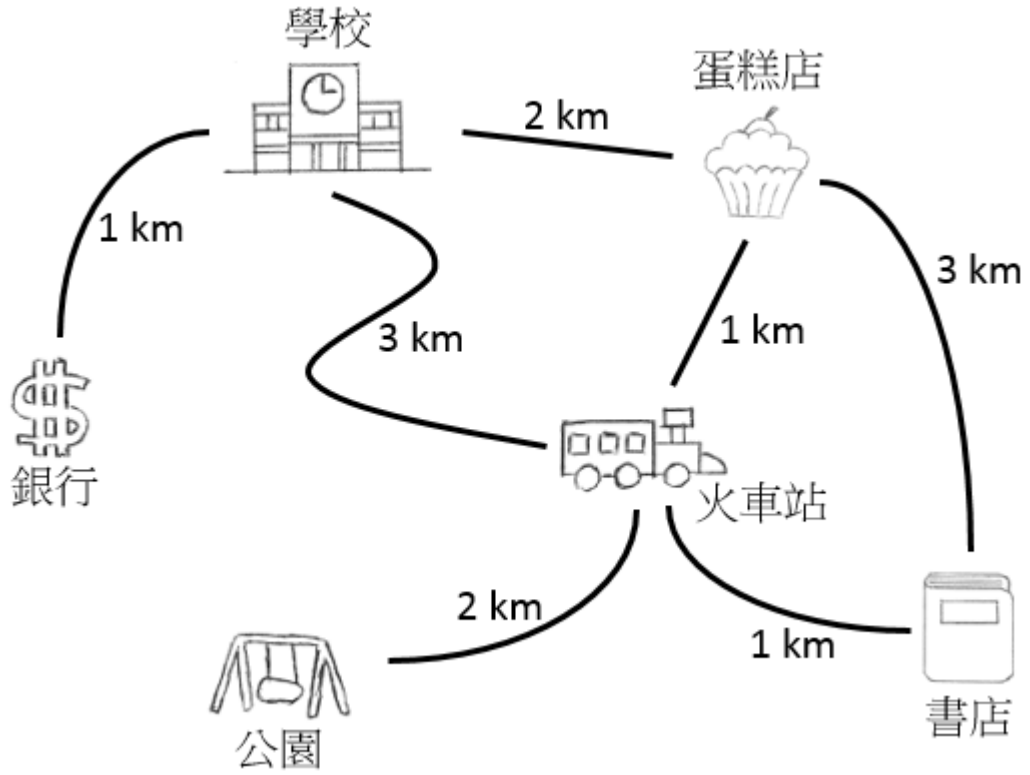
	列式：42÷7 答案：6		
2PS17	媽媽有 54 個橙，每半打裝成一袋。她最少需要袋多少個，才可把全部橙裝進袋內? 列式：54÷6 答案：9	2N6	
2PS18	 使用上面全部的泥膠和竹枝，可以製作下面哪一個角錐體? 請圈出答案。	2S1	B

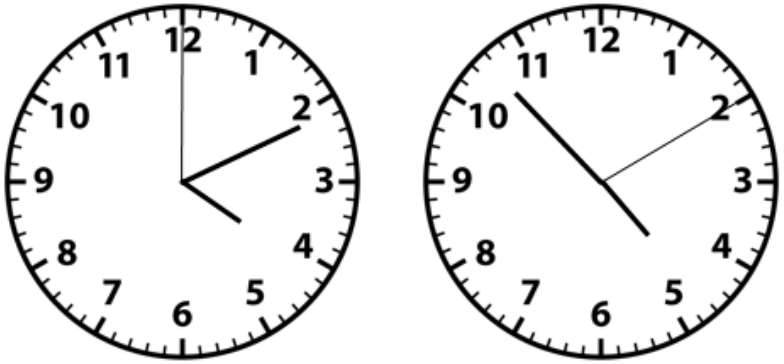
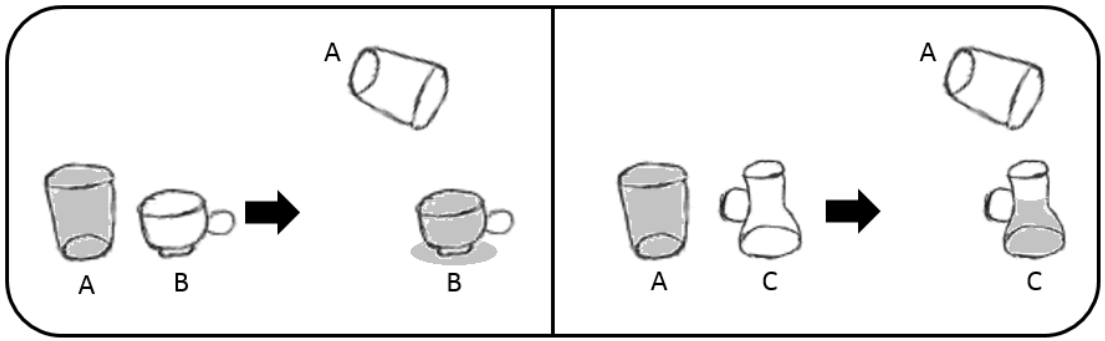
			
2PS19		2S3	A

	小美向 北 方走，會到達蛋糕店，再向 東 方走，會到達玩具店。		
2PS20	圈出答案。 用  可砌出 (正方形 / 長方形 / 菱形)。	2S4	

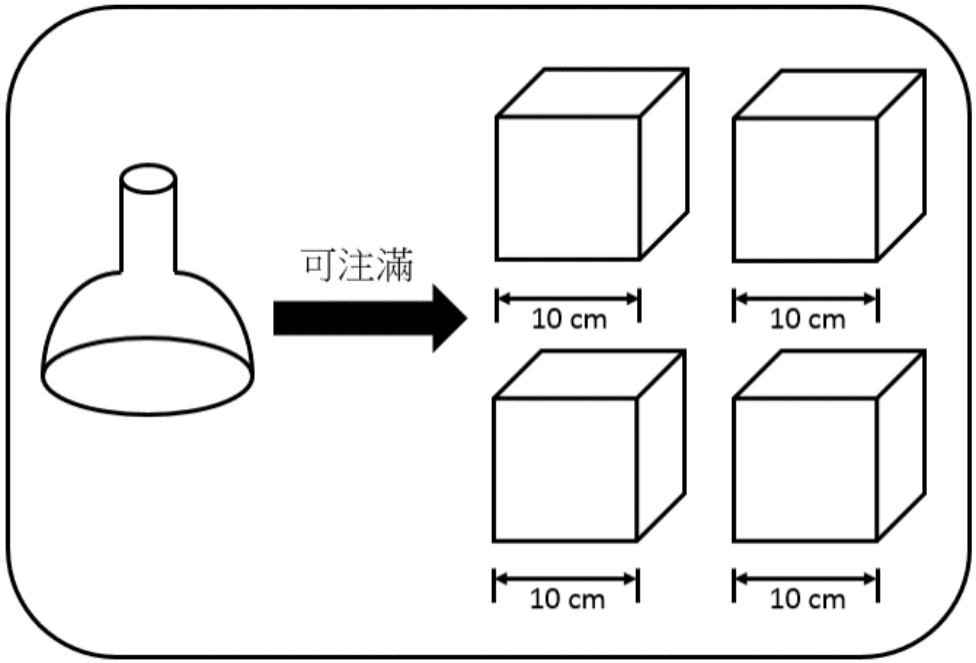
3PS1	 星期 五 的雪糕銷量是 星期 二 的 2 倍。	3D1	A

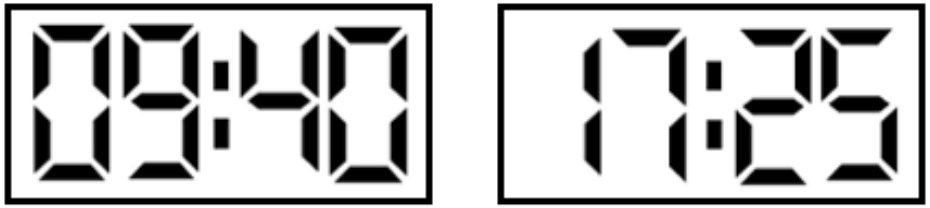
3PS2	 這個星期，便利店每日雪糕銷量平均為 3 杯。	3D1	B

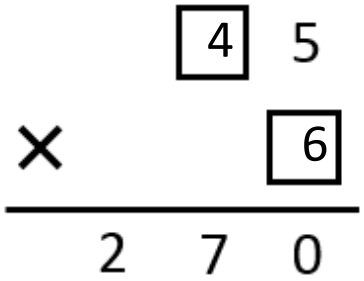
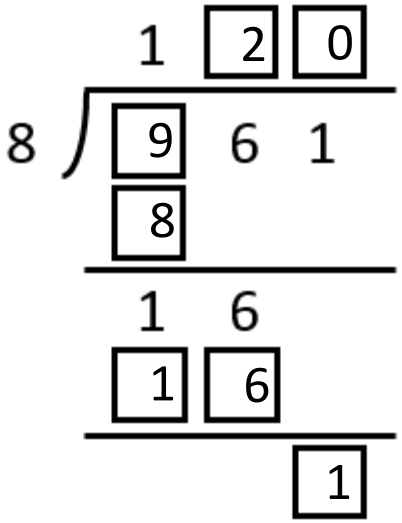
3PS3	 妹妹從銀行，經學校和蛋糕店，走到書店。妹妹所走的路比最短的路程長 1 公里。	3M1	

3PS4	以下是小美今天去公園玩耍的開始和完成時間。  開始時間　　完成時間 小美花了 <u>42</u> 分 <u>10</u> 秒在公園玩耍。	3M2	
3PS5		3M3	

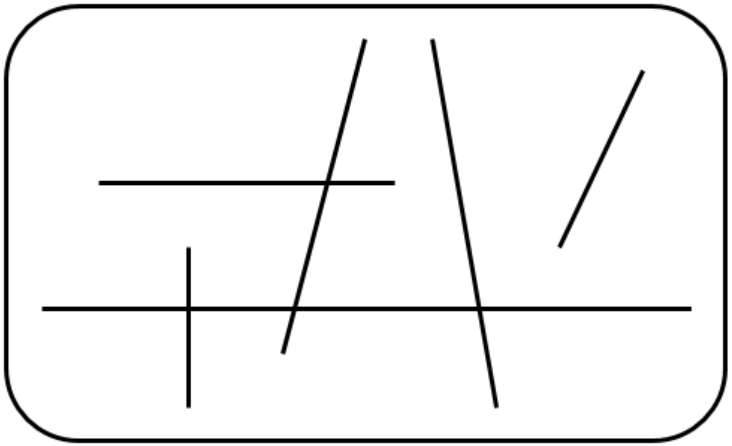
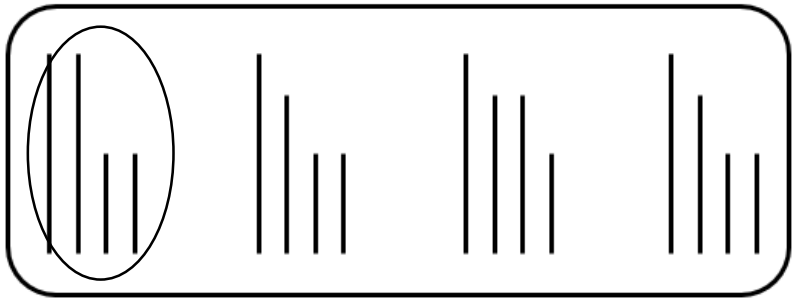
	把 A，B，C 依容量由小至大排列。 杯子 B ， 杯子 A ， 杯子 C (最小) (最大)		
3PS6	盛滿水的  可注滿  盛滿水的  可注滿  盛滿水的  可注滿  盛滿水的  可注滿 2 個 	3M3	

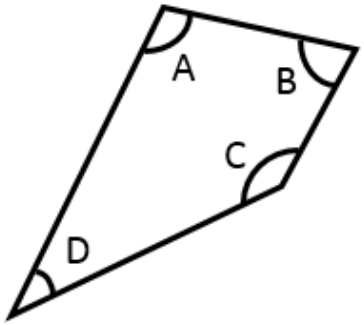
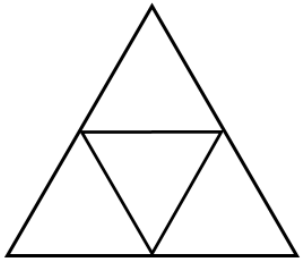
3PS7	  的容量是 4 升。	3M3	A
3PS8	以下是叔叔昨天上班和下班時間。	3M4	B

