A Project entitled

An Inquiry-based Course Pack for E-learning Class – Addition and Subtraction of Same Denominator Fractions

Submitted by

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Declaration

I, LU Yuen Wing declare that this research report represents my own work under the supervision of Dr. CHIU Mei Choi, and that it has not been submitted previously for examination to any tertiary institution.

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Abstract

This project focused on the design and construction of an e-learning course pack "Addition and Subtraction of Same Denominator Fractions". The course pack is accessible at the website: https://luyuenwing.wixsite.com/4n6-fraction. The following discussed and explained the teaching designs with design principles and objectives. The instruction of using different teaching materials is included to facilitate the application of the course pack.

Keywords: Course pack, e-learning, inquiry-based learning, fraction

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1. Introduction

Different textbook analyses show that the teaching of fractions focuses on procedural knowledge (Alajmi, 2012; Lenz et a., 2019; Son & Senk, 2010). Students are better at applying strategies without a thorough understanding. Researchers discovered there are common misconceptions of the rule of adding or subtracting fractions (Kara & Incikabi, 2018; Önal & Yorulmaz, 2017; Ni & Zhou, 2005; Soylu & Soylu, 2005; Tang, 2002) (see Figure 1).

Figure 1

Mistake made by students in adding and subtracting fractions

1. Adding or subtracting numerator with numerator and denominator with denominator separately.
2. Reversing the place of numerator and denominator.
3. Adding the integer to numerator directly.
4. Adding the integer to both numerator and denominator.
5. Subtracting the integer from the numerator directly.
6. Adding "10" to the numerator as borrowing from the integer.
7. Forgetting to minus 1 from the integer after borrowing.
8. Using the bigger digits to minus the smaller digits regardless of the original fractions.
9. Changing the arithmetic sign.

During my block practice, I have seen students make mistakes as they learn by remembering rather than understanding (Durkin & Rittle-Johnson, 2015). This package aimed to assist teachers in teaching "Fractions" with the inspiration of inquiry-based learning. Helping students develop both conceptual knowledge and procedural knowledge of fraction operations.

2. Literature review

2.1. Learning objectives

According to The Curriculum Development Council (2017), when learning "Fractions" in primary 4, the learning objectives are as follows (see Figure 2):

Figure 2

The learning objectives of "Fractions" in Primary 4

Learning Unit	Learning Objective	Remarks
4N6 Fractions (II)	1. recognise the concepts of proper fractions, improper fractions and mixed numbers	Students are required to recognise that a mixed number is the sum of a whole number and a proper fraction.
	2. perform the interconversions between improper fractions and mixed numbers	Interconversions such as "$\frac{6}{2}$ and 3" are required.
	3. recognise the concepts of expanding fractions and reducing fractions	Students are required to recognise the concept of fractions in their lowest terms.
	4. compare the magnitude of fractions with the same denominators	Comparing the magnitude of fractions and whole numbers is required.
	5. perform addition and subtraction of not more than three fractions with the same denominators	Addition and subtraction of fractions and whole numbers are required.
	6. perform mixed operations of addition and subtraction of three fractions with the same denominators	Mixed operations of addition and subtraction of fractions and whole numbers are required.
	7. solve problems	Note: (i) The result of calculations can be expressed as mixed numbers or improper fractions in the lowest terms. (ii) Students are required to recognise how to estimate the result of calculations.

This package focuses on the objectives of performing addition and subtraction of not more than three fractions with the same denominator. Solving word problems was also included in this package.

2.2. The Principles of the Teaching Design

2.2.1. Inquiry-based learning in Mathematics.

Inquiry-based learning (IBL) is a learning-centered approach where active learning is guided by authentic problem-based questions (Blessinger et al., 2015). IBL has gained great evidence in research that shows the effectiveness in learning Mathematics relative to the traditional teaching-centered approach (Capaldi, 2015; Hmelo-Silver et al.,2007; Kwon et al., 2005; Yoshinobu & Jones, 2012) (see Appendix A for further discussion).

This package focused on Structured Inquiry and Guided Inquiry (see Figure 3), where teachers provided the question for exploration. Students can make use of the prescribed procedure as well as developing their method in the exploration stage (Banchi & Bell, 2008). According to the 5E Model (Bybee & Landes, 1990), all events of the lesson are designed based on a learning-centered approach where knowledge should be built up by students rather than delivered from teachers (see Figure 4).

Figure 3

The four levels of inquiry and the information given to the student in each one

Inquiry Level	Question	Procedure	Solution
1—Confirmation Inquiry *Students confirm a principle through an activity when the results are known in advance.*	✔	✔	✔
2—Structured Inquiry ***Students investigate a teacher-presented question through a prescribed procedure.***	✔	✔	
3—Guided Inquiry *Students investigate a teacher-presented question using student designed/ selected procedures.*	✔		
4—Open Inquiry ***Students investigate questions that are student formulated through student designed/selected procedures.***			

Figure 4

5E Model of Inquiry-based Learning

2.2.2. E-learning in Mathematics.

Different e-learning platforms allow real-time evaluation for teachers to make teaching decisions based on students' immediate learning performance (Balta et al., 2017). Research have indicated that with interactive technologies, students' motivation, and participation in or outside the classroom would increase (Engin & Donanci, 2015; Fitzpatrick et a., 2011; Heaslip et al., 2014; Heflin et al., 2017, Pettit et al., 2015). Despite the convenience of technological devices in the classroom, there is evidence of the importance of traditional activities and assessments (see Appendix B for further discussion).

To maximize the effect of the teaching materials, students should be allowed and able to use individual digital tablets during and after the lesson. Hence, a primary 4 authentic classroom environment with digital supports is targeted.

3. Teaching Materials

This package includes teaching plans and teaching materials of 12 lessons (see Figure 5). All materials were organized on a website: https://luyuenwing.wixsite.com/4n6-fraction (see Figure 6).

Figure 5

The learning objectives of each 12 lessons

Addition	01	Addition of same denominator proper fractions
	02	Addition of same denominator mixed fractions
	03	Repeated Addition of same denominator fractions
	04	Word problems on addition of same denominator fractions
	05	Consolidation on addition of same denominator fractions
Subtraction	01	Subtraction of same denominator fractions without borrowing
	02	Subtraction of fractions from integers
	03	Subtraction of same denominator fractions with borrowing (horizontal form)
	04	Subtraction of same denominator fractions with borrowing (column form)
	05	Repeated subtraction of same denominator fractions with borrowing
	06	Word problems on subtraction of same denominator fractions
	07	Consolidation on subtraction of same denominator fractions

Figure 6

The website for the course pack

3.1. Real-time assessments

Assessments were created by Quizizz and Microsoft Forms, designed to recall the acquired knowledge and to facilitate the real-time evaluation of students' learning performance. Quizizz allows instructor-based quizzes, where teachers control the pace of the quiz and students participate with their tablet (see Appendix C for the Quizizz's instruction).

With the real-time collection of students' answers by the platform, the teacher would easily evaluate students' individual and overall performance (see Figure 7 and Figure 8). The real-time evaluation promotes teaching decisions. With the scoring system and leader board, students are motivated to participate in the assessment (see Figure 9). However, it may cause students to deviate from the original learning objectives, focused on exploring the fun and gaining scores (Dobler, 2015).

Figure 7 and Figure 8

Real-time evaluation from Quizizz

Figure 9

Scoring system of Quizizz

Microsoft Forms act as a more formal assessment tool. As a student-based quiz, students could answer at their own pace with an abundance of time. Teachers could review students' performance after the quiz with the charts and spreadsheets generalized by the platform (see Figure 10 and Figure 11).

