A Project entitled

Attempt to solve spin system by computational method

Subtitled

Impact of Self-Sustained Clusters(SSC) on Spin Glass

Submitted by

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A Project submitted to

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Declaration

I, Tse Shun Hang Wittiem declare that this research report represents my own work under the supervision of Dr YEUNG Chi Ho, and that it has not been submitted previously for examination to any tertiary institution.

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Abstract

In this report, I am going to describe my work, result and discussion on the honours project in semester two of the academic year 2020-2021. The objective of the project is to attempt to solve the existing spin system by the selected computational method.

The report starts with introducing the background of spin system, computational method and the objective and expected results that show the reason and motivation for doing the project. Then, the platform and environment the project is used for development and runtime are reviewed. The introduction part will give a brief look and guide to the way the project works practically.

Then, the theory and Definition of the related topic are reviewed because there is a need to cover the theory, definition and some unknown in the existing theory. After introducing the theory, the report will cover the method of study that transforms the theories into a computer simulation. As well as the way the experiment is conducted. The experiment aims to test in the project to see firstly if the self-sustained clusters existed in SK spin glass model and secondly to see if the self-sustained clusters do affect the spin glass. In this part, I will also show how the equation works and transform it into the simplified equation that works in a computer simulation.

In the Result and Discussion part, I will discuss the result the project found and the effect which may solve the existing problems. The project tries to explore the impact of Self-Sustained Clusters(SSC) on the ising spin glass system.

In the conclusion part, the research progress of the project is reviewed. The Future Work is suggested in further development of the project in the coming times.

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Chapter 1. Introduction

1.1 Motivation

It is well known that computational methods are used in solving different physics problems. Monte Carlo simulation is one of the most commonly used computational methods in simulating physical systems and providing solutions. In some simulations of spin models, even solutions are provided but it is hard to find their optimum solution. This comes to my attention to study the related problems. The project tries to examine existing theories on the Spin system, to see the existence of the phenomenon and attempt to provide evidence to support the possible explanation to the situation that the spin system cannot find its optimal state to reach its ground state of system energy at the lowest temperature.

1.2 Background

Computational Method

In physics study, numerical algorithms are often used in simulating the physical system and solving problems. One algorithm called the Monte Carlo Method was proposed in the 1950s by Metropolis (1953). The technique name comes from the famous European gambling centre - Monte Carlo. Because of the simplicity and practicality of the Monte Carlo method, even after seventy years of history, the algorithm is still widely used in simulating physical systems such as a spin system and solving problems in the physical system.

Monte Carlo method is very suitable for practising two dimensional Ising Model. (J.E. Gudmundsson, 2010) so the method and the similar spin model is selected for the project.

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Spin System

The concept of the Monte Carlo Algorithm is considering a physical system as a statistical system that satisfies the Boltzmann probability distribution function at a certain temperature. In the simulation by the Monte Carlo algorithm, the Magnetization, system energy and other related quantities can be computed by the random sample generated from many, as much as possible, random sampled states in the procedure. (H. Gould & J. Tobochnik, 1996) The Monte Carlo method is often used in simulating the physical system with ising spins. (Inagaki, Inaba, Hamerly, Inoue, Yamamoto & Takesue, 2016) This is also the algorithm used in the project to conduct simulated experiments and collect the data by computing in the selected simulated spin system.

Simulated Annealing

To find the global minimum of a function, S. Kirkpatrick, C. D. Gelatt and M. P. Vecchi (1983) suggested that the metropolis program can do the job. It is because the cost function of optimization problems is very similar to the energy of a physical system. Simulated Annealing is an algorithm that is similar to physical solid annealing. In physical solid annealing, the solid first heats to a sufficiently high temperature. In high temperature, the internal energy of the solid is high and the internal particles become disordered. Then the solid is slowly cooling down. In the process of cooling down the solid, the internal energy is decreased as well as the internal particles become ordered. Finally, it reaches equilibrium and also the internal energy of the system reaches its minimum at the lowest temperature ie. room temperature. Very similar to physical object annealing, the concept of simulated annealing is to heat the system to a very high temperature at the start. Then slowly cool it down until it reaches the lowest temperature. The benefit of Simulated Annealing is that it always provides a solution even though the solution is not always

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the optimal answers to the problem. (P. J. M. van Laarhoven & E. H. L. Aarts, 1987) In recent years, the method Quantum Annealing (QA) was invented to apply the quantum fluctuation property into classical simulated annealing methods to attempt to find the optimum solution. (P. Ray, B. K. Chakrabarti, and A. Chakrabarti, 1989))In this project, the method of simulated annealing is chosen to the simulated model of the spin system to conduct the experiment.

Self-sustained clusters (SSC)

The theory of Self-sustained clusters (SSC) suggested in the spin systems, there exists of dominant cluster. (Yeung & Saad, 2013) That means the spin in the cluster will not interact with the spins out of the cluster. The theory will be introduced in detail in the following sections. The project will focus on exploring the relation of the theory to the spin systems.

1.3 Research Objective

The Project attempts to solve spin systems by various computational methods to achieve the following objectives:

1. Do Self-sustained clusters (SSC) exist in the SK Spin Glass model?
2. If yes, what form will Self-Sustained Clusters (SSC) exist?
3. What impact Self-Sustained Clusters (SSC) have on the SK spin glass system?
4. What relation of Self-Sustained Clusters (SSC) with the system energy of SK spin glass system at the lowest temperature?

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Expected outcome

Applying the method of simulated annealing into the ising spin glass model, it is expected to be successfully approached to a solution. With doing several samples, the project is expected to analyze the data for several research outcomes:

1. Self-sustained clusters (SSC) detected in SK spin glass model
2. Measure the Self-sustained clusters (SSC) configuration in the SK spin glass model.
3. Relationship between Self-sustained clusters (SSC) and properties of the Ising Spin Glass model found.
4. If Self-sustained clusters (SSC) existed, system energy at the lowest temperature will be higher.

1.4 Development/ Runtime Environment

Environment	Platform / tool	Programming Language involved
Development Environment	Window 10 X64／Visual Studio 2017 Chrome OS／Browser-based cloud tools	C++
Runtime Environment	Window 10 X64／Visual Studio 2017	／

Some developing processes are conducted on a MediaTek Processor chromebook while most of the developing processes and all the runtime processes of the simulation are run on a 3.4Ghz Ryzen™ Processor Windows 10 PC by Visual Studio 2017.

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Visual Studio 2017

Visual Studio 2017 is an integrated development environments (IDEs) tool from Microsoft which is free software that is supported by Microsoft Windows 10. It includes most of the tools needed and supports programming languages such as C++. Version 15.9.24 of Visual Studio 2017 is currently being used for the development and runtime of the project.

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Chapter 2. Theory and Definition

2.1 Ising Model

The ising model is a magnetic model named after Ernst Ising. It is well known and well studied for its magnetic properties and phase transition in statistical mechanics. (MA, 1990) In figure 1, it shows a configuration of a 5×5 2D Ising model on square lattice. Similar to the figure In the project, we are studying a two-dimensional ising model on 10×10 square lattice.

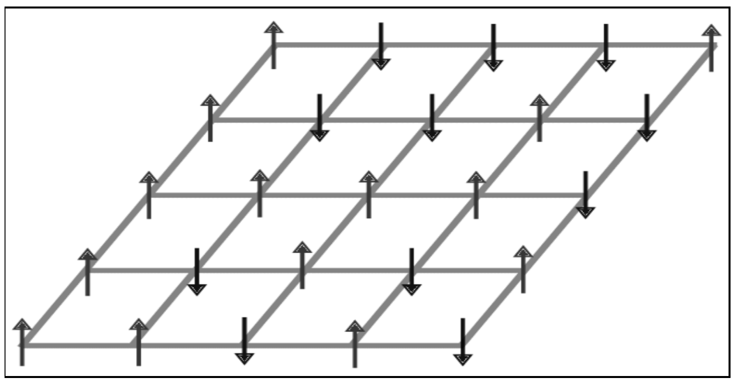

Figure 1. A configuration of a 2D Ising model on square lattice. Adapted from 'Thermalisation and Relaxation of Quantum Systems.' by W. Sascha, 2017

In the system of the ising model, there are N variables of *spins s* which can define to be two possible states, ($+1$ or -1), in short form $+$ or $-$). For any nearby *spin s* variable, there is an *interaction J*. The total energy of the system is given by *Hamiltonian H*,

$$H = -J\sum_{ij} s_i s_j - h\sum_{i} s_i \tag{1}$$

Where the h represents the external magnetic field to the spin system. In the project, the method of studying is not examining the external magnetic field so without considering the external field

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that interacts with the spin system, we have h = 0. therefore, total system energy can simplify to *Hamiltonian H*,

$$H = -J\sum_{ij} s_i s_j \tag{2}$$

In this certain state, we can measure other thermodynamics properties of the spin model such as magnetization of the system. For *magnetization M*,

$$M = \frac{\partial \log Z}{\partial B} = \sum_{S} p(S) \sum_{i} S_i \tag{3}$$

Where $p(S)$ is the probability of finding magnetisation and energy in the system at a certain state. In Monte Carlo simulation, the *magnetization M* is measured by sum up all configurations of spin so $\sum_{S} p(S) = 1$. Therefore, *magnetization M*,

$$M = \sum_{i} S_i \tag{4}$$

the magnetization per spin *m,*

$$m = \frac{M}{N} \tag{5}$$

Though the project is attempting to explore the relationship between system energy *Hamiltonian H* and spin system, the magnetization per spin *m* is also computed In the experiment part. In the next section, if a modification is made on Ising model to make the coupling variable to be random. The Ising model can be used to model the behavior of spin glasses.