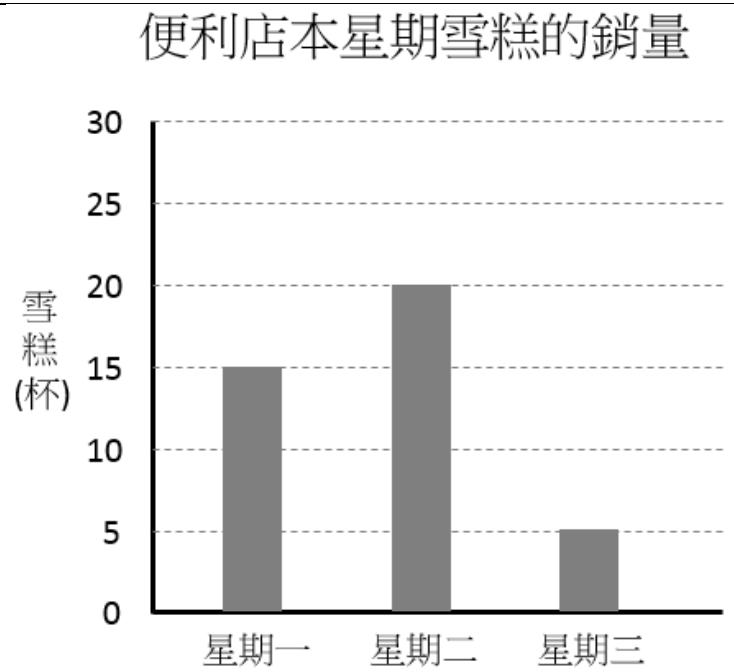
	 上班時間　　下班時間 叔叔昨天共上班 7 小時 45 分。		
3PS9	將 2，7，0，3，9 組成最小的五位雙數。 23790	3N1	
3PS10	請在方格內填寫數字，使算式成立。 5 1 5 7 − 3 6 4 4 7 9 3	3N2	B
3PS11	請在方格內填寫數字，使算式成立。	3N3	A

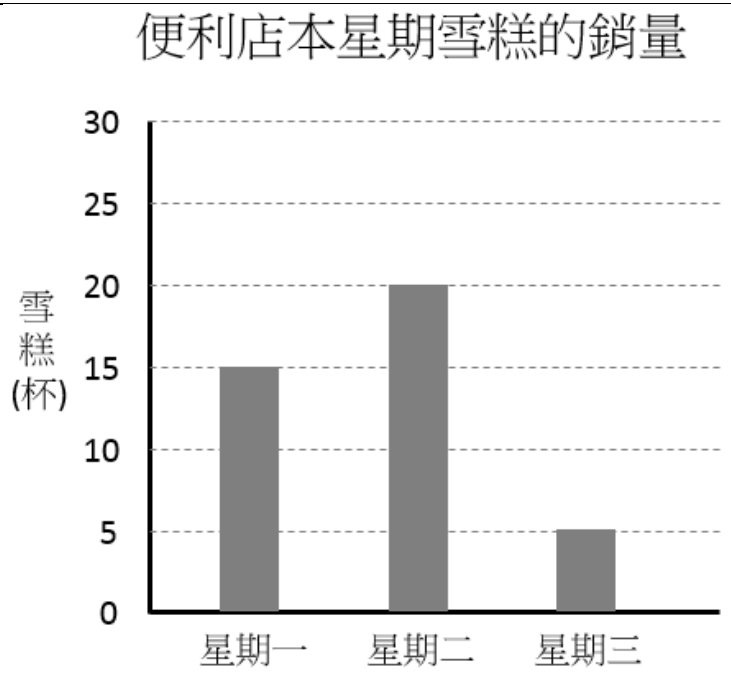
			
3PS12	請在方格內填寫數字，使算式成立。 	3N4	

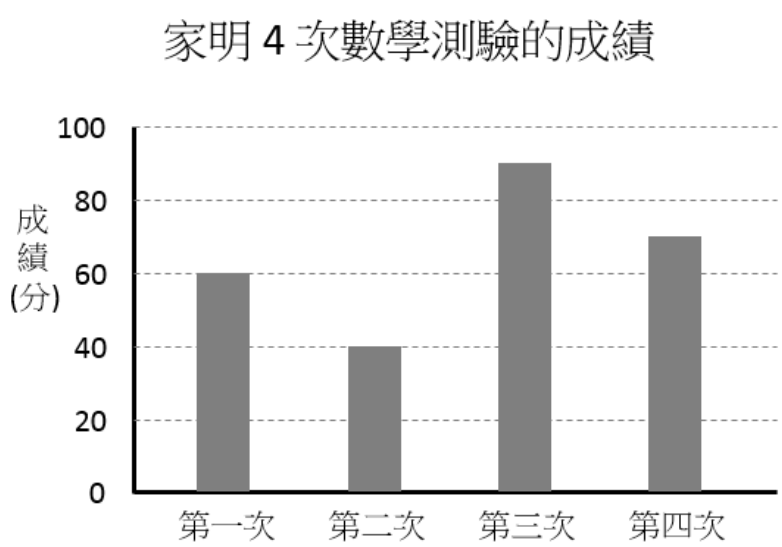
3PS13	緩跑徑長 600 米，妹妹來回跑了兩次，再多跑 125 米。妹妹共跑了多少米? 列式： 600x4+125 答案： 2525	3N5	
3PS14	貨車可載重 600 公斤，一箱蘋果重 27 公斤，一箱香蕉重 36 公斤，把蘋果和香蕉各 9 箱放進貨車會超重嗎? 為什麼? 列式：(27+36)x9 = 567 答案： 不會	3N5	
3PS15	有一盒餅乾，小明吃去 $\frac{2}{15}$ ，小美吃去 $\frac{2}{10}$ ，小欣吃去 $\frac{3}{10}$。 (小明 / 小美 / 小欣) 吃的餅乾最多。	3N6	

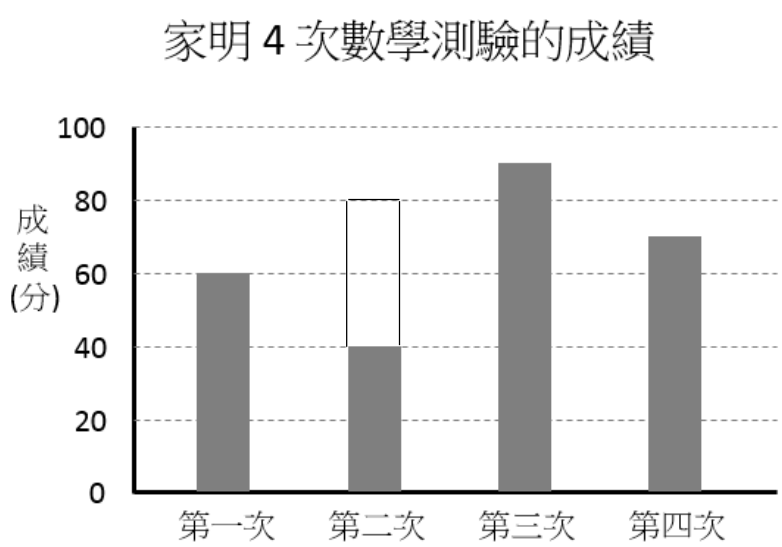
3PS16	 當中的直線，有平行線 1 對，垂直線 1 對。	3S1	
3PS17	下列哪一組線條，可以拼成一個平行四邊形? 請圈出答案。 	3S2	A

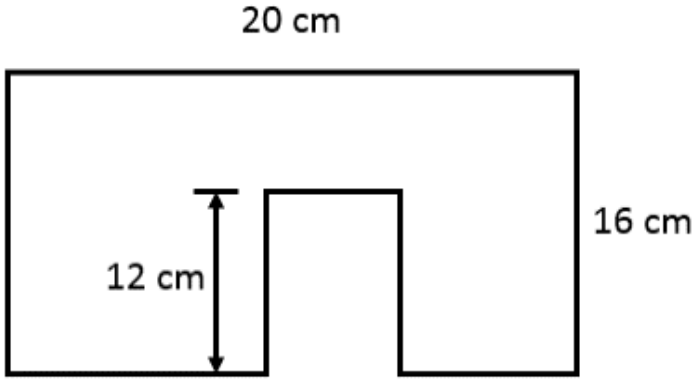
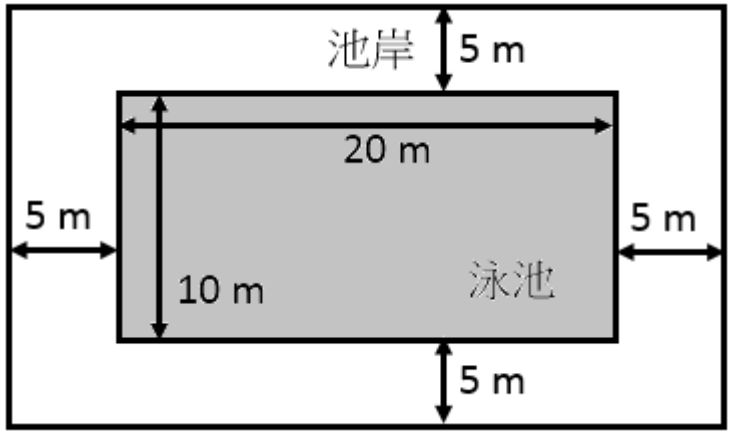
3PS18	 把角 A，B，C，D 由大至小排列。 角 C ， 角 A ， 角 B ，角 D (最大) (最小)	3S3	B
3PS19	 這個圖形可數出 5 個三角形。	3S4	

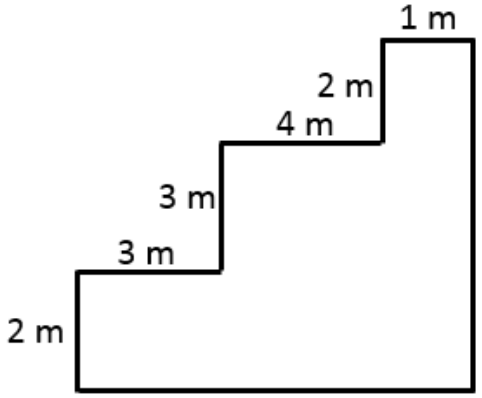
3PS20	 用以上兩條線條製作一個等腰三角形，還需要一條 5 cm 的線條。	3S4	
4PS1	 雪糕一杯售 5 元，便利店在星期一至星期三，賣出的雪糕共收得 200 元。	4D1	A

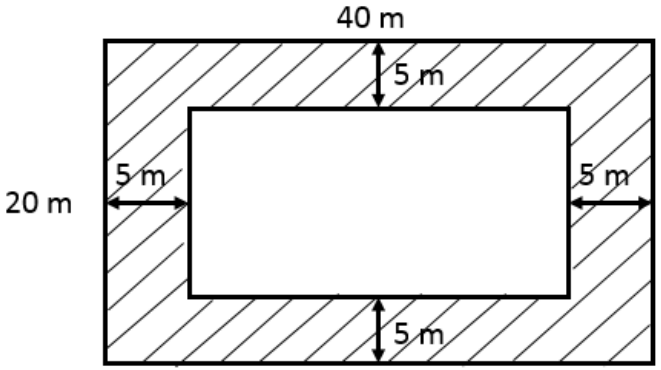
4PS2	 星期三的雪糕銷量是星期一的 $\frac{1}{3}$ 。(請在方格內填寫數字。)	4D1	B

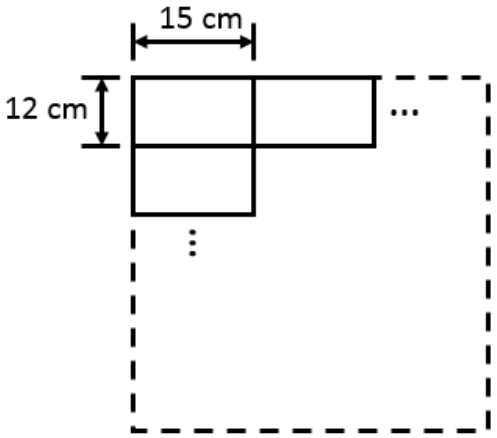
4PS3	 如果每次測驗合格的分數是 60 分，家明有 __3__ 次測驗合格。	4D1	

4PS4	 老師後來發現在計錯了家明第二次數學測驗總分，真正的分數應該比圖上面的高 40 分。請在上圖，加長第二次測驗的棒，以表示出真正的分數。	4D1	

4PS5	 這個圖形的周界為 96 cm。	4M1	A
4PS6		4M1	B

	這是一個泳池的平面圖。泳池長 20 m，闊 5 m，池岸寬 5 米。池岸外圍的周界為 <u>100</u> m。		
4PS7	 這個圖形的面積為 <u>30</u> m^2。	4M2	

4PS8	 陰影部份的面積為 500 m^2。	4M2	
4PS9	一張長 24 cm，闊 18 cm 的紙，剛好被剪成多個相同大小的正方形，而沒有剩餘。以下哪一個不可能是正方形的面積? 請圈出答案。 4 cm^2　　9 cm^2　　(16 cm^2)　　36 cm^2	4M2	
4PS10	請在橫線填上一個數字。 62 + 9<u>8</u> 的結果，可以被 2 和 5 整除。	4N1	

4PS11	312 個學生參觀飲品廠，飲品廠預備了紀念品 12 盒，每盒有紀念品 15 個。你認為這些匙扣足夠分給每一個學生嗎? 為什麼? 列式： 12x15=180 答案： 不夠	4N2	
4PS12	小進用一些長 15 厘米，闊 12 厘米的長方形拼出一個正方形 (如圖)。  這個正方形的邊長最短是 60 厘米。	4N5	