Figure 10

Evaluation from Microsoft Forms

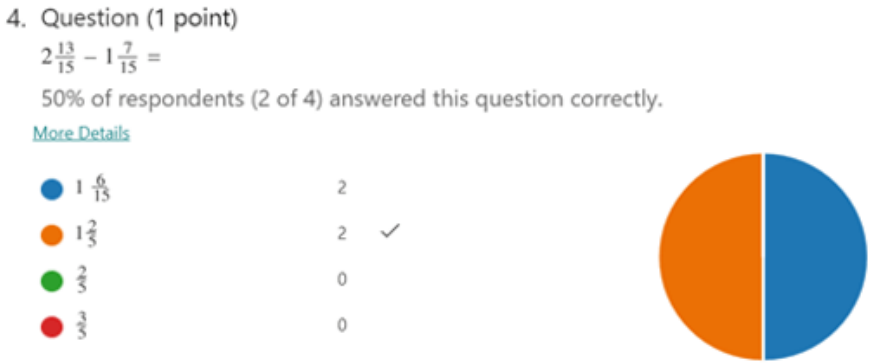


Figure 11

Spreadsheets from Microsoft Forms

	D	E	G	J	M	P	S	V	Y
1	名稱	總數	計算同分母真分數減	\frac{9}{11}-\frac{5}	計算同分母帶分數減	Question2\frac{13}{	選出最合適的下一步	選出最合適的下一步	
2	Student A	1	減分子；減分母	4	分別計算；相減	1\ \frac{6}{15}	5\frac{3}{3}−2\frac{3}{4	9\frac{9}{9}-9\frac{7}{9}	
3	Student B	5	減分子；不變	\frac{4}{11}	分別計算；相加	1\frac{2}{5}	5\frac{4}{4}−2\frac{3}{4	9\frac{10}{9}-9\frac{7}{9}	
4	Student C	4	減分子；不變	\frac{4}{11}	分別計算；相加	1\ \frac{6}{15}	5\frac{4}{4}−2\frac{3}{4	9\frac{10}{9}-9\frac{7}{9}	
5	Student D	6	減分子；不變	4	分別計算；相加	1\frac{2}{5}	5\frac{4}{4}−2\frac{3}{4	9\frac{10}{9}-9\frac{7}{9}	
6									

3.2. Inquiry-based activities

3.2.1. GeoGebra.

The GeoGebra activities are created for inquiry, guiding students to explore the concepts through different steps individually (see Figure 12). It visualizes abstract concepts and dynamically shows problem-solving processes. After exploring with pictures, students are required to review the problem-solving process in the horizontal form of calculation (see Figure 13).

Several trials of reviewing with different numbers allow students to generate the calculation method. As they perform and generalize the whole process, they could have a greater understanding of the concepts and methods developed by themselves.

Additionally, with the immediate feedback from GeoGebra, students can explore and directly review their steps, reinforcing a positive self-study attitude (Beaumont et al., 2011; Smithrud et al., 2015).

(See Appendix D and Appendix E for detailed instructions of the GeoGebra activities).

Figure 12

Exploration stage GeoGebra activity

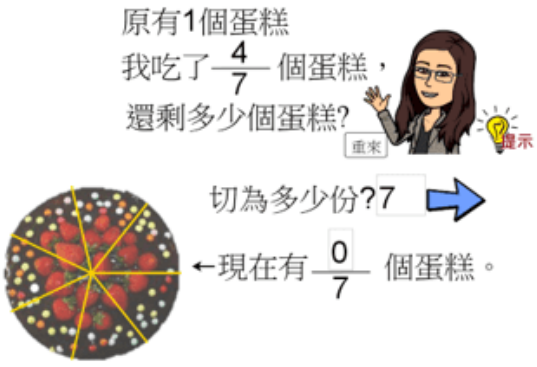


Figure 13

Review stage GeoGebra activity

3.2.2. Hands-on group activities.

Classwork such as hands-on group activities are designed to facilitate the inquiry through collaboration (see Appendix F for hands-on group activity example).

By exploring in groups, collaboration provides peer guidance and supports for tackling difficult problems (Capaldi, 2015). All stages are student-centered, students should present their discovery to others and generate the calculation method through discussion. The role of teachers is to provide a minimum amount of guidance and facilitate different stages of inquiry.

By hands-on activities, students can have a stronger understanding of the knowledge as the conceptual knowledge and calculation rules are developed by students' personal experience and discussion.

3.3. Worksheets

The in-class worksheet allows students to summarize the conceptual knowledge and perform the calculation to practice their procedural knowledge. There were a variety of worksheets for exploring, practicing, or consolidating, etc. (see Appendix G for classwork example).

There were guidelines assisting students with weaker performance (see Figure 14). And there were challenging questions for students with stronger performance after they have finished the required tasks (see Figure 15). Besides basic questions that train students' ability to calculate, there were different kinds of questions enhancing students' motivation and problem-solving skills in both classwork and homework.

Figure 14

Assistance on worksheets

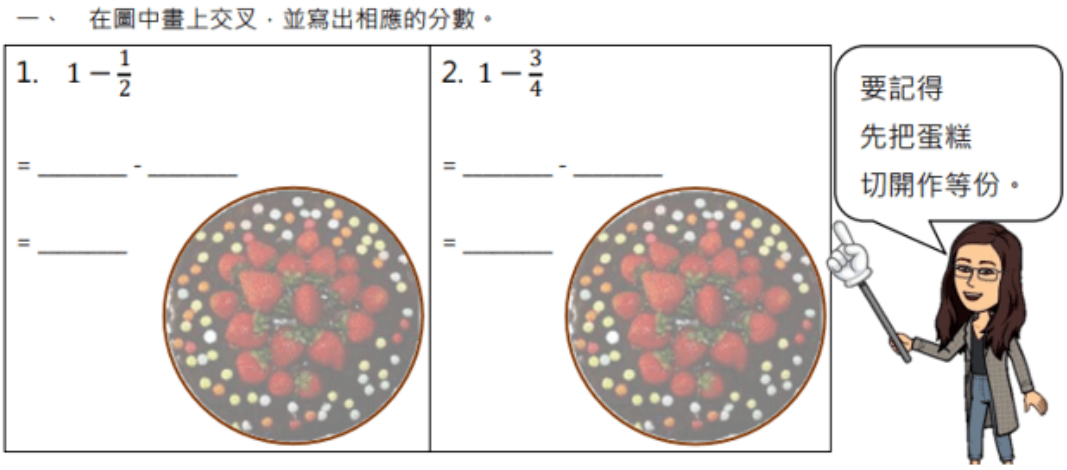
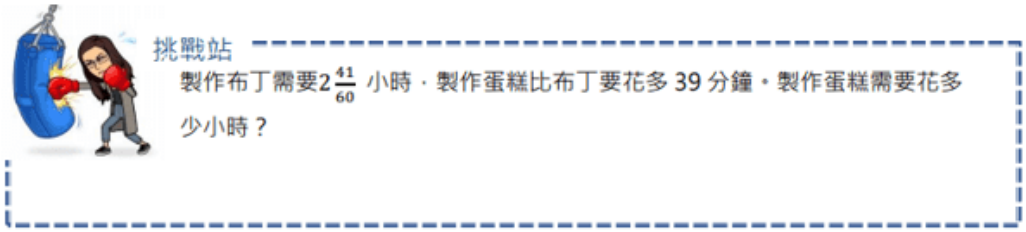