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2.2 Theory of Spin Glass & SK spin glass model

Spin glass is a magnetic system of disorder and frustration with randomness and competing interactions. (A. P. Young, 2010) To study the properties of spin glasses, set up by Edwards and Anderson (1975), most theories use a simple model called Edwards-Anderson (EA) Model. Investigated by Sherrington and Kirkpatrick (1975), the ising version of spin glass model is called SK model to study the behaviour of spin glass.

The SK model is simply an Ising EA model. Alike to (2), the total system energy in SK model is also given by *Hamiltonian H*,

$$H = -\sum_{ij} J_{ij} s_i s_j \tag{6}$$

where J_{ij}, s_i, s_j in the SK model has the same meaning as the ising model. It has the properties of ising type *spin s* that also define to be two possible states, “up” or “down” (+ or −). The difference is that the *interaction* J_{ij} between spin sites are randomly distributed.

Spin glasses are frozen and disordered spin systems at low temperature state because the competing interaction makes no configuration of spin canfavour all interaction. (the idea is called “Frustration” will be introduced in 2.3) and the interactions are partially random at least. (K.H. Fischer & J.A. Hertz, 1991) The random distributed interaction is a necessary ingredient for SK spin glass to study the behavior of spin glasses.

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2.3 Frustration and Energy Landscape

Frustration is an important concept associated with the spin glass system. When there is no spin configuration to allow all spins to have their lowest energy, frustration occurs. (Yu, 2011) There is an example of frustration below:

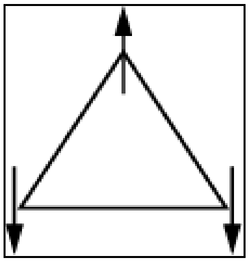

Figure 2. Frustration of spin glass in a triangle. Adapted from PHYSICS 238C. Condensed matter physics, Lecture 18 ISING MODEL, University of California, Irvine, 2011

In the triangle example of figure 2, a spin tends to point opposite from its neighbouring spins but considering the antiferromagnetic interaction between the neighbour spins, the spins on the triangle can not all be satisfied. That is frustration. In spin glass systems, the various spin configurations at low temperature, from equation 7, the system energy can be computed which are different. The relation of spin configurations to system energy can be presented by an energy graph. Many "mountains" and "valleys" will appear in the energy landscape graph. The energy landscape is present as follows.

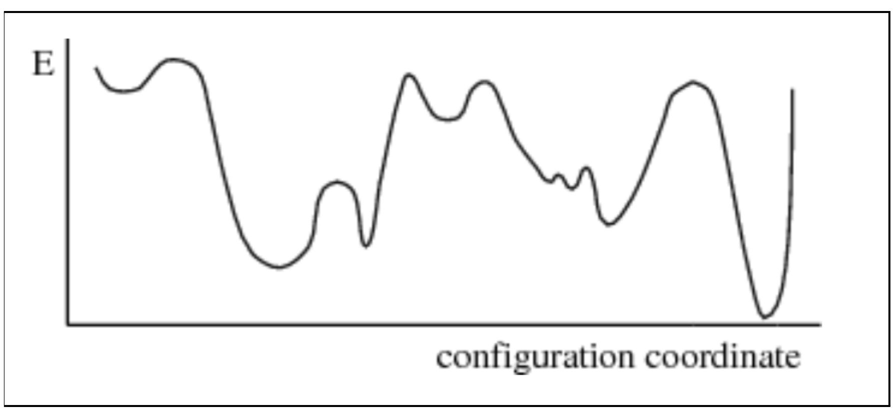

Figure 3. Energy landscape of spin glass. Adapted from PHYSICS 238C. Condensed matter physics, Lecture 18 ISING MODEL, University of California, Irvine, 2011

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The "mountains" correspond to the high energy spin configurations and the "valleys" correspond to the low energy spin configurations. In figure 3, there are some local minimums in the energy landscape that the system energy is often trapped in this kind of "valley"(local minima). (Fischer & Hertz, 2002) While, for the spin system, there is one optimum solution where the system energy reaches its minimum level. That is the ground state of system energy at the lowest temperature or what we call global minima. Normally, the best situation is that the spin system reaches its ground state of system energy at the lowest temperature. Yet, there are still many uncertainties in terms of the spin configurations that cause the interesting shape of the energy Landscape. The project focuses on investigating the spin configuration of SK spin glass, attempting to provide a possible explanation.

2.4 Self-Sustained Clusters

Self Sustained Cluster is a theory associated with the spin configuration in spin glasses, the theory suggests that there is a formation of dominant clusters in spin glass. (Yeung & Saad, 2013) In the spin variables of spin systems, a set C of spin variables is denoted where the spins are defined spins in-cluster ($i \in C$) and spins out-cluster ($i \notin C$) and corresponding magnetic fields. For in-cluster ($i \in C$) magnetic field,

$$u_i = \sum_{j \in C} J_{ij} s_j \tag{7}$$

For out-cluster ($i \notin C$) magnetic field,

$$v_i = \sum_{j \notin C} J_{ij} s_j \tag{8}$$

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The total magnetic field experienced by spin i sum up by in-cluster ($i \in C$) magnetic field and out-cluster ($i \notin C$) magnetic field equal to $-\ hi\ =\ ui\ +\ vi$. The spin set C is a self-sustained cluster, if

$$|u_i|\ >\ |v_i|, \qquad \forall i \in C. \tag{9}$$

That is, if the in-cluster (i ∈ C) magnetic field is larger than the out-cluster ($i \notin C$) magnetic field experienced by all individual spin i in set C, set C is a self-sustained cluster. (Yeung & Saad, 2013) The concept can apply to various spin systems. Following is an example of self-sustained clusters in the spin system.

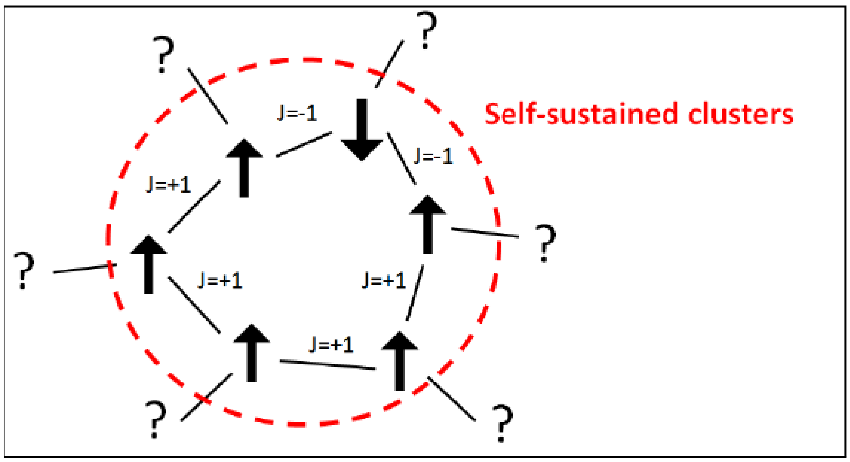

Figure 4. Self-sustained cluster in loops in 3-regular graphs. Adapted from Computational hardness in p-spin models - a microscopic perspective, Rocchi, Saad and Yeung, 2016

Figure 4 is an example of the formulation of self-sustained clusters. It is loops in 3-regular graphs, where a set of spins cluster is set that is presented with a dot-line circle. In the spin cluster, for each individual spin i, the experienced in-cluster (i ∈ C) magnetic field is larger than the out-cluster (i ∈ C) magnetic field, the spin cluster C is a self-sustained cluster.