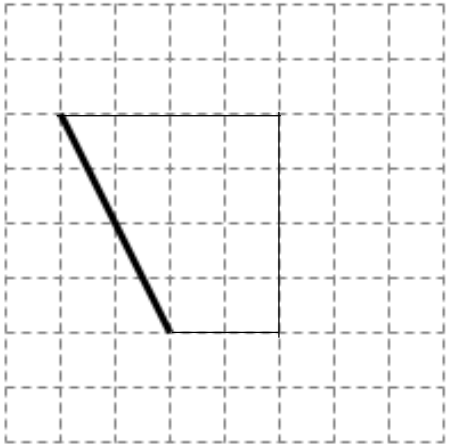
<table>
<tr>
<td>4PS13</td>
<td>以下是班際問答比賽的得分，但是其中 4D 班的答對題目的方格被塗污了。

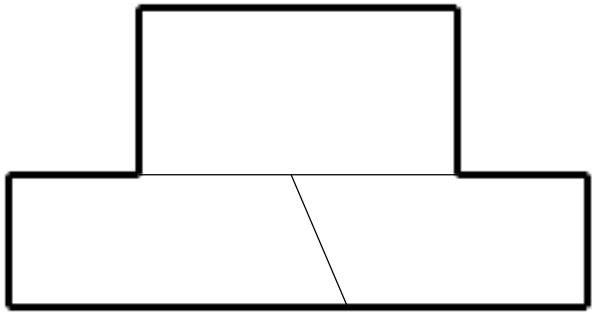
<table>
<tr><td>班別</td><td>4C</td><td>4D</td></tr>
<tr><td>答對 (題)</td><td>10</td><td>  </td></tr>
<tr><td>答錯 (題)</td><td>6</td><td>7</td></tr>
</table>

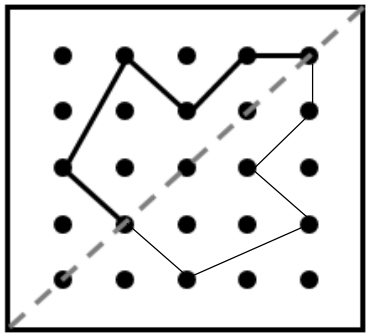
比賽問始時，每班的底分為 100 分。答對 1 題加 10 分，答錯一題扣 5 分。

如果 4D 班的總分是 225 分，那個被塗污的方格內的數字是 <u>16</u> 。</td>
<td>4N6</td>
<td>A</td>
</tr>
<tr>
<td>4PS14</td>
<td>請在下面算式中的□上，適當地加上「＋」,「－」,「×」或「÷」，使算式成立。 (也可適當地加上括號（）。)

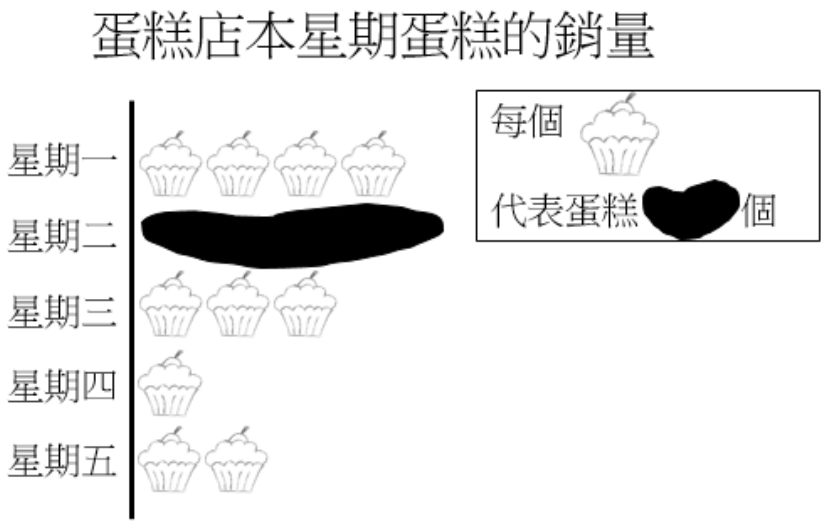
$56 \square (3 \square 4 = 8$</td>
<td>4N6</td>
<td>B</td>
</tr>
</table>

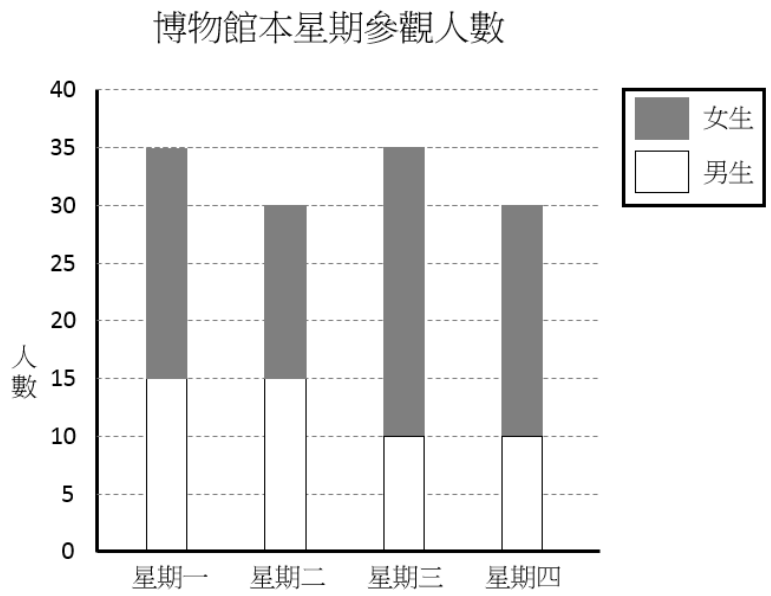
4PS15	0.2●5 在上面的小數中，其中的一個數字被塗污了。已知這個小數比 0.238 大，那麼 ●中的數字最小可以是 4 。	4N8	
4PS16	在方格紙上加上直線，作成一個有兩個直角的梯形。 	4S1	

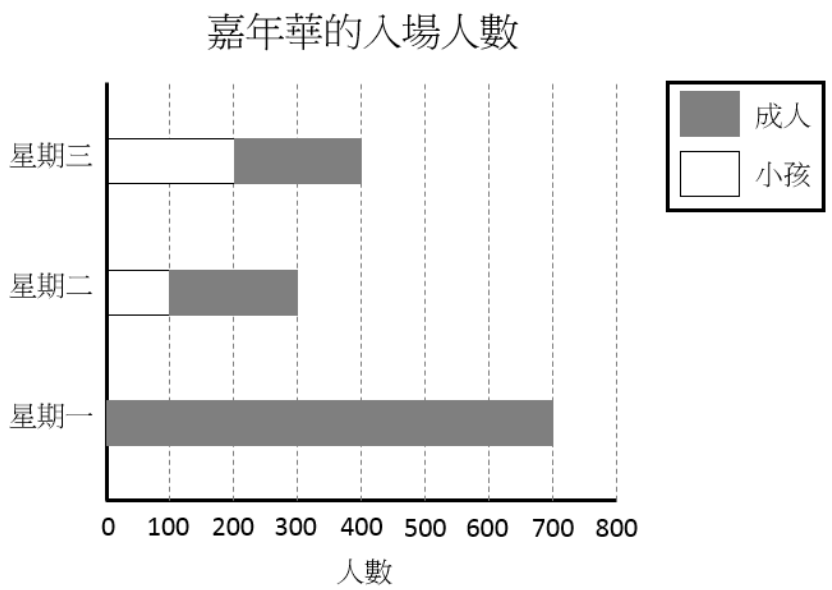
4PS17	小文有兩條 6cm 的木條，若加上兩條 3cm 的木條，可組成以下哪些圖形? 請圈出答案。 (答案可多於一個。) 平行四邊形　梯形　菱形　長方形	4S1	A
4PS18	 用直線將上面的圖形，分成 1 個長方形和 2 個梯形。	4S2	B
4PS19		4S2	

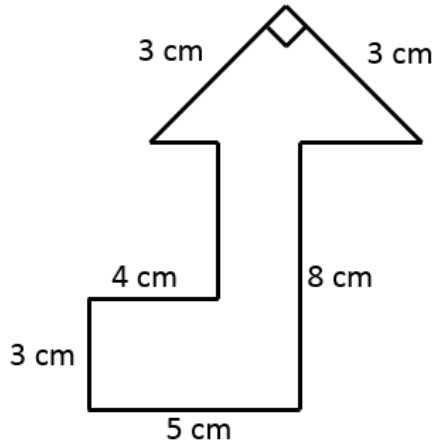
	以上兩個梯形，可砌成以下哪些圖形? 請圈出答案。 (答案可多於一個。) 平行四邊形　梯形　長方形　六邊形		
4PS20	以虛線為對稱軸，完成下面圖形。 	4S3	
5PS1	小明有 u 張貼紙，小進有的是小明的 3 倍加 6 張。小進有貼紙 3u+6 張。(用代數式表示。)	5A1	
5PS2	有糖果 240 粒，小美取去 y 粒後，再把餘下的平均分成 5 份，每份糖果有 $\frac{240-y}{5}$ 粒。(用代數式表示。)	5A1	B

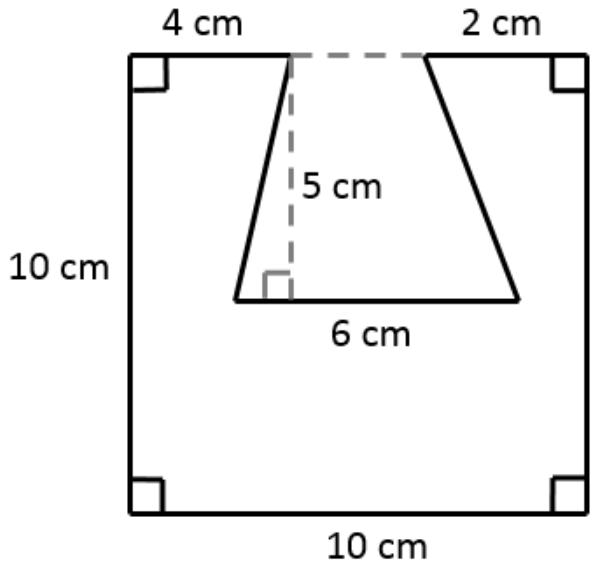
	_____		
5PS3	某數比 102 大，它與 102 的差是 9。那個數是什麼? (請用方程式解決問題。) 列式： 設 y 為那個數 y-102=9 y=111 答案： 111	5A2	A
5PS4	每包朱古力售 30 元，小軒買了 4 包後，還有 20 元。小軒原本有多少元? (請用方程式解決問題。) 列式： 設 y 為小軒原本有的錢 y-4x30=20 y=140 答案： 140	5A2	
5PS5	以下是蛋糕店本星期蛋糕銷量的象形圖。但當中有些地方被塗污了。	5D1	

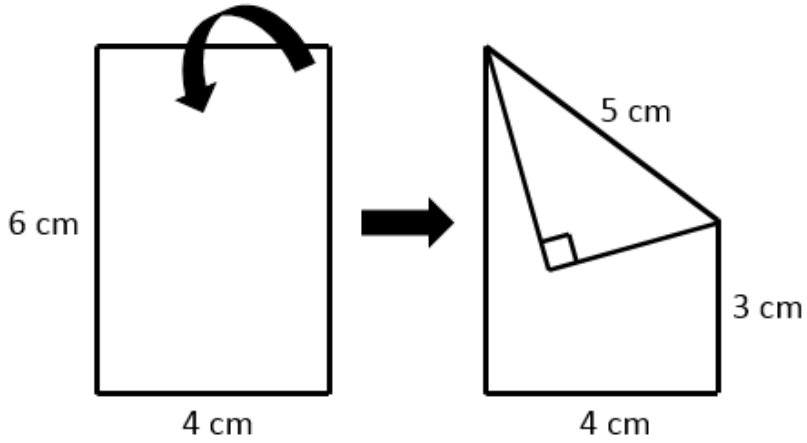
	 如果蛋糕店本星期總共賣出蛋糕 110 個，那麼星期二的項目上，可以有 0/1 個。		
5PS6	以下是博物館本星期的參觀人數。	5D2	

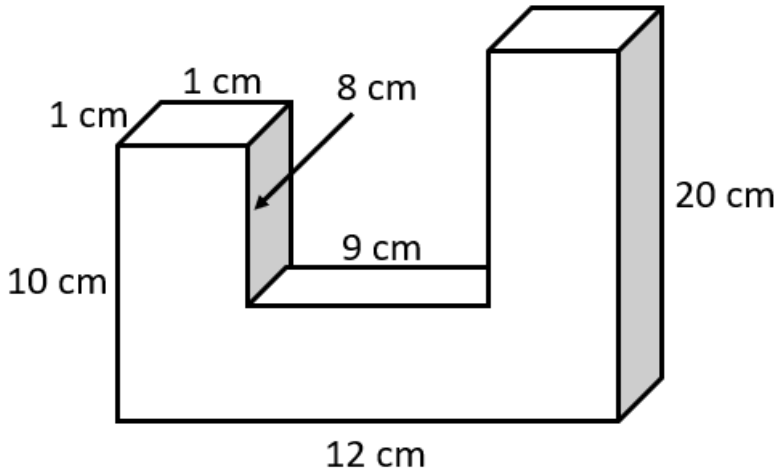
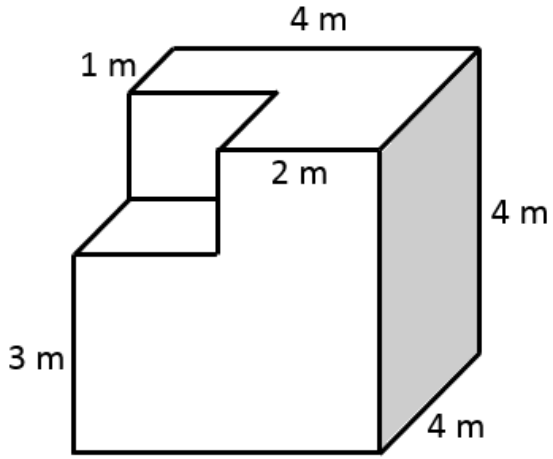
	 星期 四 當天，男生的人數剛好是女人的一半。		
5PS7	以下是嘉年華三天的入場人數。	5D2	A