Figure 15

Challenges on worksheets



3.4. After-class materials

3.4.1. Revision videos.

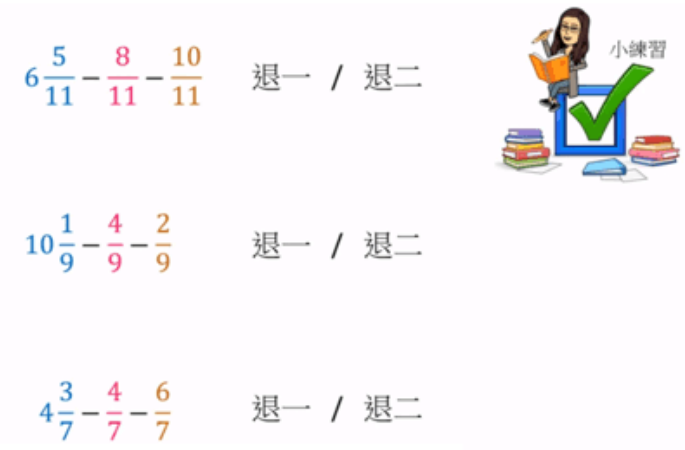
During the student-centered lesson, teachers will not be delivering and explaining knowledge as often as usual. Revision videos were created with clear demonstrations, assisting different educational needs. Each video was created based on a learning objective (see Figure 16), summarizing the concepts and calculation method of the learning objective. There were questions included for students to practice (see Figure 17). Students can review and consolidate their newly acquired knowledge.

Figure 16

The videos with different objectives

Figure 17

The questions in revision videos



The videos are organized in a Youtube playlist:

https://www.youtube.com/playlist?list=PL1TpgxHIaMn-zQCxvYXCnlAXilIlD5o-n

Students are welcomed to replay the videos at any time, with timestamps to facilitate the viewing experience (see Figure 18). For learning objectives that allow different calculation methods, there are different versions of the video. Allowing students to watch according to their preference (see Figure 19).

Figure 18

Timestamps in videos

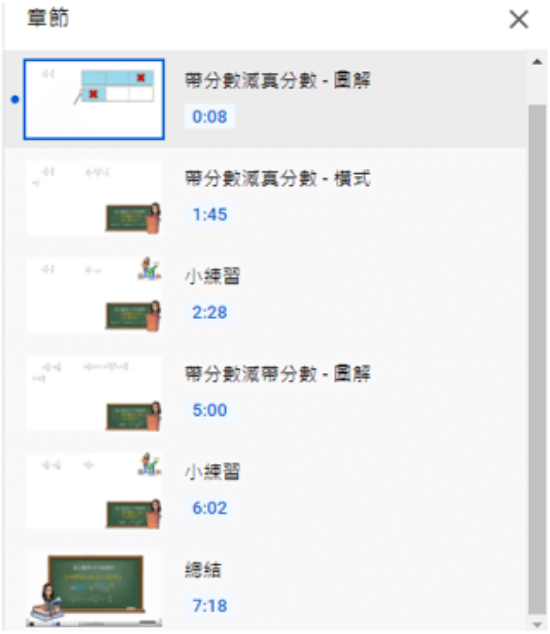


Figure 19

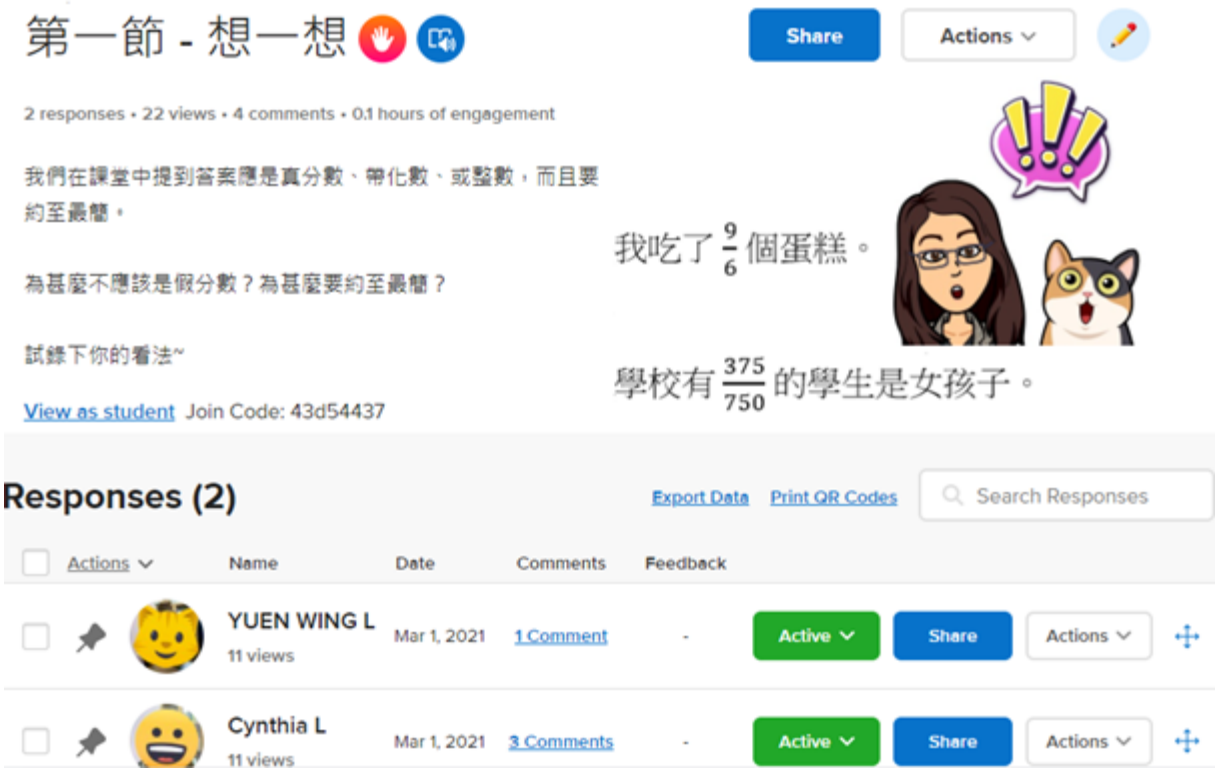
The different versions of videos

3.4.2. Online discussion.

To promote students' problem-solving skills and raise students' interest in Mathematics, there were online discussion assignments that required all students to participate. Regarding the learning objective of each lesson, students have to join a discussion or solve a challenging question on Flipgrid (see Figure 20).

Figure 20

The example of a Flipgrid discussion



Students could post their responses to teachers' questions with different types of presentation methods (see Figure 21). They could also give out comments to others (see Figure 22). It promotes the discussion of Mathematics outside the classroom. The teacher could show some students' responses and discuss them together in the next lesson. The online discussion promotes students to investigate the fun in Mathematics together, allowing students to learn from one another (see Appendix H for Flipgrid instruction).

Figure 21

The different presentation methods on Flipgrid

Figure 22

The comments section on Flipgrid

4. Package Organisation

All teaching materials including guidance and instructions were uploaded to a Wix website: https://luyuenwing.wixsite.com/4n6-fraction. With a website, readers could access all materials in an organized way (see Appendix I for the design of the website).

5. Reflection

When designing the materials, different textbooks were analyzed. Understanding the learning objectives and the difficulty of students learning in this topic. Each question and step has been fine-tuned to make the materials the most effective, allowing students to learn and practice without getting bored. The promotion in students' motivation was also considered as an important aspect of the course pack.