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2.5 Effect of Self-Sustained Clusters on Energy Landscape

One effect of self-sustained clusters is that the individual spin i in cluster C is dominated by the peer spins in cluster C. That means all individual spins inside cluster C are not interacting with the spins out of the spin cluster because the in-cluster variables dominate the state of spins ($i \in C$). At low temperatures, destabilisation of self-sustaining clusters is unlikely while the destabilizing of self-sustaining clusters are required for a macroscopic change so that resulting fluctuations do not change the state of spins inside the cluster. (Rocchi, Saad and Yeung, 2016)

Flowing back to the Energy landscape of spin glass, there are unknown reasons why the spin system is trapped in some valley and unable to reach its lowest system energy which the energy configurations correspond to the spin configurations. By the concept of self-sustained clusters, it is possible that the special spin configuration of self-sustained clusters causes the system energy higher, as well as not finding the one optimum solution. In other words, because of the existence of self-sustained clusters, the system energy might be trapped at local minima and might not climb over the mountain of energy to find the global minima in the energy landscape.

The project attempts to apply the theory of self-sustained clusters to the ising spin glass system on the square lattice, to investigate the relationship between the special spin configuration of self-sustained clusters and the energy landscape of spin glass.

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Chapter 3. Methodology

The project uses a computational method to solve the spin model. A computer algorithm is programmed to simulate the spin model - SK spin glass model. To introduce the method, the pseudocode of the simulation program is presented in the following section. By Monte Carlo Method, the programme simulated the annealing process in the SK spin glass model, providing the ability to complete the system energy, magnetization and measuring Self-Sustained Clusters in the spin configuration.

3.1 Spin Model

In the first part of the simulation of an SK spin glass system. Various important parameters of the spin model are set, there is the size of the spin model, the timestep and the temperature.

important parameters

size = 10	Length of 2D SK model
step = 100000*size *size	Timestep
T = 5.0	Temperature
Sample = 300	Number of Samples

Figure 5. important parameters of the SK spin glass model

The program is a two dimensions spin model on square lattice where the length is 10. While it is a 10×10 spins, so the total spins in the system will be 10×10 equal to 100 spins. For the timestep, the basis timestep is set to 100,000, then the timestep times the square of size equals 10,000,000. For temperature it is set to 5.0, this is the temperature where the spin system starts at. After the set up the important parameter, the program uses three two-dimensional arrays s[i][j], Ji[i][j] and Jj[i][j] to store the spin variables, horizontal coupling variables and vertical coupling

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variables separately. Then, in each sample, the program draws different random configuration of spin and coupling to conduct the experiment as follow:

```
initial states

s = randam          Random spin variables
Ji = random         Random horizontal coupling variables
Jj = random         Random vertical coupling variables
```

Figure 6. initial states of the SK spin glass model

3.2 Simulated Annealing

After initializing the spin model, it is the main part of the program where the Monte Carlo dynamic starts. The Monte Carlo dynamics work as a commonly known ising model. At each timestep, the program draws a random spin site for update and computes the energy of flipping the spin, then updates the spin variables. (LEE, 2002) Then go back and repeat the loop. The difference is that the method of simulated annealing is applied in the Monte Carlo Method of SK spin glass model. That is the temperature of the spin system changes with the step time.

```
MC dynamics

for steptime = 1 to step      Main Loop for simulated Annealing
  T = annealing by step       Annealing the temperature by linear decay
  s[rand][rand]               Choose a random spin
  deltaE                      Compute Energy change of flipping the spin
  if Flip
  end if                      Flip the spin
  next steptime               Go back and repeat
end MC dynamics
```

Figure 7. Monte Carlo dynamics in the SK spin glass model

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In the loop of the Monte Carlo dynamic, there is a row of pseudocode, T = annealing by step, the cooling schedule or what is called annealing path of the program is present below. In the process of simulated annealing in the SK spin glass program, a linear decay of temperature is selected. The temperature at each step,

$$T = \frac{Tmin - T_{max}}{total\ steptime} steptime + T_{max} \tag{10}$$

Practically, the equation simply means that at system temperature start at T = 5.0, at each timestep, the system temperature decay linearly with the timestep. I.e. in every 10000 steps, the temperature decreases linearly by 0.005.

3.3 Cooling schedule

Timestep	Temperature
0	5
10,000	4.995
20,000	4.99
...	...
9,990,000	0.005
10,000,000	0

Figure 7. Cooling schedule

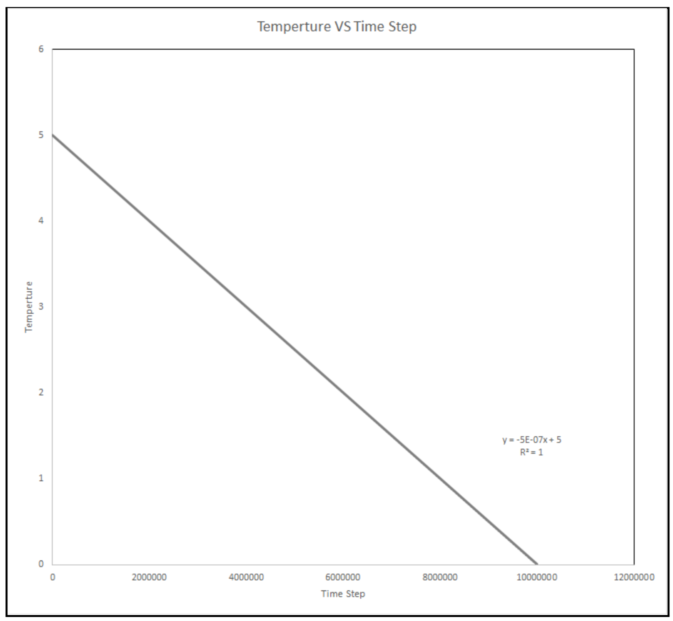

Figure 8. annealing path in Monte Carlo dynamics

In figure 7 and figure 8, the temperature decay with the time step, finally the temperature arrives, T = 0, at the end of Monte Carlo simulation, where timestep is 10,000,000. At this point, the program will compute the magnetization and the energy of the system.

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3.4 The complete program

At the end of the Monte Carlo simulation, the program will compute the magnetization and the energy of the system by equation (5) and equation (6). The following figure is the A pseudocode of the complete program of SK spin glass simulation.

```
program SK

size = 10                              Length of 2D SK model
step = 100000*size *size               Timestep
T = 5.0                                Temperature
Sample = 300                           Number of Samples

sampling
for SampleNo = 1 to sample             Loop for sampling
  Initial state
  s = randam                           Draw initial state for spin
  Ji = random                          Draw initial state for horizontal interaction
  Jj = random                          Draw initial state for vertical interaction

  MC dynamics
  for steptime = 1 to step             Main Loop for simulated Annealing
    T = annealing by step              Annealing the temperature by linear decay
    s[rand][rand]                      Choose a random spin
    deltaE                             Compute Energy change of flipping the spin
    if Flip
    end if
    next steptime                      Go back and repeat
  end MC dynamics

  compute E                            Compute system energy
  compute m                            Compute magnetization
  Measure SSC                          Measure SSC
end sampling
```

Figure 9. Pseudocode of complete program

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3.5 Annealing Process

After completing the programing, it is time to test the program function and try to simulate the annealing process of the spin glass system. The program starts the system at temperature equal to 5.0, the program starts to lower the temperature by timestep and at the end, the temperature is

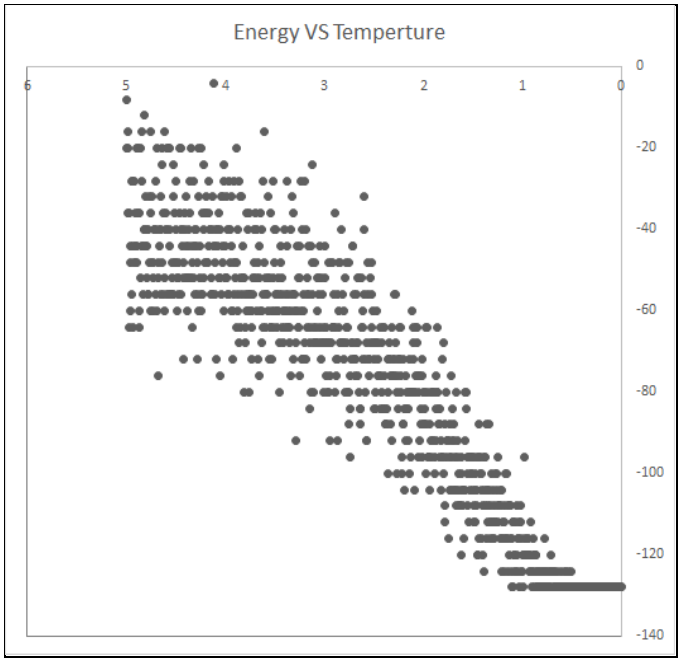

Figure 10. Annealing Process with 1 sample

equal to 0. In Figure 10, it shows one sample of annealing process via the graph of system energy against the system temperature. It is observable that the system energy is decreasing with lowering the temperature even the system energy seems to be unstable that keeps jittering. Yet the system energy at low temperature becomes stable at a point where Energy equals to around -130. Then more trial is conducted to measure the annealing process of the monte carlo method.