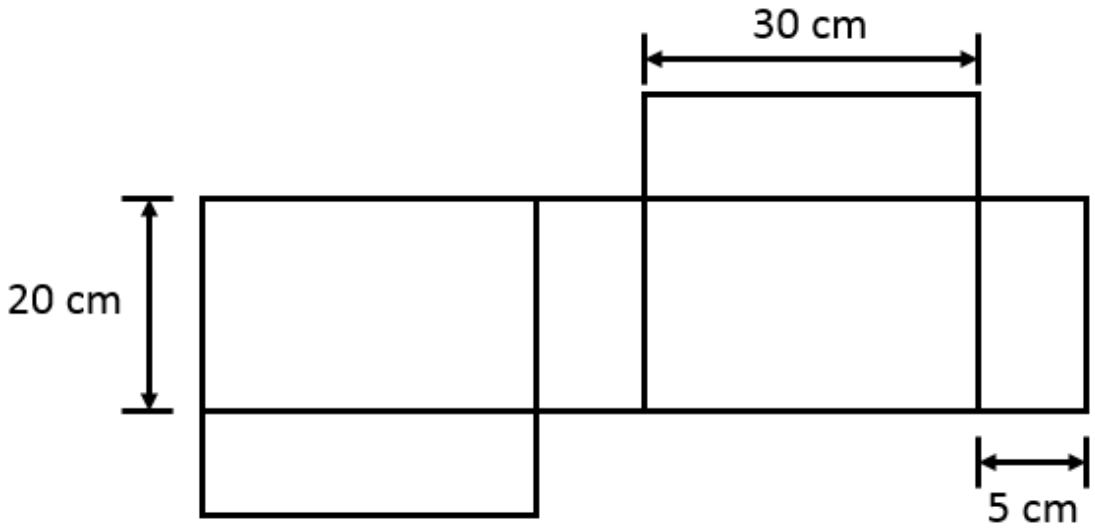
	 如果成人的票價為 20 元，小孩的票價為 10 元，嘉年華三天總共收得入場費 25000 元。		

5PS8	 這個圖形的面積為 <u>24.5</u> cm^2。	5M1	

5PS9	 這個圖形的面積為 <u>75</u> cm^2。	5M1	
5PS10	一個梯形的面積是 56 cm^2，上底是 5 cm，高是 7 cm，下底是 <u>11</u> cm。	5M1	

5PS11	 如圖所示，把一張長方形紙摺了一角，摺起的部分是一個三角形。這個三角形的面積是 <u>6</u> cm^2。	5M1	

5PS12	 這個立體的體積是 <u>68</u> cm^3。	5M2	
5PS13		5M2	A

	這個立體的體積是 58 cm^3。		
5PS14	 利用這個摺紙圖樣，摺出來的立體圖形的體積是 3000 cm^3。	5M2	B
5PS15	以下是小進三次數學小測的成績。	5N2	

數學小測	總題數	答對題數
第一次	10	4
第二次	15	12
第三次	20	17

如果每一次的小測當中，各題的分數都是一樣，小進在哪一次的成績最好?

為什麼?

列式：

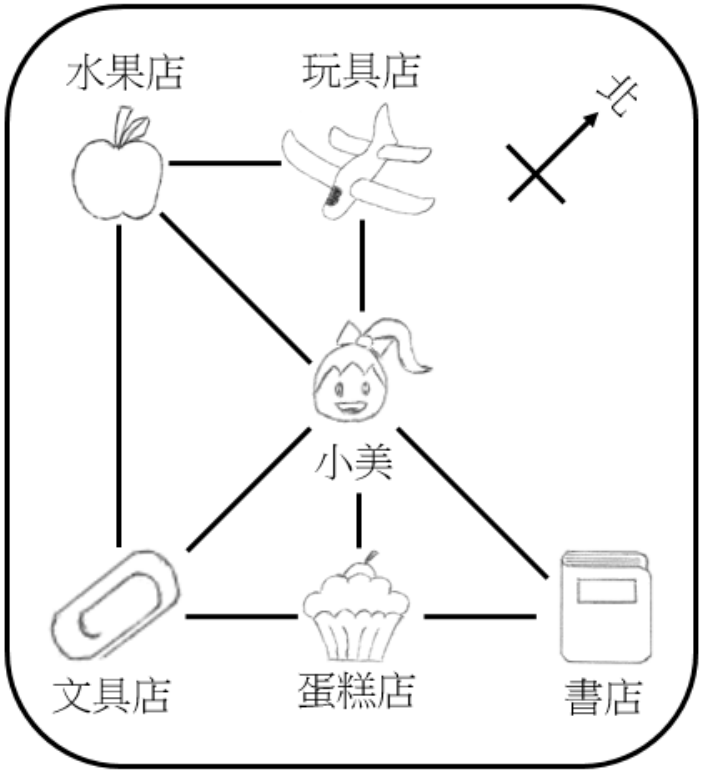
第一次: 4/10 = 0.4

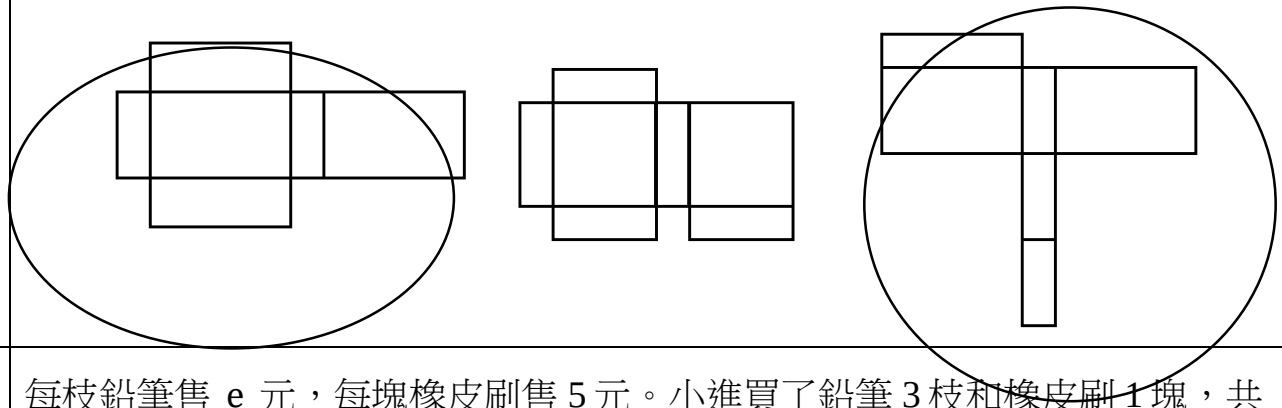
第二次: 12/15 = 0.8

第三次: 17/20 = 0.85

答案：第三次

5PS16	有 77 個蘋果，上午賣去其中的 $\frac{8}{11}$ ，下午再賣去餘下的 $\frac{2}{7}$ 。下午賣去蘋果 6 個。	5N3	
5PS17	A，B 兩點相距 80 米。小美由 A 點出發，向 B 點跑了 60.2 米後，回頭又向 A 點跑了 13.3 米。小美現在距離 A 點 46.9 米。	5N4	A
5PS18	6 個工人每天可築路 $50\frac{1}{2}$ 米。現在減少 2 名工人，每天可築路 $33\frac{2}{3}$ 米。	5N6	B

5PS19	 小美依照以下的路線走：西北，西南，東南。 小美的目的地是 <u>文具店</u> 。	5S1	

5PS20	下面的摺紙圖樣，哪些可以摺成長方體? 請圈出來。 (答案可多於一個。) 	5S2	B
6PS1	每枝鉛筆售 e 元，每塊橡皮刷售 5 元。小進買了鉛筆 3 枝和橡皮刷 1 塊，共付了 23 元。每枝鉛筆售 ___ 元。(請用方程式解決問題。) 列式： 設每枝鉛筆售 e 元 3e+5=23 e=6 答案： 6	6A1	
6PS2	外婆今年的年齡剛好是小新的 7 倍。如果 3 年後小新是 15 歲，外婆今年是多	6A1	B

<table>
<tr>
<td></td>
<td>少歲? (請用方程式解決問題。)