Examination of different software and tools is required in selecting preferred effects. It was the first time making a website as well as making videos. To reach the desired result, numerous versions have been produced.

Unfortunately, the course pack has not been applied to analyze the effect and reflection from students. The materials were tested by other teachers and student teachers, their opinions and advice promote the development of the materials. When the course pack is applied in the future, precise adjustments would be made according to students' performance and reflection.

6. Conclusion

The course pack includes teaching materials for different activities and assessments inside and outside the classroom. Throughout the whole project, I have built up more skills in different stages and became more confident in creating different teaching materials. Therefore, it benefits me in my teaching career, and I believe that I would keep on designing and constructing different materials in the future.

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Appendix A

Further Discussion on Inquiry-based Learning

IBL lessons require appropriate planning, activities, and assessments to allow teachers as the facilitators to guide the inquiry process (Blessinger et al., 2015). From engaging the situation to evaluating the experience, all stages of the inquiry are student-centered, where teachers provide a minimum amount of guidance.

Through different stages of the IBL lesson, students develop their self-regulated learning skills and learn to identify a more meaningful way to learn (Blessinger et al., 2015). Students are motivated to explore as they are allowed to apply their acquainted knowledge, critical thinking skills, and reasoning skills (Huijser et al., 2015). Moreover, IBL enhances students' "mathematical proficiency" of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Yoshinobu & Jones, 2012).

Appendix B

Further Discussion on E-learning in Mathematics

A stable platform for e-learning is applied with the mature development of mobile technology (Svela et al., 2019). Inside the classroom, efficient use of digital technology increases the opportunities for active and collaborative learning (Pettit et al.,2015). Technology provides more variety and opportunities for students to explore mathematical concepts, analyze different situations (Sawaya & Putnam, 2015). Bray and other researchers (2017) showed that technology motivates students to take on more responsibility in facilitating their mathematics learning. Besides, technological devices and software allow teachers to visualize complex or abstract concepts and show problem-solving in a dynamic way (Bray et al., 2017; Geiger et al., 2010).

Outside the classroom, students could achieve a higher score with practice in online assessments, leading to a positive relationship between the completion of online homework and academic performance (Eichler & Peeples, 2013). Moreover, the reduction in the time needed in distributing and collecting assignments, and the convenience in monitoring students' individual and overall performance benefit the teaching efficiency of teachers (Fynewever, 2008; Ismail et al., 2014; Lunsford & Pendergrass, 2016).

There are concerns about the usage of technology in education. Different devices and software may cause students to deviate from the original learning objectives, focused on exploring the fun and other items in the software (Dobler, 2015). By answering the final answer only in online assessments, it might result in students focused on getting the correct answers rather than the process of finding the answers (Mendicino, Razzaq, & Heffernan, 2009). Online assignments may introduce a lack of handwriting, reducing the learning and practice through motor memory (Smithrud et al., 2015). Hence, studies emphasized the effectiveness and the need for restricted use of technology in and outside the classroom. The importance of the implementation of traditional assessments along with online assessments was also affirmed.

Appendix C

Instruction of Quizizz

The questions of the assessments are designed to recall acquired knowledge. Multiple-choice questions and open-ended questions are enhancing the variety of answers (see Figure 23). Quizizz also allows questions such as checkbox, fill-in-the-blank, poll, and adding slides during the quiz. The assessments are recommended to be participated under an Instructor-paced live quiz (see Figure 24), allowing students to go through each question together. Teachers would also have better control over the time limit for the whole process. The following settings are recommended to facilitate the effectiveness of the quiz (see Figure 25). A Scoring system is inserted to motivate student's participation. While the timer is off, preventing students to rush through the quiz trying to gain extra points by being the first to answer. Teachers could access and apply the quiz or adjust the original design on digital devices with the hyperlink: https://quizizz.com/admin/quiz/60360a74fa3c2c001dbc4dbe.

Figure 23

Example of questions for Quizizz

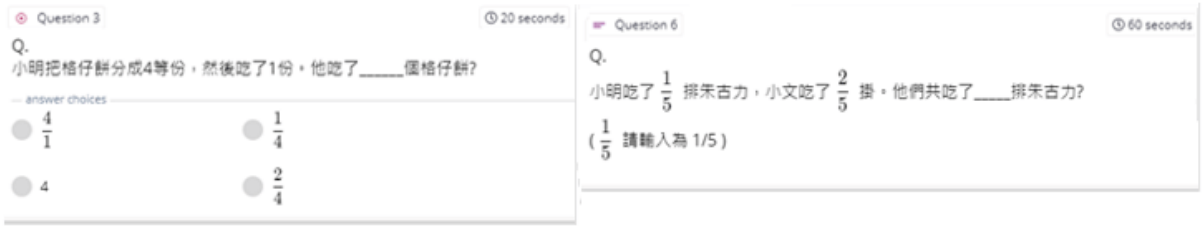


Figure 24

Format of the Quiz

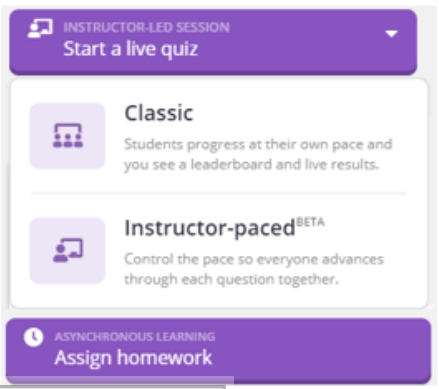
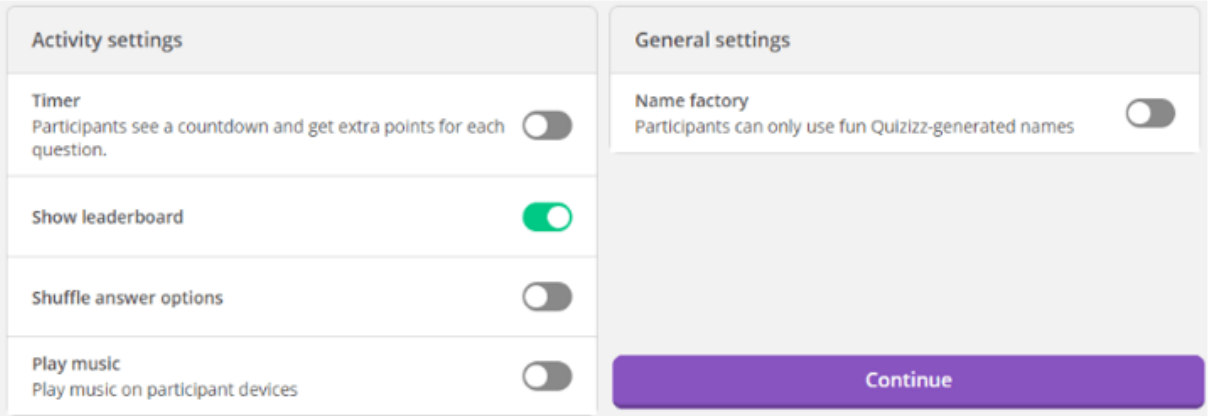


Figure 25

Recommended setting of the Quiz



Before starting the quiz, students should join the quiz by entering the specific game code at https://quizizz.com/join. Teachers could also share the hyperlink generated each time the teacher starts a quiz. Students should input their names, allowing teachers to monitor their participation (see Figure 26).

Figure 26

Teacher screen before starting the quiz

During the quiz, students should read and answer questions on their digital tablets and by pressing on the choices to answer the questions (see Figure 27). Teachers could monitor the progress of students on the digital device, pausing the quiz or skipping questions.