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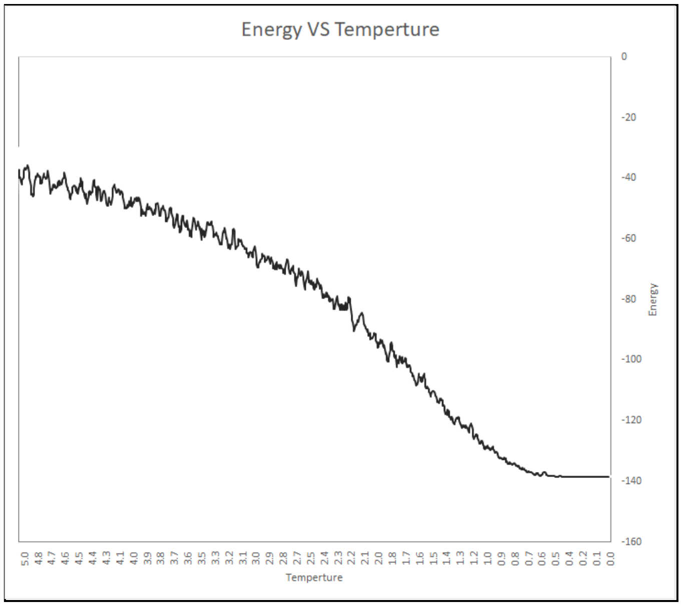

Figure 11. Annealing Process with 10 sample

In figure 11, 10 samples of the annealing process are recorded to analyze by averaging and shows a smooth graph that shows the clear path that the system energy falls along with the temperature. The system energy falls until around -140 at T = 0. This is also the spot where the program does the measurement. The more trial shows that the simulated annealing program works fine to simulate the spin system cooling down process. From this point, the program can be further modified to measure the existence, configuration and properties of self-sustained clusters on SK spin glass systems.

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Chapter 4. Measurement

4.1 Magnetization and system energy

The program does the measurement at the end of the simulated annealing where the spin system arrives at the lowest temperature. Generally, the magnetization and the energy of the system are computed by equation (5) and equation (6). To study the property of self-sustained clusters, the program is modified in some way to measure the existence of self-sustained clusters, the configuration of self-sustained clusters as well as the relationship between self-sustained clusters and system energy at the lowest temperature. In that case, there are two methods to measure the self-sustained cluster that cause two different versions of the program at three different phases.

4.2 Self-Sustained Cluster

The measurement in the project only doing in a specific set C of 2×2 spins self-sustained cluster. From equation (9), $|u_i| > |v_i|, \forall i \in C$.After choosing the cluster on the square lattice, the project starts to test if each individual spin *i* experience a larger in-cluster field then out cluster field. In figure 11, the program selects 4 spins in a 2×2 spin cluster to conduct measurement.

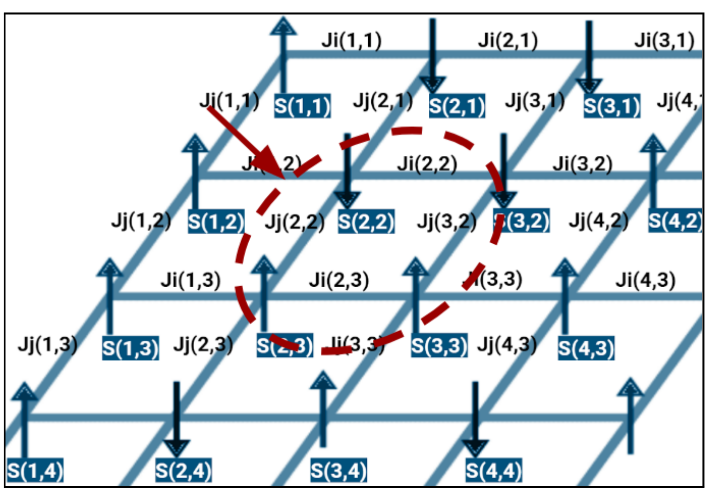

Figure 11. Measuring an Self-Sustained Clusters Set C on a 2D Ising spin glass model on square lattice.

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The four spins are separately s(2, 2), s(2, 3), s(3, 2), s(3, 3) on the square lattice. For the spin on the left top side, the in-cluster (i ∈ C) field of Spin s(2, 2), by equation (7) $u_i = \sum_{j\in C} J_{ij} s_j$ is,

$$u_i = J_i(2,2)s(3,2) + J_j(2,2)s(2,3)$$

the out-cluster (i ∉ C) field of Spin s(2, 2) ,by equation (8),

$$v_i = J_i(1,2)s(1,2) + J_j(2,1)s(2,1)$$

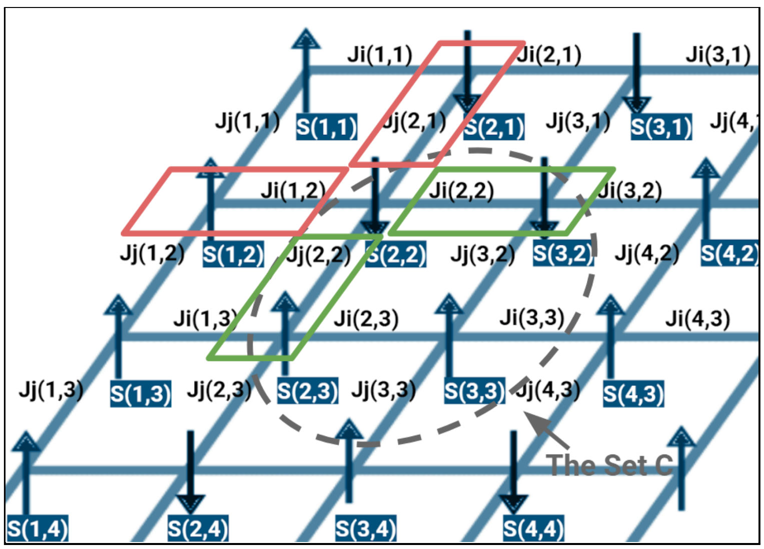

Figure 12. Calculated an individual spin *i* in a spin Clusters Set C on a 2D Ising spin glass model

For left top spin s(2,2), by equation (9), $|u_i| > |v_i|,\ \forall i \in C$.If the in-cluster field experienced by this one individual spin is larger than the out-cluster field experienced by an individual spin, the program will continue to examine other individual spins in the cluster. If left bottom spin s(2, 3), right top spin s(3, 2) and right bottom spin s(3, 3), four individual spins in the 2 × 2 cluster, all reach the condition of in-cluster field larger than the out-cluster field, the set C is measure as a self-sustained cluster.

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4.3 Phase One - Measure Specific Cluster C

At the very start of the research, it is unknown whether the self-sustained cluster existed in the SK spin glass model on the square lattice model so a special condition is decided to be set to allow the Self-Sustained Cluster to form easily. From equation (9) $|u_i| > |v_i|, \forall i \in C$. , if the in-cluster field experienced by individual spin is larger than the out-cluster field experienced by individual spin, the set C is a self-sustained cluster. To reach this condition, the in-cluster field should be adjusted to be larger. From equation (7) $u_i = \sum_{j\in C} J_{ij} s_j$, if the spin variable and coupling variable is enlarged, it will result in the in-cluster field being enlarged too.