列式： 設婆今年是 y 歲

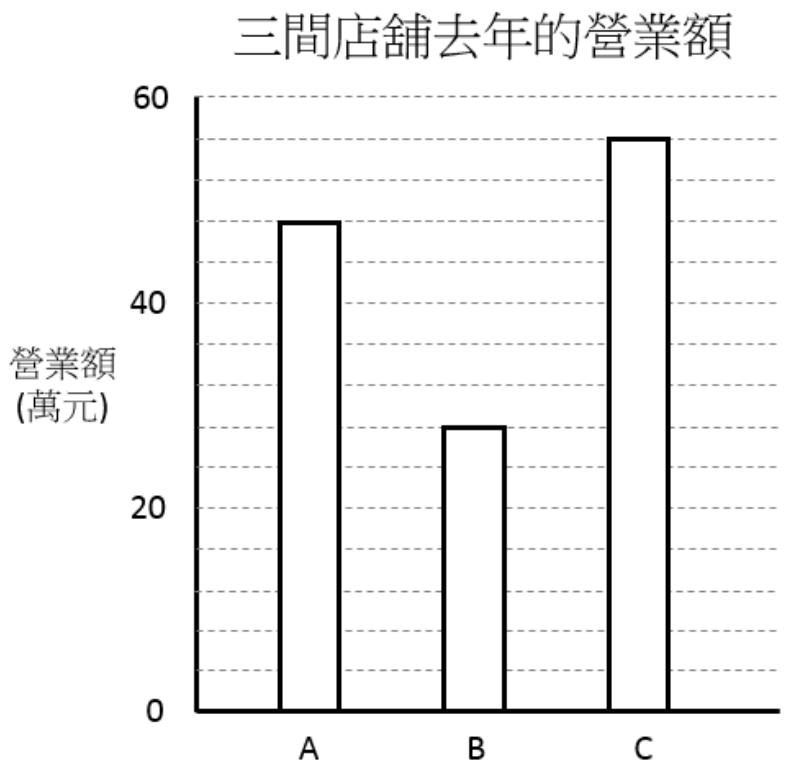
$\frac{y}{7} + 3 = 15$

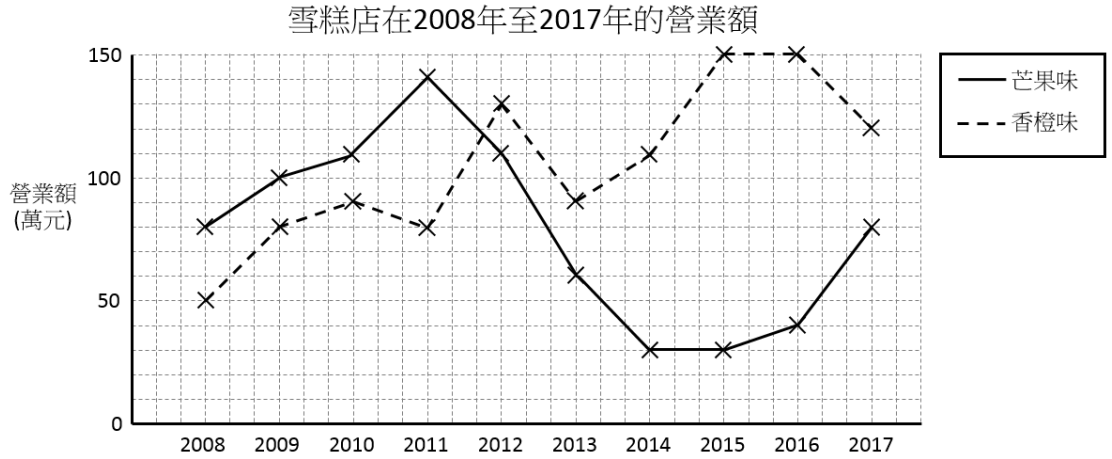
y=84

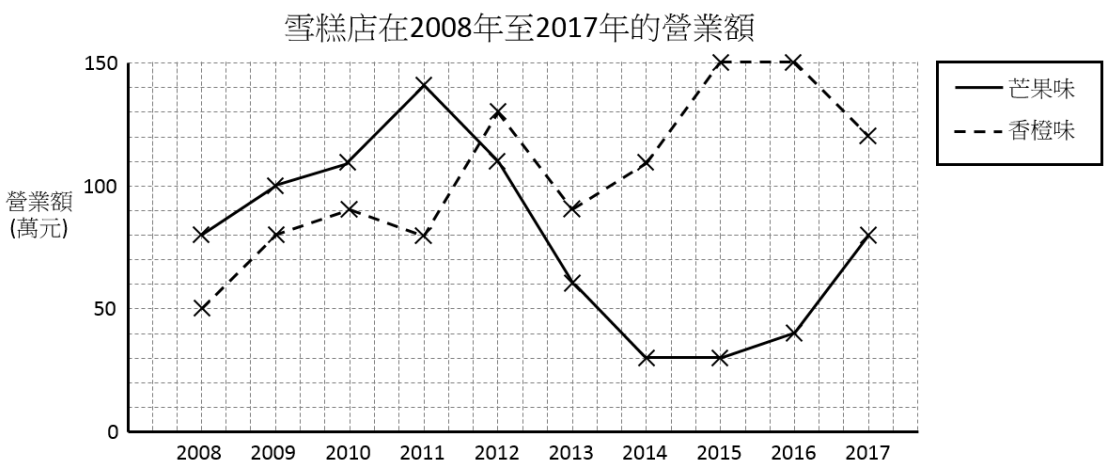
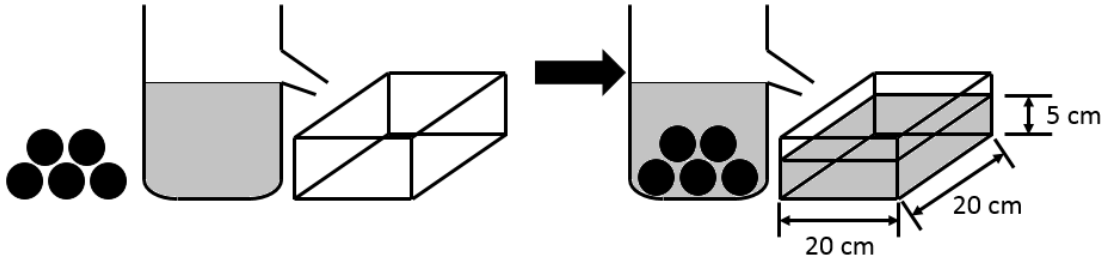
答案： 84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6PS3</td>
<td>以下是小軒玩電腦遊戲時的分數。

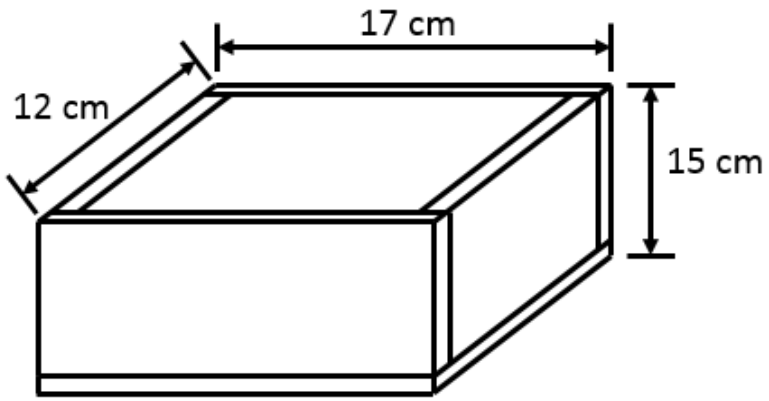
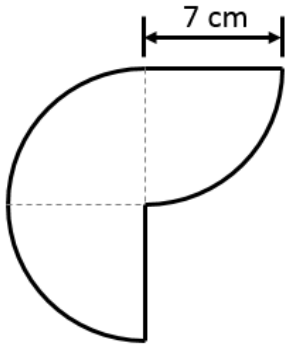
<table>
<tr><th>回合</th><th>得分</th></tr>
<tr><td>第一個</td><td>22</td></tr>
<tr><td>第二個</td><td>30</td></tr>
<tr><td>第三個</td><td>?</td></tr>
</table>

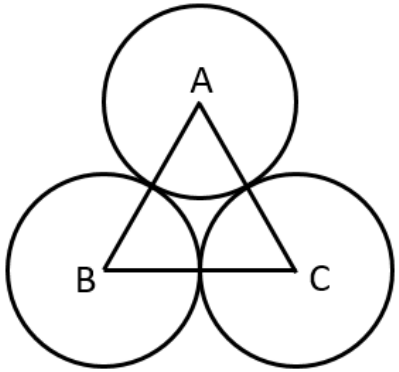
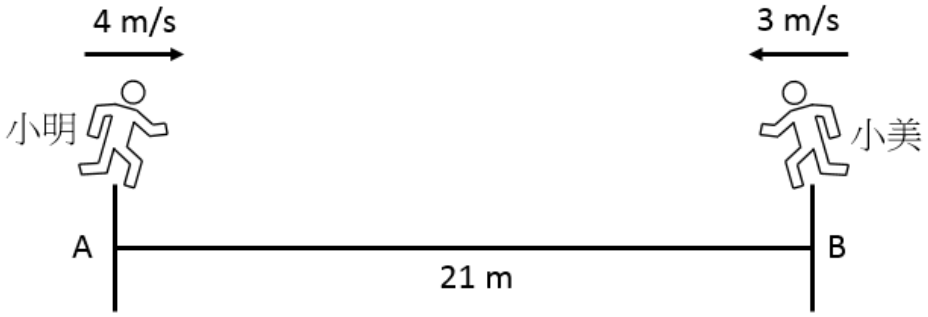
如果小軒希望平均每個回合最少取得 28 分，那麼他在第三個回合中，最少要</td>
<td>6D1</td>
<td>A</td>
</tr>
</table>

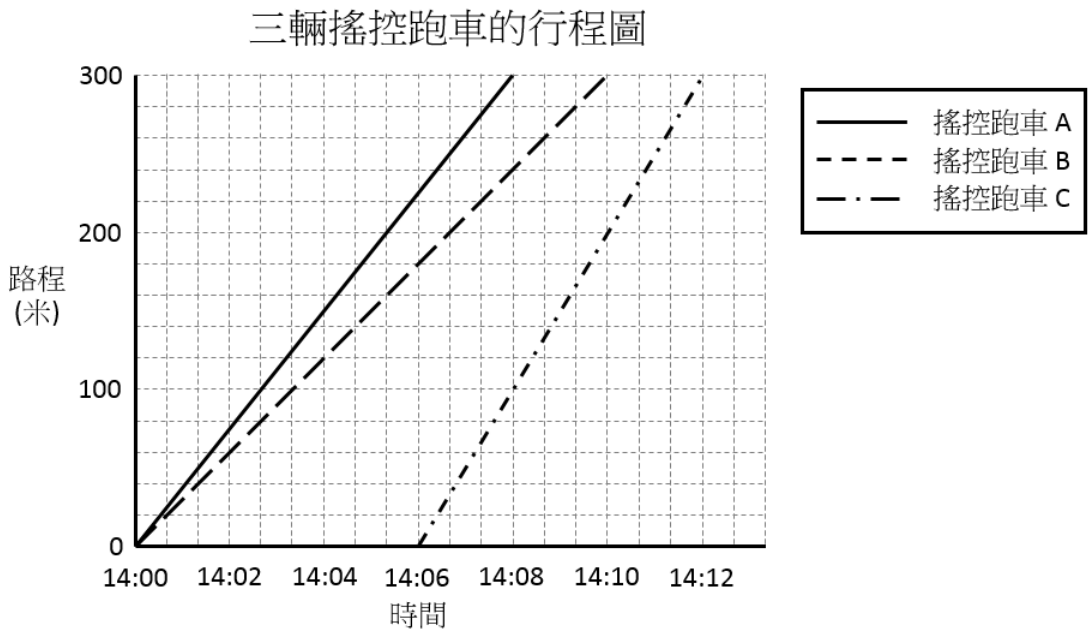
	取得 32 分。		
6PS4	以下是三間店舖去年的營業額。  	6D2	

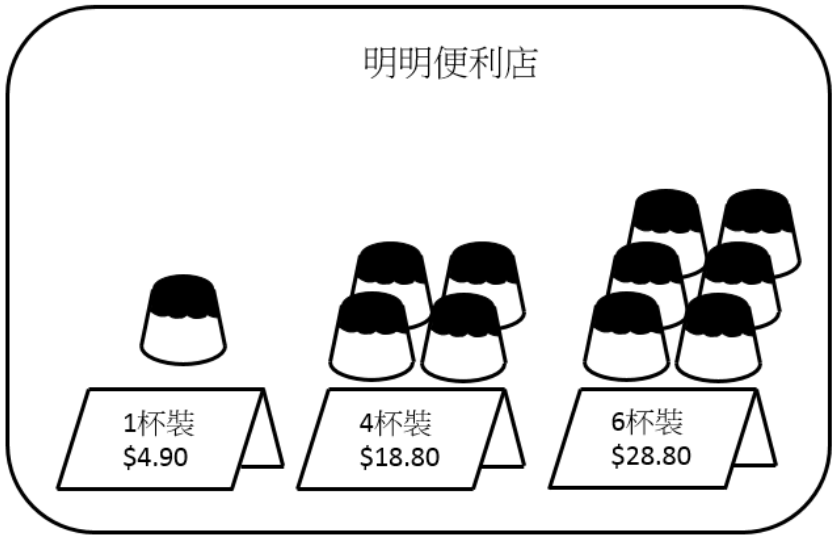
	店舖 C 的營業額約是店舖 A 的 $1\frac{1}{11}$ 倍。 (以最簡分數作答。)		
6PS5	 在 2015 年，芒果味雪糕和香橙味的營業額相差最大，約相差 1 200 000 元。	6D3	A

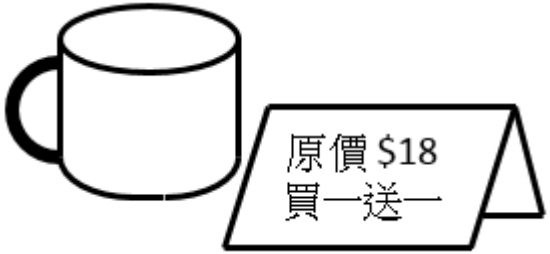
6PS6	 在 2011 年至 2014 年期間，芒果味雪糕的營業額持續下降。	6D3	
6PS7	 一個●的體積是 400 cm^3。	6M1	

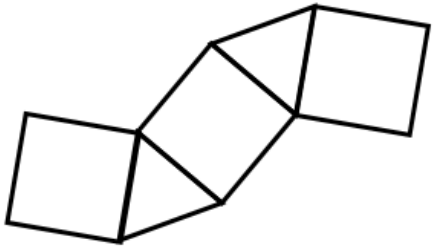
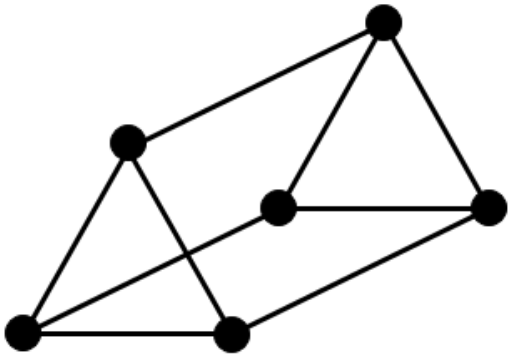
6PS8	 這個木箱由五塊厚 1 cm 的木板製成。小明把水注入箱中，如果水位高 5 cm， 箱子內水的體積是 <u>750</u> cm^3。	6M1	B
6PS9	 這個圖形的周界是 <u>47</u> cm。（取 $\pi = \frac{22}{7}$ 。）	6M2	A

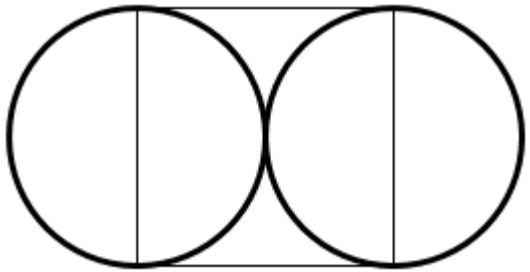
6PS10	 這個標志是由三個大小相同的圓形和一個三角形 ABC 組成。A，B，C 分別是各圓形的圓心。如果每個圓形的周界長 44 cm，三角形 ABC 的周界為 <u>42</u> cm。（取 $\pi = \frac{22}{7}$ 。）	6M2	
6PS11		6M3	A

	A 兩地相距 21 米。小明以 4 米每秒的平均速率從 A 跑向 B，小美在同一直路上，同時以 3 米每秒的平均速率從 B 跑向 A。兩人會在幾秒後相遇? 86，列式： 21÷(3+4) 答案： 3		
6PS12		6M3	B

	搖控跑車 B 的平均速率是 0.5 米每秒。		
6PS13	 小進要買 10 杯布甸，他該怎樣買才最便宜? 列式： 18.8x2+4.9x2=47.4 答案： 2 個 4 杯裝和 2 個 1 杯裝	6N1	
6PS14	將 4.78，409.8%，$4\frac{16}{21}$由小至大排列。	6N3	