After each question, teachers review students' overall performance easily with a bar chart (see Figure 7 in Chapter 3.1). Giving out reminders before answering the following questions. Teachers could also evaluate after the quiz, as the portion of correct answers was shown (see Figure 8 in Chapter 3.1). After the quiz, a leader board would be generated to show the individual scores of students (see Figure 8 in Chapter 3.1). Teachers could connect the ranking to the regular rewards system to further motivate students' participation. Quizizz also provides a spreadsheet for teachers to download and review after the live quiz (see Figure 28), further promoting the teaching design of the following lessons.

Figure 27

Students screen

Figure 28

Spreadsheet of Quizizz

Quizizz: 整數減分數回顧

Quiz started on: Fri 19, Mar 11:25 AM Total Attendance: 2 Average Score: 2920

Questions	Class Level			Player Level	
	# Correct	# Incorrect	# Unattempted	Student B	Student A
根據_____決定整數要均分為多少份？	1	1	0	分母	分子
1−25=1−\frac{2}{5}=1−52=	2	0	0	55−25\frac{5}{5}-\frac{2}{5}55−52	55−25\frac{5}{5}-\frac{2}{5}55−52
1−5161−\frac{5}{16}1−165	0	2	0	585\frac{5}{8}85	111111
3−17=3−\frac{1}{7}=3−71=	1	1	0	277−172\frac{7}{7}-\frac{1}{7}277−71	77−17\frac{7}{7}-\frac{1}{7}77−71
5−313=5−3\frac{1}{3}=5−331=	2	0	0	4 33− 3134\ \frac{3}{3}-\ 3\frac{1}{3}4 33− 331	4 33− 3134\ \frac{3}{3}-\ 3\frac{1}{3}4 33− 331
8 − 435=8\ −\ 4\frac{3}{5}=8 − 453=	1	1	0	855− 4358\frac{5}{5}-\ 4\frac{3}{5}855− 453	755− 4357\frac{5}{5}-\ 4\frac{3}{5}755− 453
Total	7	5	0	3220	2620

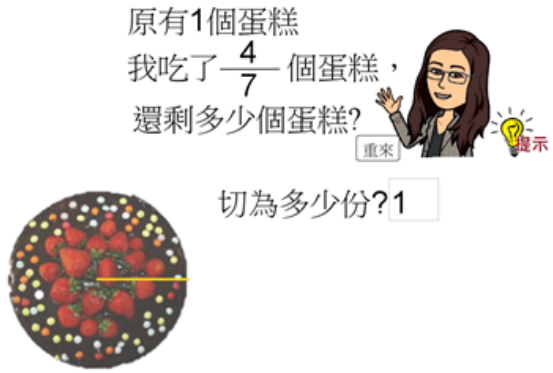
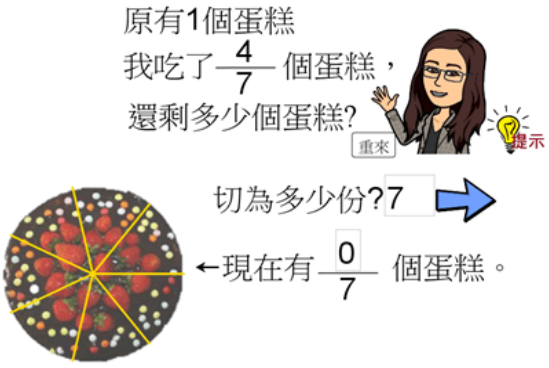
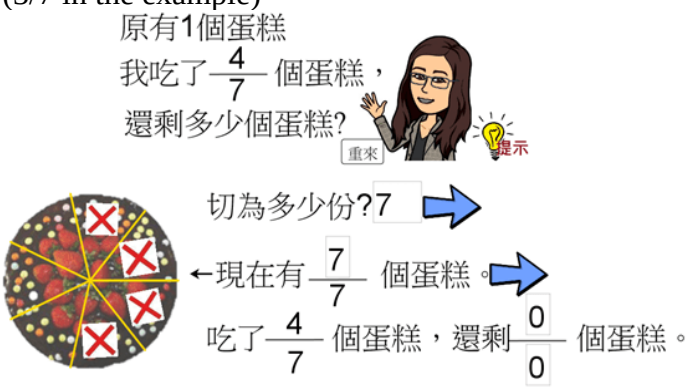
Appendix D

Instruction of GeoGebra Activity "Subtracting fraction from a whole"

This GeoGebra activity aimed to assist students in the inquiry of "Subtracting fraction from a whole". The goal of the GeoGebra is to find out the remaining fraction of cake after a portion of cake has been distributed.

Before starting the inquiry, teachers should guide the students to discuss a story. When we have a cake and would only want to eat half of it, what should we do? We have to cut the cake into two pieces. Then students should discuss the relation between the number of pieces and the desired fraction.

Then students could enter https://www.geogebra.org/m/pdc2pbbw to begin the inquiry. The inquiry is divided into two sections, exploring with diagrams and reviewing in horizontal form. The steps of the GeoGebra are as follow:

<table>
<tr><td rowspan="4">Explore</td><td>Type the number of equal parts that the cake should be divided into.
(7 in the example)
 
</td></tr>
<tr><td>Press the “提示” button to get a clue on the number.
 
</td></tr>
<tr><td>Type the numerator of the fraction.
(7 in the example)
 
</td></tr>
<tr><td>Type the remaining number of the cakes in fractions.
(3/7 in the example)
 
</td></tr>
</table>

Review	Recall that the cake has been divided into equal parts. And type the number of cakes in improper fractions. (7/7 in the example) 
	Perform simple subtraction of subtraction of same denominator fraction. And type the number of remaining cakes in fractions. (3/7 in the example) 
	Restart the exploration with different numbers by pressing on the “重來” button. 