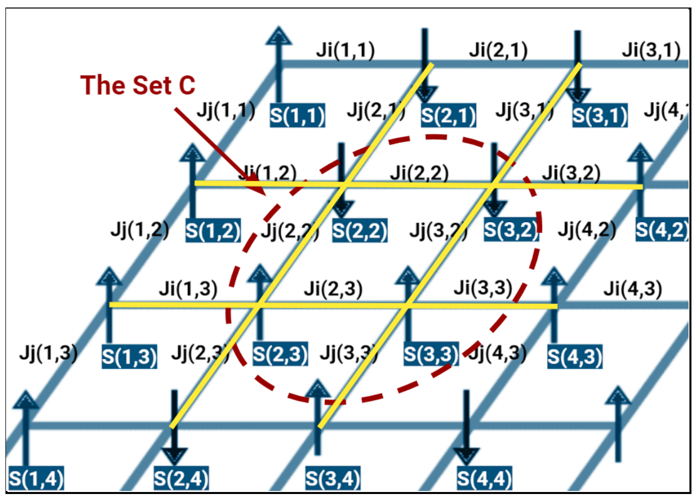

Figure 13. Special condition setup to a spin Clusters

Set C on a 2D Ising spin glass model

By theory, a special condition is set up in the initial state. In phase one of the project, the nearest coupling variables are set to be +1. Under the special condition, the in-cluster field should tend to be larger than the out-cluster field. By setting up the condition, it is expected that the Self-Sustained Cluster is easier to be found in the SK spin glass model.

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4.4 Phase Two - Measuring the Configurations

In phase 2, it is simply to expand the measuring area to the whole square lattice and without setting special conditions in the initial state to explore the original nature of Self-Sustained Clusters on the SK spin glass model. The program prints out which spin clusters are Self-Sustained Clusters at the lowest temperature. It is expected that configurations of Self-Sustained Cluster are visualised in this phase.

4.5 Phase Three - Measuring the Whole Square Lattice

In phase three, It is also worth noting that the program uses the same coupling configuration that draws the first sample at the initial state to do the sampling. In that case, it is believed that the side factor is cancelled. From Figure 9, it is simply put the drawing coupling variable out of the sampling loop, we have the program for phase three.

```
program SK

...                                 ...
Ji = random                         Draw initial state for horizontal interaction
Jj = random                         Draw initial state for vertical interaction

sampling
for SampleNo = 1 to sample          Loop for sampling
  Initial state
  s = randam                        Draw initial state for spin

  MC dynamics                       Main Loop for simulated Annealing
  ...                               ...
```

Figure 10. A modify on the program

The program is adjusted to lock down the coupling variable to all samples. It allows the experiment to be conducted hundreds of times with the same coupling configuration.

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Chapter 5. Result and Discussion

5.1 Phase One - Existence of Self-Sustained Clusters

In phase one, the program sets all neighbouring couplings to be +1, *(See 4.3 Phase 1 Existence of Self-Sustained Cluster)* after that 280 sample is done to measure a specific 2 $\times$ 2 spin cluster C. (See Appendix 1 - Data of Measurement of SSC at specific set)

SSC existed	SSC NOT existed
136	144
P(SSC existed)	48.6%

Under the special condition of setting all neighbouring couplings J to +1, self-sustained clusters are detected on the SK spin glass model. As expected, it appears that self-sustained clusters become easy to occur under special conditions. There are 48.6% of samples where self-sustained clusters exist. It shows that it is not rare that self-sustained cluster found in the SK spin model under special conditions. At this point, the result shows that 2 $\times$ 2 self-sustained cluster exists under special condition. The result also gives a motivation for the project to further investigate the existence of Self-Sustained Clusters without setting up special conditions to move on to phase two where the program does not set up any beneficial environment for the self-sustained cluster as well as the formation of self-sustained clusters exist on SK spin glass model.

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5.2 Phase Two - Configuration of self-sustained cluster

In phase two, the program is set to measure all sets C on the available area on the whole square lattice. Also, with the motivation from phase one, the measurement in phase two is conducted without setting special conditions to explore the original property of the SK spin glass model. *(See 4.4 Phase 2 Measuring the Whole Square Lattice)* As expected, Self-sustained clusters (SSC) detected in the Ising Spin Glass model with no special condition. The Configuration of the self-sustained cluster shown as below: *(See Appendix 2 - Data of Self-Sustained Clusters configuration)*

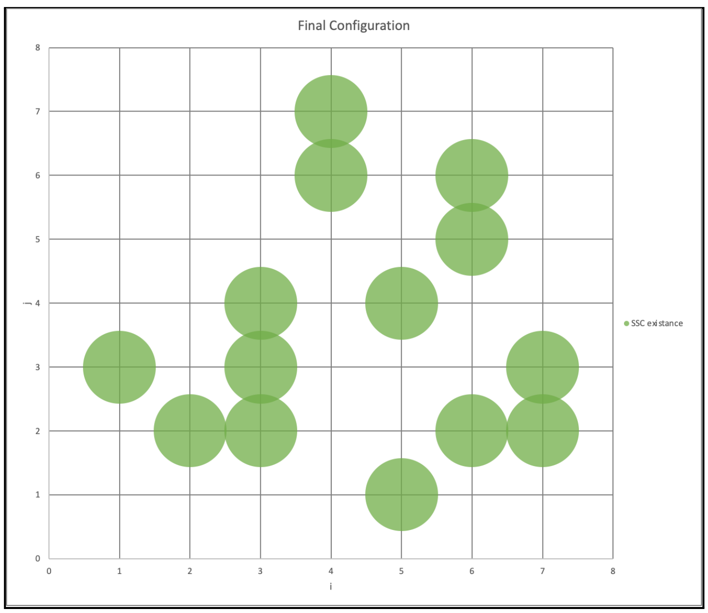

Figure 14. Configuration of 14 Self-sustained clusters found on SK spin glass on square lattice

The green dot is spin s(i, j) selected for measuring self-sustained clusters. That is, if a spin(i, j) is show green in the figure 14, the set C of 2 × 2 spin cluster: s(i, j), s(i, j+1), s(i+1, j), s(i+1, j+1), is a self-sustained cluster. In the result of figure 14, 14 of (2 × 2) Self-Sustained Clusters(SSC) is

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detected, out of 49 available areas for detection. The result shows that at 28.6% detected area found 2 × 2 Self-Sustained Clusters(SSC) without setting up special condition.

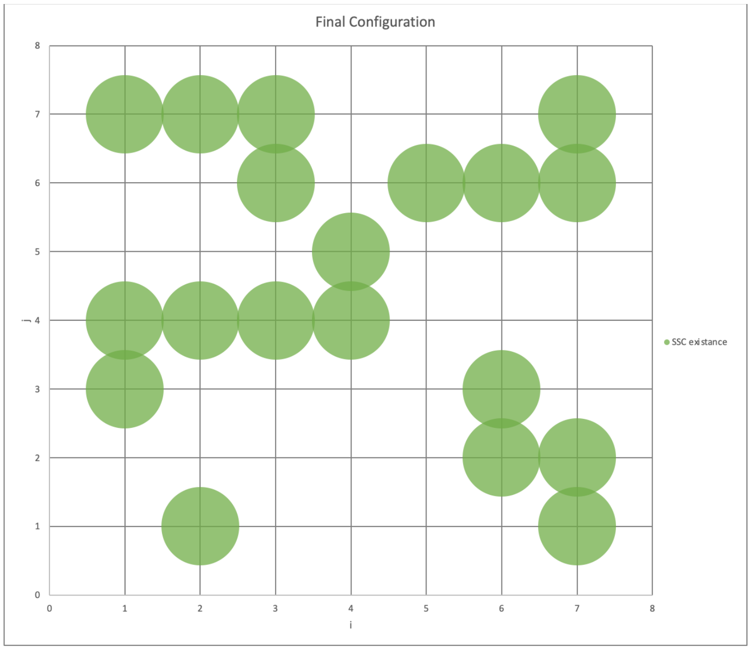

Figure 15. Configuration of 19 Self-sustained clusters found on SK spin glass on square lattice

In the result of figure 15, in SK spin glass on square lattice 19 of 2 × 2 Self-Sustained Clusters is detected, out of 49 available areas for detection. The result shows that at 38.8% detected area found 2 × 2 Self-Sustained Clusters(SSC) without setting up special condition. From various samples of measuring self-sustained clusters without setting up Special conditions, there are multiple self-sustained clusters found on SK spin glass on square lattice. In 400 samples, 34.1% of detected areas found Self-Sustained Clusters. That is 16.72 detected areas out of 49 available areas. As expected, self-sustained clusters detected in the Ising Spin Glass model. The result also visualised the formation of self-sustained clusters on the SK spin glass model that helped us to understand the configuration that self-sustained cluster existed in the SK spin glass model.