	409.8% ， $4\frac{16}{21}$ ， 4.78 (最小) (最大)		
6PS15	哥哥有 49 本圖書，妹妹有 28 本。哥哥有的圖書數量是妹妹的百分之幾? 列式： $\frac{49}{28} \times 100\%$ 答案： 175%	6N4	
6PS16	 媽媽買半打上面的杯子，可享有的優惠相當於多少折? 列式： $\frac{6\times18}{12\times18} \times 100\%$ 答案： 五折	6N4	

6PS17	 這個摺紙圖樣可以摺出的立體是 三角柱體 。	6S1	
6PS18	 如果要將這個三角柱體，改造成一個三角錐體，要移走木棒 3 枝和膠珠 2 粒。	6S1	
6PS19	一個正方體的截面，可以是以下哪種圖形? 試圈出答案。(答案可多於一個。) 三角形 正方形 長方形 平行四邊形	6S1	A

6PS20	 這個圖形是由兩個大小相同的圓形，加上一個正方形所組成的。如果圓的半徑是 6 cm，那麼正方形的周界是 48 cm。	6S2	B
JSCPS1	$y = \frac{z+5}{5-z}$ 如果 $y = 19$，那麼 z =4.5。	JSC	
JSCPS2	一個手袋，成本為 200 元，標價比成本高 10%，後來以九折賣出。 賣出這個手袋是（ 獲利 / (虧蝕) ） 1 %。	JSC	
JSCPS3	3 個蛋糕和 5 袋曲奇，價值 250 元。	JSC	

	8 個蛋糕和 2 袋曲奇，價值 440 元。 蛋糕價值為 50 元，曲奇價值為 20 元。		
JSCPS4	在地圖上的 1cm，實際長度為 50000cm。那麼實際邊長為 30 米的正方形廣場，在地圖上的面積為 0.0036cm^2 。	JSC	

Appendix C. Summaries of Rasch analysis of Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS) after first item selection

Arithmetic (AR)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1AR5	-6.26	0.17	1.21	2.11	1.04	0.23
1AR8	-6.00	0.17	1.00	0.03	0.88	-0.49
1AR11	-6.23	0.17	0.84	-1.81	0.99	0.04
1AR13	-5.86	0.15	1.25	2.76	3.47	4.90
1AR14	-4.77	0.12	1.19	2.73	2.12	4.28
2AR2	-2.98	0.13	1.13	1.81	1.31	2.08
2AR3	-3.34	0.14	1.14	1.74	1.13	0.80
2AR4	-2.03	0.12	1.03	0.60	0.96	-0.33
2AR5	-4.49	0.13	0.95	-0.58	0.90	-0.36
2AR6	-4.08	0.12	0.80	-3.25	1.26	1.26
2AR7	-4.50	0.18	0.87	-1.04	0.61	-1.44
2AR8	-1.94	0.12	1.02	0.44	1.00	0.06
2AR9	-2.58	0.10	1.17	3.36	1.66	4.15
2AR10	-0.95	0.10	1.13	2.83	1.12	0.80
2AR11	-3.82	0.16	0.78	-2.46	0.50	-2.73
2AR12	-2.96	0.10	0.76	-4.88	0.61	-3.31
2AR13	-1.82	0.09	0.91	-2.13	0.82	-1.44
2AR14	-1.74	0.12	0.88	-2.63	0.84	-1.70
3AR1	-3.32	0.17	1.09	0.95	1.19	0.78
3AR2	-2.90	0.16	0.98	-0.18	1.33	1.52
3AR3	-2.14	0.15	1.16	2.20	1.42	2.46
3AR4	-1.29	0.14	1.08	1.26	1.10	0.70
3AR5	-0.03	0.10	1.29	5.24	1.51	2.91
3AR6	-0.66	0.10	1.11	2.37	1.24	1.69
3AR7	-1.63	0.14	0.91	-1.45	0.79	-1.55
3AR8	-0.55	0.15	0.86	-2.05	0.68	-1.75
3AR9	0.40	0.11	0.89	-2.01	0.86	-0.79
3AR10	0.16	0.11	0.78	-4.51	0.62	-2.63
3AR11	-0.11	0.16	0.83	-2.35	0.75	-1.01
3AR13	0.63	0.11	1.00	-0.01	1.09	0.54
3AR14	2.00	0.15	1.04	0.43	0.61	-1.53

4AR1	1.37	0.19	1.15	1.70	1.09	0.43
4AR2	0.02	0.13	0.95	-0.93	0.92	-0.37
4AR3	1.04	0.13	1.05	0.78	0.97	-0.11
4AR4	1.15	0.19	0.86	-1.72	0.72	-1.27
4AR5	1.75	0.20	0.79	-2.22	0.62	-1.36
4AR6	2.01	0.21	0.74	-2.59	0.57	-1.52
4AR7	1.74	0.13	0.92	-1.10	0.93	-0.25
4AR8	2.54	0.16	0.93	-0.64	0.67	-1.13
4AR11	0.64	0.18	0.97	-0.35	0.91	-0.44
4AR12	0.59	0.13	0.92	-1.50	0.72	-1.39
4AR13	1.53	0.13	0.93	-1.02	0.78	-1.06
4AR14	2.28	0.22	0.88	-0.96	0.73	-0.81
5AR1	0.12	0.21	1.22	2.28	1.37	1.19
5AR2	2.28	0.16	1.36	3.61	1.51	2.34
5AR3	1.79	0.16	1.27	3.13	1.26	1.42
5AR4	3.84	0.27	0.88	-0.80	0.61	-0.83
5AR5	2.48	0.17	0.69	-3.79	0.52	-2.95
5AR6	2.87	0.23	0.85	-1.25	0.59	-1.64
5AR7	2.71	0.23	0.83	-1.42	0.58	-1.86
5AR8	3.85	0.21	0.95	-0.32	0.60	-1.22
5AR10	4.07	0.28	0.79	-1.37	0.76	-0.34
5AR11	6.51	0.45	1.04	0.24	0.45	-0.78
5AR12	1.58	0.21	0.94	-0.55	0.98	-0.03
5AR13	1.98	0.21	0.71	-2.76	0.79	-1.10
5AR14	3.14	0.19	1.09	0.80	1.17	0.70
6AR1	3.92	0.34	0.78	-1.13	0.47	-1.46
6AR2	4.41	0.29	0.91	-0.51	0.49	-1.24
6AR5	4.83	0.30	0.65	-2.06	0.29	-1.59
6AR11	4.16	0.36	0.84	-0.77	0.44	-1.34
6AR13	4.57	0.38	0.91	-0.32	0.50	-0.87
Mean	0.00	0.18	0.97	-0.30	0.94	-0.20
S.D.	3.10	0.07	0.16	2.10	0.47	1.70

Math Knowledge (MK)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1MK1	-5.13	0.18	1.06	0.54	1.12	0.52
1MK2	-4.94	0.15	1.16	1.59	1.16	0.68
1MK3	-3.67	0.12	1.27	3.91	1.38	1.86
1MK4	-4.22	0.15	1.07	0.88	1.07	0.49
1MK5	-2.43	0.13	0.93	-1.34	0.95	-0.48
1MK6	-2.35	0.10	1.22	4.71	1.34	3.60
1MK7	-2.30	0.10	1.19	3.90	1.46	3.98
1MK8	-3.62	0.14	0.88	-1.77	0.93	-0.57
1MK9	-2.83	0.13	1.09	1.48	1.07	0.70
1MK10	-3.95	0.15	0.92	-1.12	0.92	-0.53
1MK11	-4.78	0.15	1.02	0.23	1.61	1.94
1MK12	-4.41	0.13	0.93	-0.92	0.89	-0.46
1MK14	-2.68	0.13	1.03	0.59	1.01	0.17
1MK15	-3.43	0.14	0.93	-1.11	0.84	-1.50
1MK16	-4.46	0.16	0.82	-2.15	0.67	-2.02
1MK17	-3.49	0.11	1.05	0.74	1.54	2.67
1MK18	-3.24	0.10	0.91	-1.67	1.05	0.45
1MK19	-4.20	0.15	0.89	-1.44	0.72	-1.93
1MK20	-3.68	0.14	0.83	-2.59	0.72	-2.50
2MK1	-2.43	0.09	1.30	5.80	1.78	4.61
2MK2	-0.78	0.09	1.19	4.60	1.35	3.51
2MK4	-3.93	0.18	0.95	-0.35	1.01	0.11
2MK5	-2.87	0.14	1.08	1.04	1.06	0.47
2MK6	-1.69	0.12	1.00	0.10	0.99	-0.12
2MK7	-1.98	0.09	1.04	0.85	0.99	-0.09
2MK8	-0.29	0.09	1.10	2.25	1.02	0.17
2MK9	-3.08	0.14	0.91	-1.01	1.06	0.39
2MK10	-3.19	0.15	0.82	-2.19	0.74	-1.50
2MK11	-1.72	0.09	1.02	0.52	1.18	1.65
2MK12	-2.74	0.10	0.79	-4.34	0.83	-1.37
2MK13	-0.49	0.12	1.19	3.70	1.18	1.82
2MK14	-2.81	0.14	0.82	-2.61	0.63	-2.86
2MK15	0.16	0.13	1.27	4.07	1.31	2.16
2MK16	-1.44	0.11	0.78	-5.16	0.71	-4.16

2MK17	-1.15	0.11	0.77	-5.56	0.77	-3.26
2MK18	-1.17	0.09	0.84	-4.37	0.88	-1.44
2MK19	-1.31	0.09	0.82	-4.94	0.87	-1.45
2MK20	-1.30	0.11	0.78	-5.43	0.82	-2.43
3MK1	-0.80	0.10	1.31	6.69	2.28	9.09
3MK2	-0.92	0.10	1.16	3.62	1.56	4.79
3MK3	-2.58	0.17	1.03	0.29	1.43	1.73
3MK4	-1.70	0.15	0.81	-2.67	0.74	-1.93
3MK5	-1.46	0.10	0.86	-3.05	0.87	-0.97
3MK6	-0.74	0.14	1.05	0.95	1.05	0.49
3MK7	0.30	0.14	0.88	-1.93	0.85	-0.93
3MK8	0.92	0.11	1.03	0.46	0.92	-0.55
3MK9	0.57	0.15	0.82	-2.66	0.67	-2.07
3MK10	-1.76	0.15	0.87	-1.70	0.75	-1.76
3MK11	1.51	0.11	0.78	-3.60	0.55	-3.29
3MK12	1.60	0.18	0.79	-2.20	0.50	-2.12
3MK13	1.51	0.17	0.87	-1.35	0.63	-1.53
3MK14	0.38	0.10	0.80	-4.14	0.83	-1.52
3MK15	0.02	0.14	0.83	-2.95	0.74	-1.92
3MK16	0.06	0.14	1.02	0.37	1.02	0.20
3MK17	-0.70	0.14	0.92	-1.39	0.92	-0.70
3MK18	0.13	0.10	0.87	-2.70	0.95	-0.40
3MK19	1.30	0.11	1.18	2.73	1.28	1.79
3MK20	1.93	0.19	0.91	-0.77	0.81	-0.54
4MK1	-0.25	0.20	1.02	0.19	1.29	1.32
4MK2	0.79	0.13	0.80	-3.27	0.67	-2.27
4MK3	1.28	0.19	0.82	-2.28	0.73	-1.84
4MK5	2.02	0.14	0.81	-2.95	0.60	-2.23
4MK6	0.73	0.19	1.04	0.56	1.06	0.42
4MK7	-0.09	0.20	1.01	0.15	1.25	1.26
4MK8	-0.33	0.21	0.74	-2.67	0.67	-1.63
4MK9	0.60	0.12	1.21	3.26	1.75	4.47
4MK10	4.91	0.29	1.05	0.30	0.32	-1.71
4MK11	2.33	0.20	1.03	0.32	0.93	-0.18
4MK12	0.94	0.19	0.71	-3.89	0.59	-3.23
4MK13	1.90	0.19	0.65	-4.40	0.53	-2.70
4MK14	1.50	0.19	0.85	-1.84	1.44	2.31