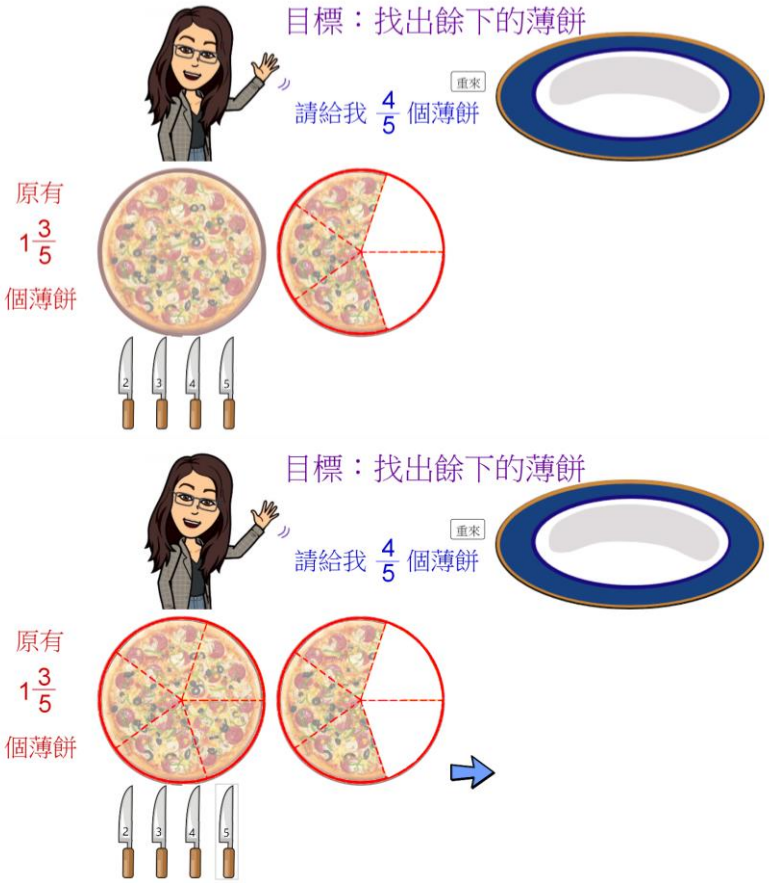
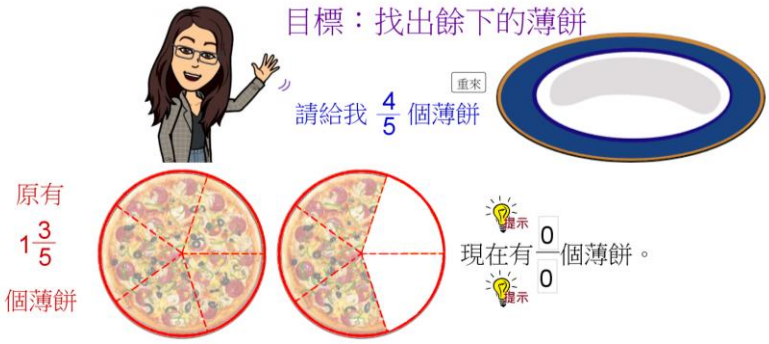
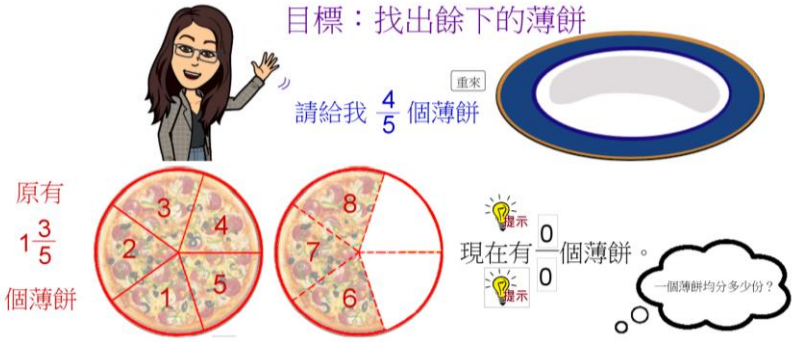
Appendix E

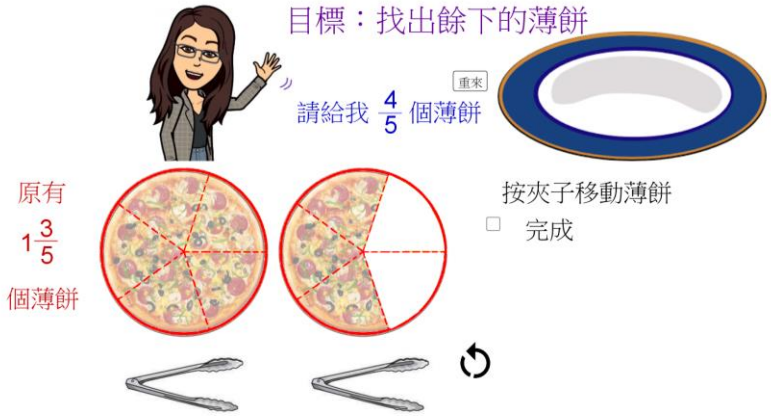
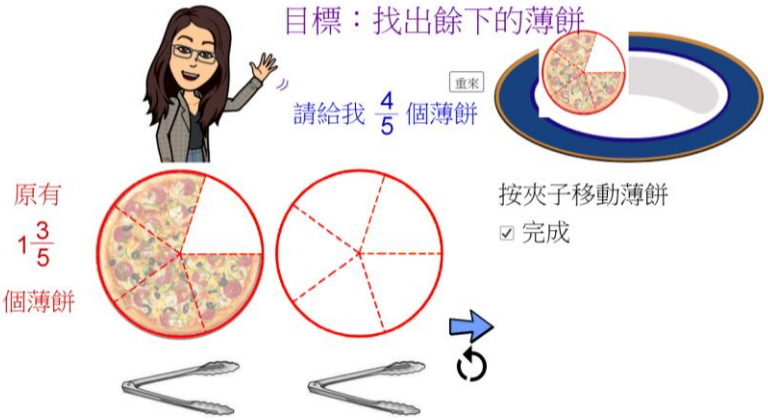
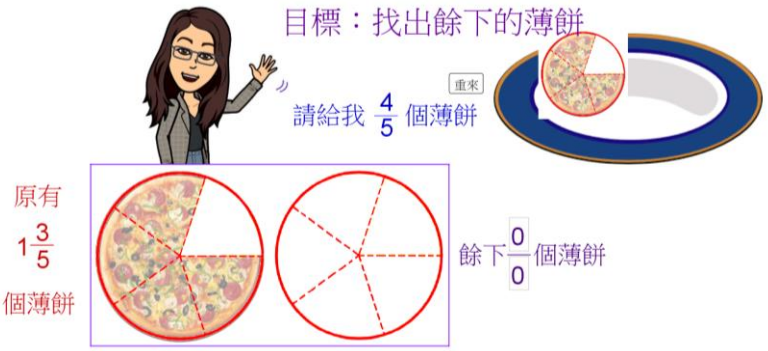
Instruction of GeoGebra Activity

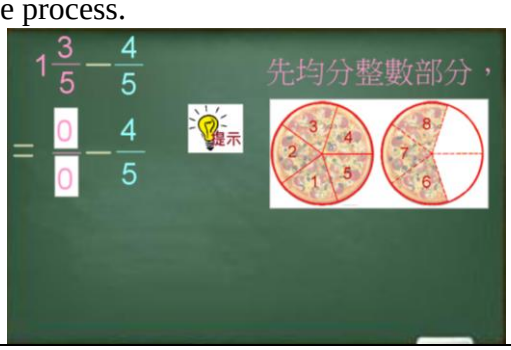
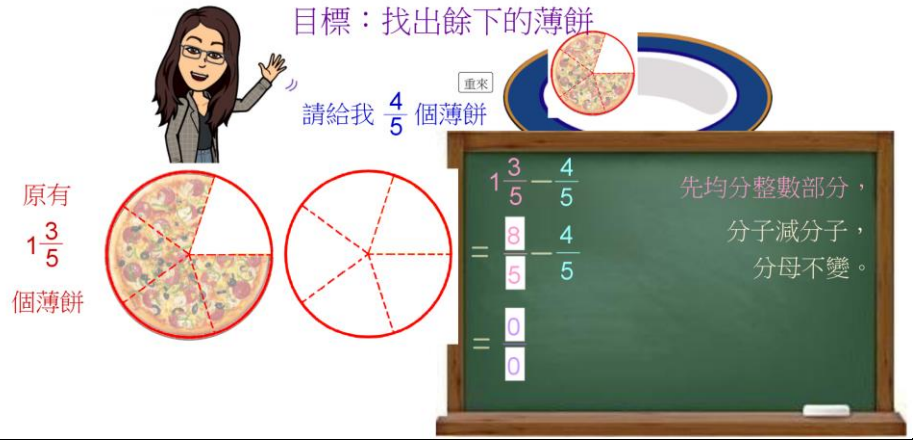
"Borrowing from the whole number in fraction subtraction"

This GeoGebra activity aimed to assist students in the inquiry of "Borrowing from the whole number in fraction subtraction". The goal of the GeoGebra is to find out the remaining fraction of pizza after a portion of pizza has been distributed. The Geogebra could be downloaded through https://drive.google.com/file/d/1O2miTZ2E-_a2GJlVSXbF7NngvXFkT0EU/view?usp=sharing.