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Accordingly, in an SK spin glass system, a self-sustained cluster is not an alone phenomenon, it exists multiply as many of 2×2 self-sustained clusters overlap with each other. Are the overlapping 2×2 self-sustained clusters forms a larger size of self-sustained clusters? certainly, it remains uncertain. Yet, the result shows that the Self-Sustained Cluster is not a rare event in the SK spin glass system so it is possible to further explore the impact of self-sustained clusters on the SK spin glass system.

5.3 Phase Three - Relation of Self-Sustained Cluster to System Energy

In exploring the relationship between self-sustained cluster and system energy at the lowest temperature, the program is adjusted. (see 4.5) With different coupling variables at the initial state in each sample, the range of system energies is too large to compare. After that, the coupling variable is fixed for all samples. That means the program uses the same coupling configuration for multiple samples. At this point, it is suggested that doing sampling with the same coupling configuration eliminates uncontrolled variables in the simulated experiment.

So that the same coupling configuration is saved as the first sample, then the same coupling configuration will be tested hundreds of times. In most cases, when the coupling variable is locked, all samples tend to find only one solution at the lowest temperature, for instance, system energy is -142 in 300 samples. However, in some cases, the simulation finds more than one solution at the lowest temperature. For example, system energies are -136 with 146 samples or -140 with 154 samples in the same 300 sample size experiment. *(see Appendix 3)*

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no of SSC (Average)	Energy
16.875	-142
15.316	-146

Sample Size: 35

Standard Error: 0.440561

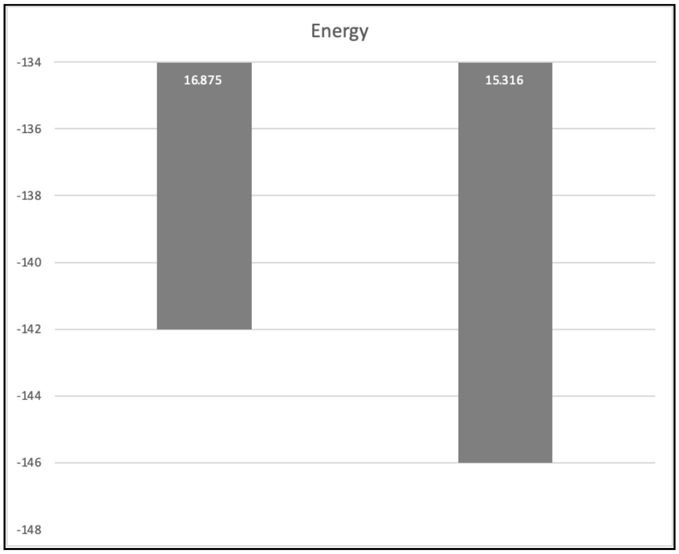

Figure 16 Energy level of the trial

In the experiments, the coupling arrangement is fixed to do the experiment. After the process of simulated annealing, the program measures the system energy and the number of self-sustained clusters at the lowest temperature. In the energy graphs of figure 16, there is the result of the trial where the program only runs about 35 samples. In the trial, while the number of self-sustained clusters in the spin system is higher, the system energy at the lowest temperature appears to be higher. This is the first time an expected result of the project related to the relation of Self-Sustained Cluster to System Energy was observed. However, considering the sample size is too sample and the standard error of mean is 0.440561 which is large so the relationship may not be that strong so the sample size is decided to increase by a lot to reduce the error of the result. Therefore, simulated experiments with a 10 times larger sample size are set up for further tests.

By simulating experiments with 300 samples using the same coupling configurations, the standard error of mean at around 0.12 to 0.16 is relatively lower than the standard error of mean of the trial with only 35 samples. At this point, the results are more representative.

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Trial *35 sample*		**Experiment 1** *300 sample*		**Experiment 2** *300 sample*	
no of SSC (Average)	Energy	no of SSC (Average)	Energy	no of SSC (Average)	Energy
16.875	-142	17.253	-136	12.554	-132
15.316	-146	15.680	-140	11.441	-136
$SE_{\bar{x}}$ = 0.440561		$SE_{\bar{x}}$ = 0.126644		$SE_{\bar{x}}$ = 0.131907	
Experiment 3 *300 sample*		**Experiment 4** *300 sample*		**Experiment 5** *300 sample*	
no of SSC (Average)	Energy	no of SSC (Average)	Energy	no of SSC (Average)	Energy
17.670	-138	18.801	-138	17.87	-138
16.656	-142	14.857	-140	16.646	-142
/	/	14.856	-144	/	/
$SE_{\bar{x}}$ = 0.163299		$SE_{\bar{x}}$ = 0.153043		$SE_{\bar{x}}$ = 0.137797	

＊$SE_{\bar{x}}$ is the standard error of mean

Figure 17 Energy level of the trial

In experiment 1, at the lowest temperature, when the system energy is -136, in the spin system, there are 17.253 self-sustained clusters in the spin system; when the system energy is -140, there are 15.680 self-sustained clusters. The same relation is observed, that is, when the number of self-sustained clusters in the SK spin glass is higher, the system energy at the lowest temperature appears to be higher. After that, more experiments under the same conditions are simulated and the results are plotted to an energy bar chart. In experiment 2, the same phenomenon occurred as the project's expected results. In figure 18 and figure 19, the same energy graph is plotted to show the relationship between the number of self-sustained clusters and system energy at the

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lowest temperature. At these two energy graphs with much larger sample size, it appears that when there are more average numbers of self-sustained clusters, the system energy at the lowest temperature will be higher as well.

no of SSC (Average)	Energy
17.253	-136
15.680	-140

Sample Size: 300

Standard Error: 0.126644

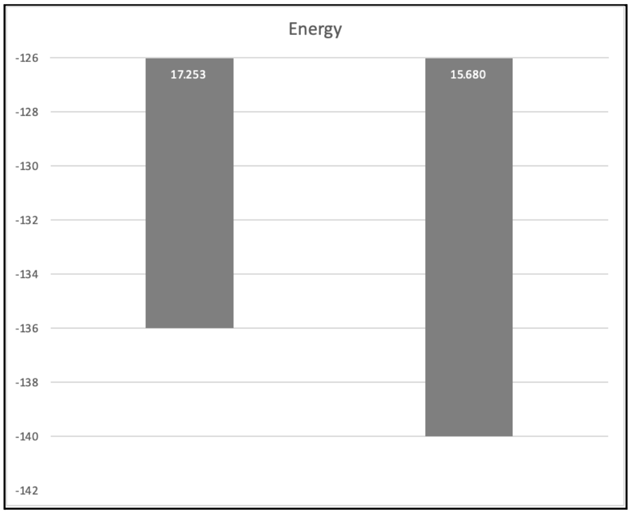

Figure 18 Energy level graph of EXP. 1

no of SSC (Average)	Energy
12.554	-132
11.441	-136

Sample Size: 300

Standard Error: 0.131907

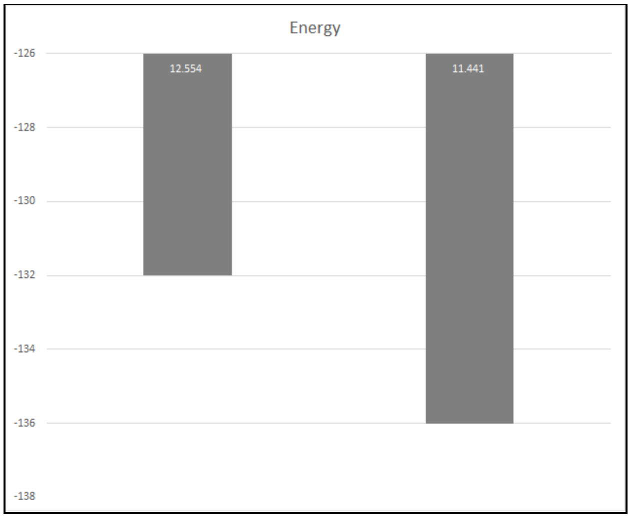

Figure 19 Energy level graph of EXP. 2

That comes to a small conclusion in phase three, among all experiments and the trial, it all shows the same result. While there are more self-sustained clusters, the system energy at the lowest

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temperature will be higher. Same as the expected outcome, the relationship between self-sustained clusters and the system energy at the lowest temperature is found.