4MK15	2.79	0.15	0.76	-3.11	0.43	-2.57
4MK16	2.21	0.15	0.81	-2.54	0.61	-1.76
4MK17	1.39	0.19	0.95	-0.62	1.03	0.25
4MK18	1.27	0.13	1.10	1.39	1.48	2.43
4MK19	2.93	0.16	0.90	-1.20	0.63	-1.33
4MK20	3.19	0.24	0.86	-1.04	0.67	-0.80
5MK1	1.64	0.23	1.13	1.17	0.94	-0.12
5MK3	3.09	0.18	1.15	1.71	1.19	0.61
5MK4	2.00	0.23	0.99	-0.09	0.74	-0.83
5MK5	0.62	0.24	0.70	-2.27	0.84	-0.68
5MK6	1.85	0.23	1.28	2.41	1.02	0.17
5MK7	2.63	0.23	0.85	-1.58	0.53	-1.19
5MK8	2.63	0.23	0.77	-2.39	0.73	-0.57
5MK9	3.23	0.18	0.87	-1.50	0.58	-1.15
5MK10	2.01	0.17	0.87	-1.55	0.70	-1.46
5MK12	1.36	0.17	2.20	9.90	4.37	9.90
5MK13	3.33	0.19	0.85	-1.72	0.60	-1.02
5MK17	2.72	0.17	1.14	1.73	1.50	1.49
5MK18	2.85	0.19	0.96	-0.44	0.72	-0.87
5MK19	4.34	0.31	1.04	0.27	0.83	0.01
6MK1	4.64	0.32	0.76	-1.38	0.42	-0.70
6MK3	3.18	0.45	1.03	0.21	0.60	-0.65
6MK4	6.37	0.47	1.07	0.32	0.58	-0.37
6MK5	4.17	0.45	0.86	-0.53	0.51	-0.51
6MK6	3.58	0.44	0.93	-0.19	0.56	-0.65
6MK9	4.44	0.31	0.71	-1.77	0.38	-0.89
6MK11	2.39	0.44	0.73	-1.02	0.57	-0.70
6MK12	5.63	0.40	0.99	0.02	0.44	-0.58
6MK15	5.62	0.48	1.21	0.92	0.58	0.12
Mean	0.00	0.18	0.97	-0.50	0.96	-0.10
S.D.	2.74	0.09	0.20	2.70	0.49	2.20

Math Problem Solving (PS)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1PS1	-4.90	0.17	0.99	-0.07	1.33	1.35
1PS2	-4.90	0.14	1.34	3.44	3.75	6.37
1PS3	-1.90	0.10	1.10	2.22	1.22	2.33
1PS4	-2.66	0.14	1.01	0.13	0.96	-0.32
1PS5	-3.02	0.14	1.08	1.29	1.16	1.46
1PS6	-1.35	0.10	1.04	0.95	1.07	0.67
1PS7	-2.59	0.10	1.32	5.94	1.62	5.30
1PS8	-4.23	0.15	1.16	2.06	1.20	1.19
1PS9	-5.65	0.20	1.04	0.35	1.21	0.71
1PS10	-1.48	0.15	0.89	-1.53	0.73	-1.52
1PS11	-2.81	0.14	0.84	-2.73	0.74	-2.59
1PS12	-2.39	0.14	0.75	-4.35	0.63	-3.48
1PS13	-2.96	0.14	0.76	-4.37	0.66	-3.51
1PS14	-2.67	0.10	0.88	-2.56	0.85	-1.60
1PS15	-2.07	0.10	0.88	-2.85	0.99	-0.13
1PS16	-3.51	0.14	0.91	-1.42	0.84	-1.39
1PS17	-4.53	0.16	0.84	-1.98	0.67	-1.85
1PS18	-1.27	0.16	1.07	0.88	0.99	0.02
1PS19	-2.28	0.10	1.15	3.18	1.33	3.33
1PS20	0.39	0.12	1.19	2.49	1.47	1.89
2PS1	-2.17	0.09	1.25	5.34	3.07	9.90
2PS2	-3.56	0.11	0.92	-1.15	1.02	0.18
2PS3	-3.44	0.16	0.97	-0.27	0.97	-0.08
2PS4	-0.67	0.11	1.07	1.51	1.07	0.92
2PS5	-1.26	0.11	0.90	-2.38	0.83	-2.63
2PS6	-3.47	0.16	1.03	0.32	1.39	1.67
2PS7	0.38	0.13	1.01	0.22	0.88	-0.95
2PS8	-2.99	0.10	0.97	-0.45	1.00	0.02
2PS9	-1.21	0.09	0.88	-3.23	0.80	-2.63
2PS10	-1.47	0.11	1.14	2.96	1.29	3.66
2PS11	-0.21	0.12	0.91	-1.78	0.83	-1.93
2PS12	-2.63	0.13	0.89	-1.62	0.92	-0.59
2PS13	-0.29	0.09	0.87	-3.22	0.75	-2.51
2PS14	-0.09	0.09	0.97	-0.74	0.82	-1.41

2PS16	-0.07	0.12	0.79	-4.10	0.65	-3.94
2PS17	-0.58	0.11	0.74	-5.91	0.67	-4.86
2PS18	-1.99	0.09	1.04	0.84	1.18	2.00
2PS19	-1.72	0.09	0.99	-0.31	1.05	0.69
2PS20	-1.74	0.12	1.12	2.43	1.10	1.20
3PS1	-0.78	0.09	0.96	-0.90	0.91	-1.03
3PS2	0.28	0.10	0.86	-3.05	0.81	-1.79
3PS3	-0.58	0.14	0.92	-1.32	0.87	-1.00
3PS4	0.70	0.16	1.05	0.59	0.89	-0.39
3PS5	-1.66	0.14	1.00	0.05	0.92	-0.61
3PS6	-0.52	0.14	1.10	1.68	1.34	2.38
3PS7	0.65	0.10	1.15	2.75	1.94	5.69
3PS8	1.40	0.12	1.04	0.60	0.77	-1.24
3PS9	0.06	0.15	1.15	2.22	1.07	0.44
3PS10	0.75	0.11	0.99	-0.09	0.91	-0.61
3PS11	2.20	0.15	0.92	-0.86	0.62	-1.71
3PS12	1.15	0.18	0.84	-1.62	0.61	-1.49
3PS14	2.54	0.27	0.93	-0.26	0.55	-1.03
3PS15	-0.42	0.14	1.03	0.58	1.12	0.90
3PS16	0.30	0.15	0.99	-0.15	0.92	-0.38
3PS17	-1.16	0.09	1.05	1.14	1.16	1.66
3PS18	-0.34	0.10	1.13	2.99	1.30	3.11
3PS19	-0.85	0.14	1.00	0.02	1.01	0.13
3PS20	0.21	0.15	0.92	-1.10	0.92	-0.39
4PS1	1.00	0.13	0.93	-1.18	0.72	-1.89
4PS2	0.69	0.13	0.84	-2.68	0.73	-1.61
4PS3	-1.84	0.23	0.83	-1.25	0.65	-1.22
4PS4	-1.17	0.21	0.80	-2.01	0.66	-1.80
4PS5	2.77	0.18	0.91	-0.77	0.49	-1.89
4PS6	2.08	0.16	0.90	-1.08	1.06	0.29
4PS7	2.68	0.27	0.91	-0.50	0.59	-0.92
4PS8	2.42	0.25	0.89	-0.72	0.65	-0.85
4PS9	2.48	0.25	1.15	0.98	1.23	0.65
4PS10	0.20	0.19	0.81	-2.63	0.70	-2.32
4PS11	2.55	0.26	1.00	0.07	0.93	-0.03
4PS12	3.08	0.30	0.99	0.03	0.99	0.16
4PS13	4.00	0.25	1.16	0.89	0.83	-0.25

4PS14	2.65	0.19	0.99	-0.02	0.74	-0.76
4PS15	0.51	0.19	0.91	-1.16	0.83	-1.15
4PS17	0.64	0.12	1.12	1.96	1.18	1.28
4PS18	0.55	0.13	1.02	0.36	1.03	0.21
4PS19	1.74	0.21	0.87	-1.24	0.74	-0.92
5PS3	2.49	0.19	0.91	-0.90	0.61	-1.53
5PS5	1.42	0.23	1.08	0.75	0.96	-0.11
5PS6	1.17	0.22	1.38	3.20	1.64	2.58
5PS7	2.60	0.19	0.94	-0.53	0.60	-1.51
5PS8	3.86	0.34	0.87	-0.54	0.53	-0.45
5PS9	2.23	0.24	0.89	-0.93	0.56	-1.41
5PS14	2.51	0.20	0.80	-1.90	0.72	-0.84
5PS18	5.83	0.48	0.90	-0.15	1.94	1.35
6PS2	4.53	0.37	0.72	-1.24	0.22	-1.20
6PS5	4.40	0.34	0.80	-0.97	0.40	-0.93
6PS6	2.97	0.39	1.01	0.10	0.78	-0.43
6PS7	3.60	0.41	0.75	-1.20	0.46	-1.10
6PS9	6.42	0.59	0.99	0.12	0.29	-0.79
6PS10	3.28	0.39	0.78	-1.03	0.45	-1.34
6PS18	3.76	0.41	1.26	1.15	2.07	1.62
6PS20	3.92	0.33	0.81	-0.94	0.33	-1.12
Mean	0.00	0.18	0.97	-0.30	0.97	-0.10
S.D.	2.55	0.10	0.14	2.00	0.50	2.20

Appendix D. Summaries of Rasch analysis of Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS) after second item selection

Arithmetic (AR)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1AR1	-8.62	0.27	1.25	1.27	5.46	6.50
1AR2	-7.41	0.19	1.17	1.37	2.02	2.13
1AR4	-6.83	0.19	1.15	1.42	1.40	2.04
1AR5	-6.26	0.18	1.15	1.70	1.29	2.04
1AR8	-6.00	0.17	1.02	0.21	0.97	-0.23
1AR11	-6.23	0.17	0.90	-1.14	0.80	-1.61
1AR13	-5.86	0.14	1.17	2.13	2.41	3.16
1AR14	-4.77	0.13	1.14	2.10	2.27	4.57
2AR2	-2.98	0.13	0.88	-2.07	0.82	-1.66
2AR3	-3.34	0.13	0.88	-1.80	0.74	-2.07
2AR5	-4.49	0.12	0.93	-0.91	0.74	-1.12
2AR8	-1.94	0.12	0.89	-2.11	0.81	-1.87
2AR14	-1.74	0.12	1.00	0.02	0.98	-0.15
3AR4	-1.29	0.15	1.17	2.23	1.16	0.72
3AR6	-0.66	0.11	1.13	2.51	1.31	1.77
3AR7	-1.63	0.16	0.85	-2.00	0.69	-1.40
3AR9	0.40	0.11	0.84	-2.92	0.86	-0.70
3AR11	-0.11	0.16	0.74	-3.48	0.54	-1.79
4AR1	1.37	0.19	1.02	0.28	0.93	-0.33
4AR2	0.02	0.13	0.90	-1.79	0.79	-1.04
4AR7	1.74	0.14	0.91	-1.35	0.73	-1.23
4AR11	0.64	0.18	0.92	-1.11	0.87	-0.97
4AR14	2.28	0.22	0.84	-1.34	0.65	-1.23
5AR2	2.28	0.17	1.13	1.45	1.19	1.25
5AR3	1.79	0.16	1.10	1.39	1.19	1.11
5AR6	2.87	0.23	0.90	-0.83	0.93	-0.24
5AR10	4.07	0.28	0.73	-1.67	0.73	-0.57
5AR14	3.14	0.19	1.06	0.54	1.23	1.02
6AR1	3.92	0.36	0.64	-2.00	0.42	-1.77
6AR2	4.41	0.30	0.88	-0.69	0.50	-1.18
6AR5	4.83	0.30	0.68	-1.95	0.33	-1.59

6AR11	4.16	0.37	0.94	-0.23	0.53	-1.12
6AR13	4.57	0.39	0.87	-0.56	0.46	-1.04
Mean	-0.84	0.19	0.96	-0.34	1.11	0.04
S.D.	3.99	0.08	0.15	1.63	0.91	1.97

Math Knowledge (MK)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1MK1	-5.13	0.18	0.97	-0.20	0.89	-0.40
1MK4	-4.22	0.15	1.02	0.32	0.94	-0.38
1MK8	-3.62	0.14	0.83	-2.66	0.77	-2.18
1MK10	-3.95	0.15	0.92	-1.12	0.80	-1.61
1MK14	-2.68	0.14	0.97	-0.54	1.03	0.30
1MK16	-4.46	0.16	0.75	-3.15	0.58	-2.82
1MK18	-3.24	0.11	0.89	-2.18	0.98	-0.14
2MK6	-1.69	0.12	0.96	-0.72	0.88	-1.45
2MK7	-1.98	0.09	1.03	0.76	1.03	0.29
2MK8	-0.29	0.09	1.07	1.56	0.95	-0.39
2MK12	-2.74	0.10	0.78	-4.47	0.72	-2.16
2MK13	-0.49	0.12	1.19	3.55	1.22	2.23
2MK18	-1.17	0.09	0.83	-4.41	0.82	-2.14
2MK19	-1.31	0.09	0.79	-5.52	0.84	-1.65
3MK5	-1.46	0.10	0.90	-1.94	0.95	-0.31
3MK6	-0.74	0.14	1.05	0.77	1.18	1.57
3MK8	0.92	0.11	0.95	-0.85	0.86	-1.02
3MK10	-1.76	0.16	0.88	-1.47	0.79	-1.28
3MK14	0.38	0.10	0.81	-4.04	0.90	-0.93
3MK16	0.06	0.14	1.05	0.80	1.04	0.33
3MK19	1.30	0.11	1.08	1.35	1.02	0.15
4MK3	1.28	0.19	0.79	-2.82	0.68	-2.44
4MK5	2.02	0.13	0.84	-2.58	0.64	-2.17
4MK6	0.73	0.19	0.99	-0.07	0.94	-0.37
4MK7	-0.09	0.20	0.92	-0.76	0.99	0.00
4MK11	2.33	0.20	0.98	-0.20	0.91	-0.30
4MK17	1.39	0.19	0.84	-2.05	0.82	-1.21
4MK19	2.93	0.15	0.85	-1.75	0.70	-1.14
5MK1	1.64	0.23	1.08	0.73	1.06	0.36
5MK3	3.09	0.18	1.02	0.30	0.94	-0.15
5MK6	1.85	0.23	1.21	1.92	1.22	1.06
5MK8	2.63	0.23	0.77	-2.40	0.84	-0.51
5MK9	3.23	0.19	0.87	-1.44	0.64	-1.42
5MK18	2.85	0.19	1.00	0.08	0.96	-0.08