Before starting the inquiry, students should recall the acquired knowledge to divide the whole into equal parts according to the denominator to facilitate the following calculation. The inquiry is divided into two sections, exploring with diagrams and reviewing in horizontal form. The steps of the GeoGebra are as follow:

Explore	Press on the numbered knife to divide the pizza into equal parts. (5 in the example) 
	Type the numerator and denominator of the improper fractions. (8/5 in the example) 
	Press the “提示” button to get clue on both numbers. 

Explore	Click on the tongs to distribute the pizza. 
	Click on the check box after distribution. 
	Type the remaining amount of pizza in fractions. 

Review	Recall that the whole pizza has been divided into equal parts. And type the number of pizzas in improper fractions. (8/5 in the example) 
	Press on the “提示” button to visualize the process.  
	Perform simple subtraction of subtraction of same denominator fraction. And type the number of remaining pizzas in fractions. (3/5 in the example) 
	Restart the exploration with different numbers by pressing on the “重來” button. 

Appendix F

Example of Hands-on Group Activity

For the inquiry of mixed fractions subtraction without borrowing, a hands-on group activity is designed. The activity materials including the props, worksheet, and instruction were posted on the website:

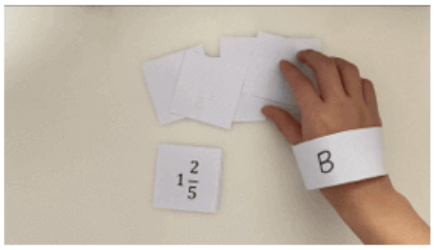
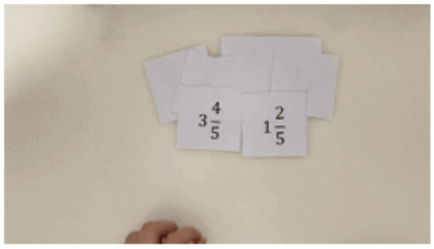
https://luyuenwing.wixsite.com/4n6-fraction/%E7%AC%AC%E4%B8%80%E7%AF%80-%E4%B8%8D%E6%B6%89%E5%8F%8A%E9%80%80%E4%BD%8D%E7%9A%84%E5%90%8C%E5%88%86%E6%AF%8D%E5%88%86%E6%95%B8%E6%B8%9B%E6%B3%95

Teachers should read through the instruction or watch the instruction video and print out the necessary props and worksheets before the lesson (see Figure 29).

Figure 29

Materials for the activity

Students work in two to participate in the inquiry. The inquiry stages are as follows:

1. Students pick a random fraction card respectively.	
2. Compare the two fractions. Students with the larger fraction act as the baker. Students with the smaller fraction act as the customer.	
3. Baker uses the cake cards to make the required amount of cake according to the fraction card.	
4. Both students verify the amount.	
5. Customer takes away the cake cards according to the fraction card.	
6. Both students verify the amount.	
7. Count the remaining amount of cake.	
8. Record the process in fraction on the activity worksheet.	
Repeat the above steps twice.	

The exploration stage allows students to apply their acquired knowledge in their problem-solving progress. Working in groups allows students to collaborate and learn from each other. For weaker students, collaborating provides peer guidance and supports for tackling difficult problems (Capaldi, 2015). Students with stronger performance can benefit from group discussion as they can see how others face struggles and by providing suitable advice, guiding the team towards the conclusion (Capaldi, 2015).

After exploration, students should explain the new concepts and knowledge they have learned. Groups that have success in the process were chosen to present their method in front of other students. Through the discussion and presentation, students could generalize the calculation method.

Teachers act as the facilitator during the inquiry process, providing a minimum amount of guidance. For example, guiding students to compare the whole number and the fraction part separately.

By hands-on activities, students could promote understanding through motor memory. The learning of conceptual and procedural knowledge is promoted as the rules of calculation are developed by themselves through exploration and discussion.

Appendix G

Design of Classwork

There are a variety of worksheets, the following will introduce the design of the consolidation in-class worksheet in the addition of same denominator fractions.

The first page is a task of "Little Detectives find the mistakes" (see Figure 30). Students are required to find out the mistakes in different calculations. By discovering the mistakes, students would revise their knowledge and be more aware of the common mistakes in this topic. Then, students are required to explain the mistakes and present the correct calculation methods to others, reinforcing their knowledge through demonstration.

Figure 30

The task "Little Detectives find the mistakes"

一、小偵探找錯處

以下的回答中出現了錯誤導致答案也不正確，

圈出錯處並寫上正確做法。

$\frac{3}{5}+\frac{1}{5}$ $=\frac{3+1}{5+5}$ $=\frac{4}{10}$ $=\frac{2}{5}$ 正確做法：	$1\frac{2}{4}+3\frac{3}{4}$ $=(1+3)+(\frac{2}{4}+\frac{3}{4})$ $=4+\frac{5}{4}$ $=4\frac{1}{4}$ 正確做法：	$2\frac{7}{10}+7+7\frac{3}{10}$ $=(2+7)+\left(\frac{7}{10}+\frac{3}{10}\right)$ $=9+\left(\frac{7}{10}+\frac{3}{10}\right)$ $=9+\frac{10}{10}$ $=10$ 正確做法：

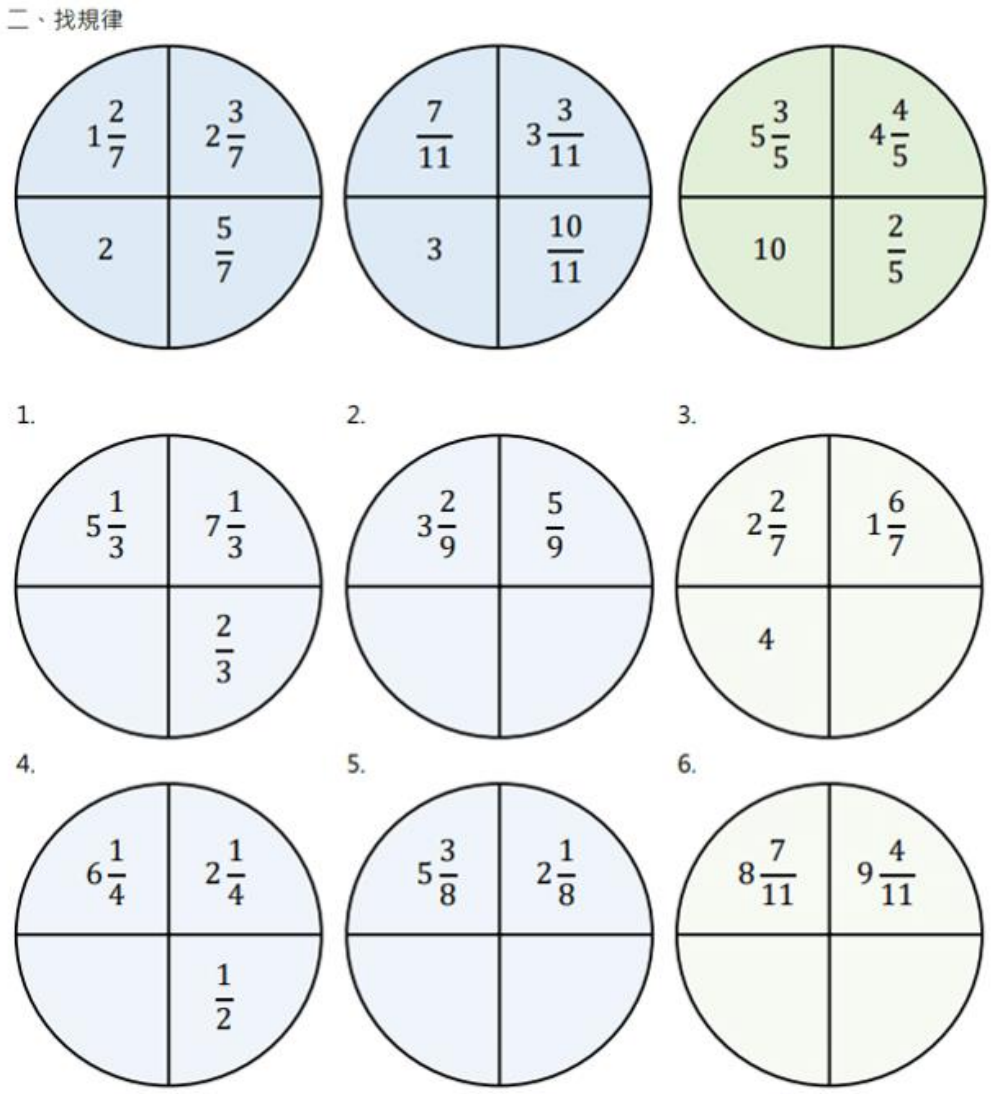
The second page is a task of "Finding patterns" (see Figure 31). Students work in groups to fill in the blanks according to the patterns:

The bottom left is the whole number of the sum of the above factions.

The bottom right is the fraction part of the sum of the above factions.

Figure 31

The task "Finding patterns"



Students could consolidate their knowledge of the addition of the the same denominator fractions through investigation and discussion. Then by demonstrating the problem-solving progress, students can consolidate their conceptual and procedural knowledge of the addition of mixed fractions with the same denominator.

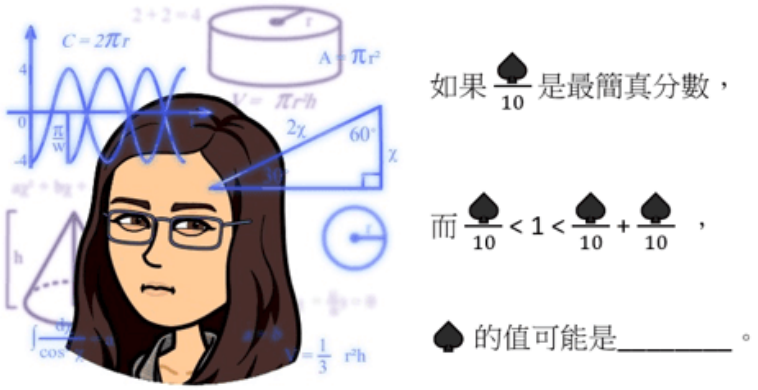
Appendix H

Instruction of Flipgrid

With Flipgrid, teachers could post questions for discussion after the lesson. For example, discuss why the final answer should be in irreducible form and why an improper fraction is not recommended (see Figure 20 in chapter 3.4.2). Teachers could also post a challenging question, allowing students to further explore the topic (see Figure 32). The discussion topics have been designed for the lessons, teachers could access and apply the topic or make adjustments to the hyperlink: https://flipgrid.com/lu5771.

Figure 32

The example of a challenging question



With different types of recording methods allowed in Flipgrid, teachers could design different types of questions, And students could respond in their most comfortable way (see Figure 21 in Chapter 3.4.2). Students could reply and give likes to others to facilitate the discussion (see Figure 22 in Chapter 3.4.2). The online platform allows students to join the discussion at any time and anywhere. Hence, even for students with weaker performance, they can watch other students' solutions and develop their own opinion. In the next lesson, teachers could choose and present some student's responses, gaining attention and stimulating recall of the prior learning.

The Flipgrid discussion allows students to discuss and investigate the fun in Mathematics together outside the classroom. Promoting students' mathematical thinking skills and problem-solving skills raises students' interest in Mathematics.

Appendix I

Design of the Website

When arriving at the website, viewers can press the button to view the content page (see Figure 33). The lists of materials arrange all teaching materials according to the lessons and the purposes (see Figure 34). By clicking on the icons, the pdf file of the worksheets or guidelines would appear. The hyperlinks to different online materials were also linked to the icons. The revision videos were also inserted on the website (see Figure 35).

Figure 33

The content page of the website

Figure 34

The list of materials

Figure 35

The page of revision videos

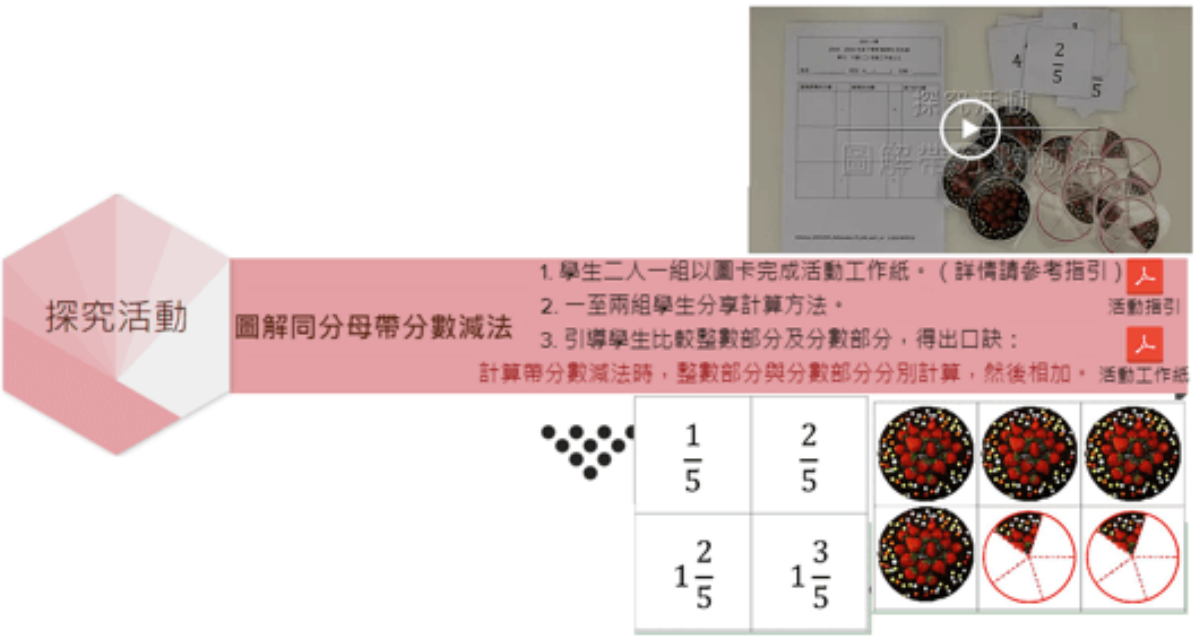
Viewers could also view the teaching materials according to the lessons. The detailed page provides the learning objectives and was arranged by the teaching plan of the specific lesson (see Figure 36). When the mouse moves to different events of the lesson, the corresponding brief introduction, and the preview will appear (see Figure 37).

Figure 36

The page of a specific lesson

Figure 37

The brief introduction of events



The website facilitates the viewing experience of the teaching materials in an organized and detailed way, viewers could easily access and download the teaching materials for the application.