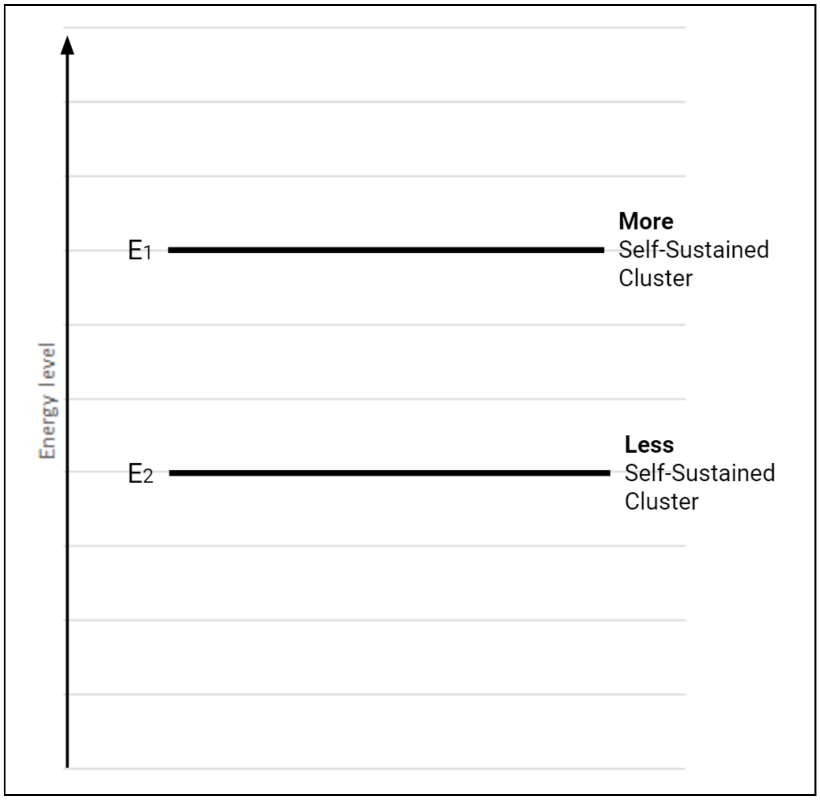

Figure 20. Relation of self-sustained cluster to system energy at lowest temperature

The results in phase three (Figure 20.) are summarized in one figure below. In chapter 6, the result will be discussed.

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Chapter 6 Discussion on Major Result

In the previous part, the data and graph in phase one and phase two are discussed which show interesting results. In this section, the chapter will focus on discussing the properties of self-sustained clusters in SK Spin Glass by the result in phase three where the method of locking on coupling variables is applied. The relation of self-sustained clusters to system energy at lowest temperature observed, the impact of self-sustained clusters to system energy and according to the result, the property of self-Sustained clusters in SK Spin Glass will be discussed which is lastly connected to the existing theory about the energy landscape in spin glass.

6.1 Lock on coupling variables

To eliminate uncontrolled variables in the experiment in phase three, one key factor is that the experiment is conducted using the same coupling arrangement. In normal situations of simulated experiment, the program draws random spin variables and coupling variables at the initial state of each sample. That means in a single experiment all samples start with different coupling configurations then start the simulated annealing process. The randomness of spin variables and coupling variables in the initial state resulting in the system energy falls to a huge range of numbers that are very hard to compare with each other.

Therefore, in phase three of the experiment, the program is adjusted to draw the random coupling variables in the first place, then use the same coupling variables and random spin variables to conduct the experiment. By this method of experiment, the same coupling configuration

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eliminates uncontrolled variables, the system energy often falls to one state of energy after the process of simulated annealing.

6.2 Impact of Self-Sustained Cluster on System Energy

After many trials with lock on coupling variables, it is observed that the experiment with the same coupling configuration, in most cases, the spin system only finds one energy state after the annealing process. By some probability, the spin system finds more than one energy state after the simulated annealing. For instance, finding two energy states at the lowest temperature. Also, very interestingly, some spin systems even find three energy states in experiment 4 in phase three. (See Figure 17.) Yet, the corresponding energy may be hard to find because there is a very few chance to find the third energy state. In that case, the sample size of the third energy state is not enough for analyzing so it is worth to enlarge the sample size to further study the three energy states annealing from the spin system with the same random generated coupling variables.

Therefore, the project focuses on the data of the experiments with the spin system at the low temperature with two energy states where the sample numbers are enough for both energy states. By analyzing results from these multiple experiments, the same result shows, it appears that when there are more average numbers of self-sustained clusters, the system energy at the lowest temperature will be higher as well. Among 1,500 samples in 5 experiments all show the same property, in SK spin glass, **the higher number of self-sustained clusters cause the system energy at lowest temperature to be higher.**

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6.3 Property of Self-Sustained Cluster in SK Spin Glass

To discuss the property of Self-Sustained Cluster in SK Spin Glass, it is necessary to introduce more character of Self-Sustained Clusters. It can exist in different forms of spin glasses. While other structures are more difficult to be optimised, square lattice is a structure that can be easily optimized.

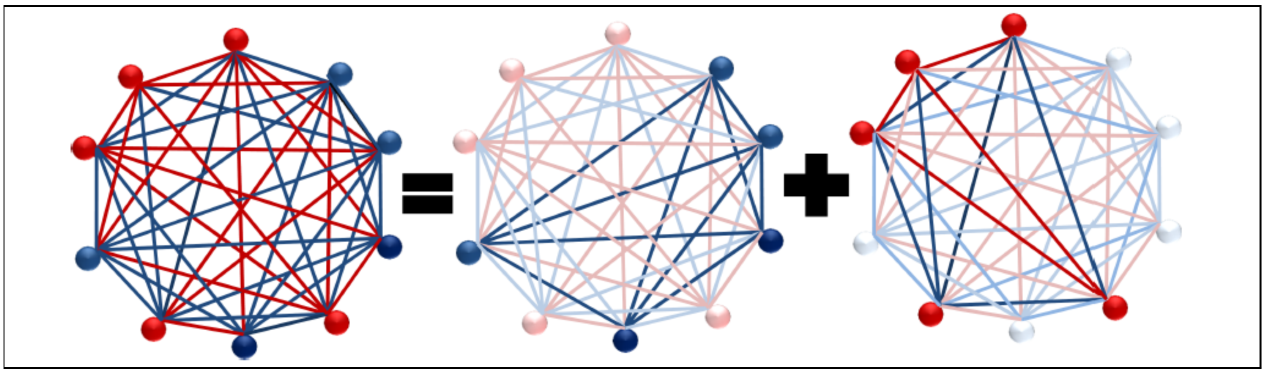

Figure 21. Self-Sustained Cluster on a circle shape network. Adapted from Computational hardness in p-spin models - a microscopic perspective, Rocchi, Saad and Yeung, 2016

The figure shows a network with red and blue nodes in a circle shape, in the presence of self-sustained cluster on spin system, because the spins in the cluster are not interacting with spin out of the cluster, some node in the spin structure can be considered to be absent when the self-sustained cluster existed. By the same concept, if a self-sustained cluster existed in square lattice, which is the spin structure that the program simulated and examined, the corresponding spin variable can also be considered absent. In the result in phase two, 34.1% of detected areas found Self-Sustained Clusters. It means about one-third of the spin clusters are not interacting with the spins out of the cluster and are considered to be absent from the square lattice.

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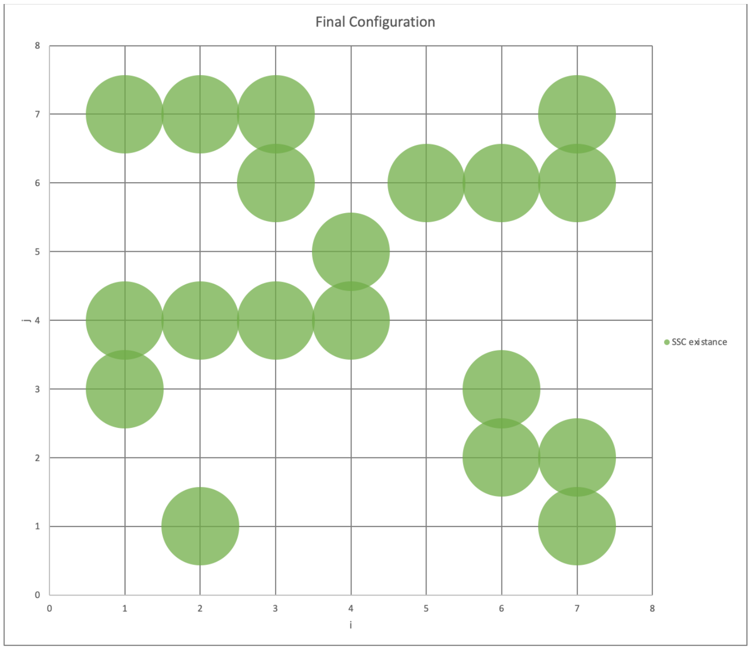

Figure 15. Configuration of 19 Self-sustained clusters found on SK spin glass on square lattice

In figure 15, the green dots on the square the lattice detected self-sustained clusters which can be considered to be absent on the square lattice. In the presence of self-sustained clusters on square lattice, because in-cluster spins are dominated which are not interacting with spins out-cluster, some spin on lattice is absent. The effect of self-sustained clusters cause the SK spin glass system on square lattice can not easily be optimised, which cause the system energy at the lowest temperature cannot be minimized.