5MK19	4.34	0.31	0.92	-0.36	0.75	-0.20
6MK1	4.64	0.31	0.67	-2.19	0.36	-0.79
6MK3	3.18	0.47	1.26	1.01	1.05	0.26
6MK4	6.37	0.48	1.01	0.13	0.90	0.21
6MK5	4.17	0.45	0.80	-0.94	0.60	-1.04
6MK6	3.58	0.46	0.93	-0.21	0.78	-0.43
6MK12	5.63	0.40	0.95	-0.15	1.00	0.24
6MK15	5.62	0.48	1.04	0.26	0.73	-0.17
Mean	0.60	0.20	0.94	-0.90	0.87	-0.58
S.D.	2.97	0.12	0.13	1.85	0.17	1.05

Math Problem Solving (PS)						
Item	Measure	Model S.E.	Infit		Outfit	
			MNSQ	ZSTD	MNSQ	ZSTD
1PS4	-2.66	0.14	0.94	-0.90	0.94	-0.57
1PS5	-3.02	0.14	1.04	0.65	1.07	0.68
1PS6	-1.35	0.10	1.01	0.26	1.00	0.02
1PS8	-4.23	0.15	0.96	-0.47	0.99	-0.01
1PS9	-5.65	0.20	0.93	-0.52	0.88	-0.27
1PS15	-2.07	0.10	0.93	-1.62	0.93	-0.71
1PS16	-3.51	0.14	0.86	-2.42	0.75	-2.40
1PS18	-1.27	0.16	1.00	0.01	1.18	0.90
2PS4	-0.67	0.12	1.04	0.86	1.02	0.24
2PS7	0.38	0.13	0.94	-0.91	0.79	-1.70
2PS9	-1.21	0.09	0.88	-3.05	0.80	-2.54
2PS10	-1.47	0.12	1.10	1.98	1.18	2.15
2PS11	-0.21	0.12	0.88	-2.21	0.82	-2.08
2PS13	-0.29	0.09	0.88	-2.78	0.80	-1.89
2PS18	-1.99	0.09	1.03	0.72	1.09	1.04
2PS19	-1.72	0.09	1.00	0.11	1.08	0.97
3PS1	-0.78	0.10	0.91	-2.16	0.89	-1.28
3PS3	-0.58	0.14	0.90	-1.82	0.84	-1.79
3PS9	0.06	0.15	1.08	1.21	1.03	0.29
3PS10	0.75	0.11	0.99	-0.13	0.95	-0.41
3PS12	1.15	0.18	0.84	-1.58	0.63	-1.86
3PS15	-0.42	0.14	1.06	0.93	1.13	1.32
3PS16	0.30	0.15	1.02	0.35	1.06	0.47
3PS19	-0.85	0.14	0.99	-0.17	0.94	-0.61
4PS1	1.00	0.13	0.87	-2.24	0.70	-2.47
4PS6	2.08	0.16	0.89	-1.18	0.64	-1.67
4PS9	2.48	0.25	1.23	1.40	1.35	1.04
4PS11	2.55	0.25	0.98	-0.08	0.85	-0.33
4PS15	0.51	0.18	0.89	-1.61	0.82	-1.49
4PS17	0.64	0.12	0.99	-0.23	1.00	0.03
4PS18	0.55	0.13	0.92	-1.39	0.86	-1.09
4PS19	1.74	0.21	0.76	-2.31	0.57	-2.19
5PS3	2.49	0.18	0.84	-1.57	0.61	-1.95
5PS5	1.42	0.23	1.06	0.57	0.93	-0.30

5PS6	1.17	0.23	1.37	3.15	1.50	2.34
5PS7	2.60	0.19	0.85	-1.46	0.58	-2.05
5PS8	3.86	0.32	0.76	-1.20	0.46	-0.98
5PS9	2.23	0.24	0.82	-1.61	0.58	-1.86
5PS14	2.51	0.20	0.78	-2.08	0.60	-1.77
5PS18	5.83	0.46	0.86	-0.30	1.26	0.57
6PS2	4.53	0.36	0.65	-1.79	0.25	-1.39
6PS5	4.40	0.33	0.72	-1.52	0.33	-1.09
6PS6	2.97	0.39	0.97	-0.05	0.74	-0.60
6PS7	3.60	0.41	0.77	-1.03	0.48	-1.13
6PS9	6.42	0.56	0.88	-0.18	0.22	-0.96
6PS10	3.28	0.40	0.78	-0.98	0.52	-1.18
6PS18	3.76	0.41	1.20	0.90	1.95	1.55
6PS20	3.92	0.32	0.74	-1.41	0.35	-1.58
Mean	0.73	0.20	0.93	-0.66	0.85	-0.64
S.D.	2.57	0.11	0.13	1.31	0.32	1.25

Appendix E. Standardized factor loadings in confirmatory factor analysis (from Grade 1 to 3)

Standardized factor loadings (Grade 1)			
	Standardized Estimate	Standard Error (s.e.)	p-value
Construct indicator			
Math Fluency (MF)			
Simple addition	0.725	0.026	0.000
Simple subtraction	0.808	0.027	0.000
Arithmetic (AR)			
1AR1	0.847	0.057	0.000
1AR2	0.829	0.048	0.000
1AR4	0.717	0.055	0.000
1AR5	0.773	0.044	0.000
1AR8	0.802	0.039	0.000
1AR11	0.876	0.032	0.000
1AR13	0.965	0.025	0.000
1AR14	0.914	0.027	0.000
Math Knowledge (MK)			
1MK1	0.67	0.076	0.000
1MK4	0.591	0.072	0.000
1MK8	0.709	0.047	0.000
1MK10	0.661	0.059	0.000
1MK14	0.571	0.061	0.000
1MK16	0.742	0.06	0.000
1MK18	0.806	0.046	0.000
Math Problem Solving (PS)			
1PS4	0.614	0.06	0.000
1PS5	0.631	0.053	0.000
1PS6	0.513	0.07	0.000
1PS8	0.572	0.066	0.000
1PS9	0.591	0.089	0.000
1PS15	0.795	0.043	0.000

1PS16	0.704	0.053	0.000
1PS18	0.541	0.071	0.000

Standardized factor loadings (Grade 2)			
	Standardized Estimate	Standard Error (s.e.)	p-value
Construct indicator			
Math Fluency (MF)			
Simple addition	0.829	0.025	0.000
Simple subtraction	0.851	0.026	0.000
Arithmetic (AR)			
2AR2	0.545	0.076	0.000
2AR3	0.479	0.076	0.000
2AR5	0.438	0.099	0.000
2AR8	0.516	0.066	0.000
2AR14	0.618	0.071	0.000
Math Knowledge (MK)			
2MK6	0.659	0.052	0.000
2MK7	0.521	0.064	0.000
2MK8	0.584	0.055	0.000
2MK12	0.711	0.061	0.000
2MK13	0.15	0.07	0.032
2MK18	0.875	0.035	0.000
2MK19	0.792	0.04	0.000
Math Problem Solving (PS)			
2PS4	0.468	0.058	0.000
2PS7	0.566	0.055	0.000
2PS9	0.67	0.051	0.000
2PS10	0.477	0.061	0.000
2PS11	0.635	0.053	0.000
2PS13	0.541	0.057	0.000
2PS18	0.693	0.045	0.000
2PS19	0.618	0.048	0.000

Standardized factor loadings (Grade 3)			
	Standardized Estimate	Standard Error (s.e.)	p-value
Construct indicator			
Math Fluency (MF)			
Simple addition	0.729	0.026	0.000
Simple subtraction	0.908	0.025	0.000
Arithmetic (AR)			
3AR4	0.537	0.064	0.000
3AR6	0.719	0.048	0.000
3AR7	0.754	0.045	0.000
3AR9	0.885	0.035	0.000
3AR11	0.837	0.04	0.000
Math Knowledge (MK)			
3MK5	0.708	0.066	0.000
3MK6	0.586	0.059	0.000
3MK8	0.832	0.039	0.000
3MK10	0.759	0.049	0.000
3MK14	0.699	0.05	0.000
3MK16	0.553	0.06	0.000
3MK19	0.615	0.062	0.000
Math Problem Solving (PS)			
3PS1	0.76	0.047	0.000
3PS3	0.693	0.054	0.000
3PS9	0.448	0.065	0.000
3PS10	0.561	0.064	0.000
3PS12	0.825	0.052	0.000
3PS15	0.567	0.063	0.000
3PS16	0.523	0.069	0.000
3PS19	0.546	0.064	0.000

Appendix F. Developmental score tables, Grade percentile score tables of Math Fluency (MF), Arithmetic (AR), Math Knowledge (MK), Math Problem Solving (PS), and Grade percentile score table of Composite score

Developmental score table of Math Fluency (MF)	
Raw Score	Developmental Score (Normed)
0	448
1	450
2	453
3	455
4	458
5	460
6	463
7	465
8	468
9	470
10	473
11	475
12	478
13	480
14	483
15	485
16	488
17	490
18	493
19	495
20	498
21	500
22	503
23	505
24	508
25	511
26	513
27	516
28	518
29	521

30	523
31	526
32	528
33	531
34	533
35	536
36	538
37	541
38	543
39	546
40	548
41	551
42	553
43	556
44	558
45	561
46	563
47	566
48	567
49	570
50	573
51	576
52	577
53	580
54	583
55	586
56	588
57	590
58	597
59	596
60	597
61-65	601
66-70	621
71-75	629
76-80	629
81-85	651
86-90	672

Percentile score table of Math Fluency (MF)						
Grade Percentile	P1	P2	P3	P4	P5	P6
10	453	468	476	484	484	497
25	464	481	489	498	503	518
50	476	494	503	513	525	541
75	489	508	517	529	546	564

Developmental score table of Arithmetic (AR)	
Raw Score	Developmental Score (Normed)
0	190
1	233
2	264
3	285
4	302
5	318
6	334
7	349
8	366
9	383
10	400
11	416
12	430
13	444
14	458
15	470
16	482
17	494
18	505
19	517
20	528
21	539
22	550
23	561
24	571
25	582
26	594
27	605
28	616
29	629
30	643
31	660
32	686
33	724

Grade percentile score table of Arithmetic (AR)						
Grade Percentile (%)	P1	P2	P3	P4	P5	P6
10	257	376	400	458	481	546
25	299	407	441	484	517	580
50	346	440	485	512	556	617
75	393	474	530	541	596	654

Developmental score table of Math Knowledge (MK)	
Raw Score	Developmental Score (Normed)
0	278
1	319
2	345
3	362
4	377
5	389
6	400
7	410
8	420
9	430
10	439
11	447
12	455
13	463
14	470
15	477
16	485
17	492
18	500
19	506
20	514
21	520
22	527
23	534
24	540
25	547
26	554
27	561
28	568
29	575
30	582
31	589
32	597
33	605

34	614
35	622
36	632
37	643
38	655
39	670
40	688
41	715
42	756

Grade percentile score table of Math Knowledge (MK)						
Grade Percentile (%)	P1	P2	P3	P4	P5	P6
10	354	408	453	466	506	547
25	383	435	480	495	533	587
50	414	465	511	528	563	632
75	446	495	541	560	593	678

Developmental score table of Math Problem Solving (PS)	
Raw Score	Developmental Score (Normed)
0	282
1	326
2	357
3	377
4	392
5	404
6	414
7	424
8	432
9	440
10	446
11	453
12	459
13	465
14	471
15	476
16	481
17	487
18	492
19	497
20	502
21	507
22	512
23	517
24	522
25	527
26	532
27	537
28	542
29	547
30	552
31	558

32	564
33	569
34	575
35	580
36	586
37	593
38	599
39	606
40	614
41	622
42	630
43	640
44	652
45	666
46	684
47	711
48	753

Grade percentile score table of Math Problem Solving (PS)						
Grade Percentile (%)	P1	P2	P3	P4	P5	P6
10	341	413	430	448	474	547
25	371	439	456	474	505	587
50	404	468	484	503	539	632
75	438	497	512	532	573	678

Grade percentile score table of Composite score						
Grade Percentile (%)	P1	P2	P3	P4	P5	P6
10	364	427	449	472	494	536
25	386	446	471	492	519	565
50	410	467	496	514	546	597
75	435	488	521	536	573	629

Appendix G. Calculation of probabilities of correct with respect to Arithmetic (AR), Math Knowledge (MK) and Math Problem Solving (PS)

	AR	MK	PS
Mean item difficulty difference (logit)			
Probabilities of correct with item difficulty above ___ mean higher	0.54	0.38	0.33
1	0.37	0.41	0.42
2	0.25	0.32	0.34
3	0.17	0.24	0.27
4	0.10	0.18	0.21
5	0.06	0.13	0.16
6	0.04	0.09	0.12
7	0.02	0.07	0.09
8	0.01	0.05	0.07
9	0.01	0.03	0.05