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6.4 Energy graph of self-sustained cluster and SK spin glass

By the result of phase three, an energy graph is summarized to present the result and effect of self-sustained clusters on the SK spin glass system on square lattice.

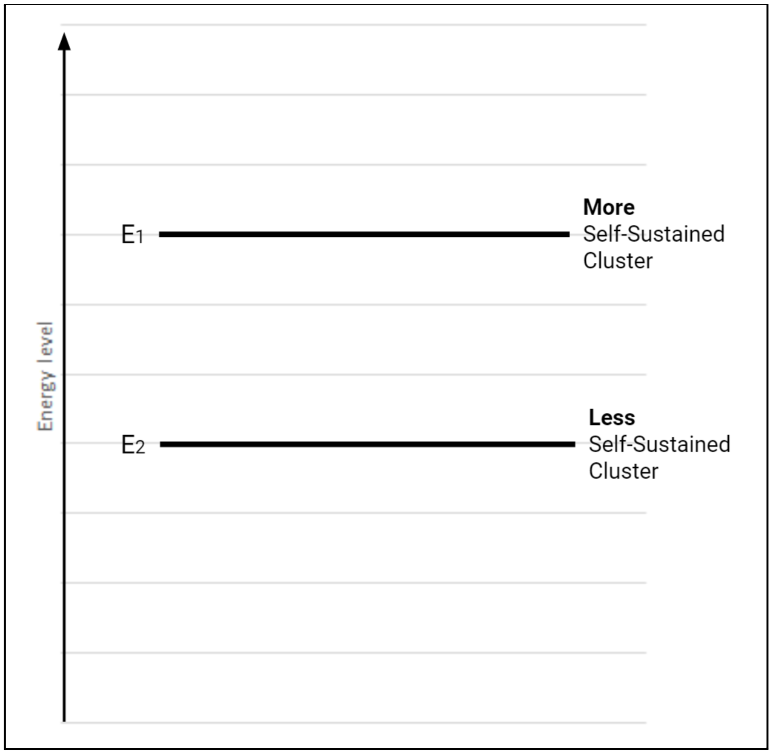

Figure 20. Relation of self-sustained cluster to system energy at lowest temperature

In figure 20, where E_1 represents the system energy of the spin system where there are more self-sustained clusters and E_2 represents the spin system where there are fewer self-sustained clusters. In the energy graph, it is observed that there is an energy difference between spin systems because of the existence of the number of self-sustained clusters. If there are more self-sustained clusters, the system energy is relatively higher, if there are fewer self-sustained clusters, the system energy is relatively lower. **The result of the project supports the**

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self-sustained cluster is one factor that causes the SK spin glass system unable to reach its lowest possible energy. This is a major result of the project. Also, the result may be a possible explanation for the existing theory - the energy landscape in spin glass.

6.5 Relation of Self-Sustained Clusters to Energy Landscape

In this section, the property of self-sustained clusters is associated with the energy landscape of spin glass. Let recap the energy landscape in spin glasses.It is a many-valley diagram where there are many possible stable states. (figure 3) There are many local minima and one deepest global minima with energy barriers separating them. (Fischer & Hertz, 2002) In the deepest valley, it corresponds to the spin configuration which has the lowest system energy while for the valley they represent low energy spin configurations. For the mountains, they correspond to the spin configurations with high system energy.

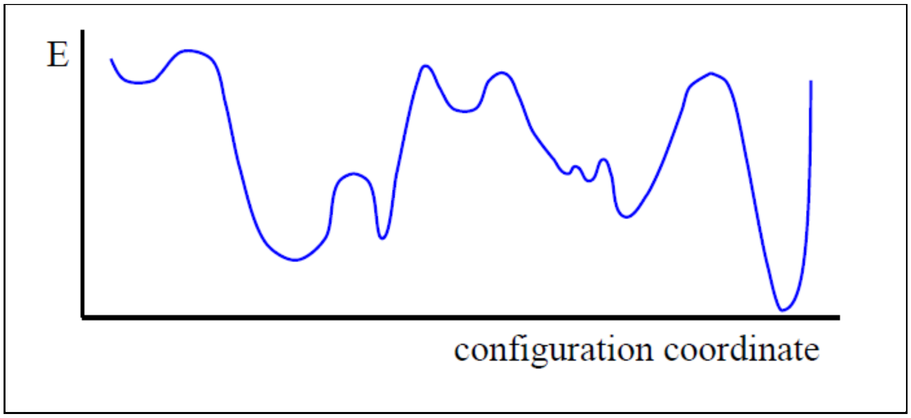

Figure 3. Energy landscape of spin glass. Adapted from PHYSICS 238C. Condensed matter physics, Lecture 18 ISING MODEL, University of California, Irvine, 2011

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The feature of competing interaction is frustration. It is an inability for all bonds to satisfy simultaneously (Touluse, 1977) Also, no spin configuration has their lowest possible interaction energy when no way all spins in the spin system can be satisfied, frustration occurs too. (YU, 2011) The concept of frustration is often associated with the energy diagram, where the spin configuration can not have a clear ground state where the system energy is lowest.

To associate the founding of the project into the energy landscape of spin configurations, apart from frustration, the concept of self-sustained clusters can also help us to understand and explain why the spin systems do not have their lowest system energy that fall into the local minima where the system energy is higher. The properties of self-sustained clusters on the spin glass system found in the project, when self-sustained clusters exist in the spin glass system, the in-cluster spins are dominated by the variables inside the set cluster C so the not interacting in-cluster spins cause the spin system to be harder to optimise and the system energies of the system are more likely to be unable to have lower system energy. From the result of phase three and applied the concept of the impact of self-sustained clusters to system energy of SK spin glass at lowest temperature. (figure 20) Because the system energy is higher when the number of self-sustained clusters is higher. In the spin systems where the system energy falls to local minima, it is probable that more self-sustained clusters exist in these systems when more in-cluster spins are not interacting with the out-cluster spins, the spin system is harder to be optimised. It resulted in higher system energy so the spin system could not find the optimum solution.

If there are lesser self-sustained clusters exist in the spin system, the system energy will be relatively lower so the spin system will find a local minimum with relatively lower system energy. At the global minimum, there are likely the least self-sustained clusters in the spin

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systems. When there are fewer in-cluster spins that are not interacting with the out-cluster spins, the spin system is easier to optimise so the system energy will be lowest as well. It shows the self-sustained clusters in SK spin glass is one probable factor for the inability of system energy to reach ground state in the energy landscape.

In terms of the energy barrier in the energy landscape that makes the spin system trapped in local minima, it may be likely that there is some relation of the formation of self-sustained clusters to the barrier of the energy landscape. The destabilisation of self-sustained clusters is unlikely in low temperature. (J. Ricchi, D. Saad & C. H. Yeung, 2016) Also, self-sustained clusters are one probable factor to the spin system to be unable to find the lowest temperature. It is suggested to further explore the properties of self-sustained clusters to have better understanding about the phenomena that the system energy of spin glasses to drop into local minima and cannot overcome the energy barrier to find the lowest energy.

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Chapter 7. More Attempts on Exploring

In this section, apart from major results, more results of attempts to explore the impact of self-sustained clusters on SK spin glass systems are recorded. In this part, the project tries to investigate the spin systems in terms of the cooling time and the system energy with different spin and coupling in the experiment.

7.1 Cooling time

Firstly, it comes to the study on relation on the number of self-sustained clusters with cooling time step. In the Monte Carlo dynamics, the system temperature decays linearly with the timestep growth. *(see 3.3)* It is known that the system energy of the spin system decreases when the system temperature decreases. *(see 3.5)* So in this part, the timestep need for the system energy to decrease to a certain level is measured. Also, at the end of the annealing process, the number of self-sustained clusters at the lowest temperature is measured. The result as below:

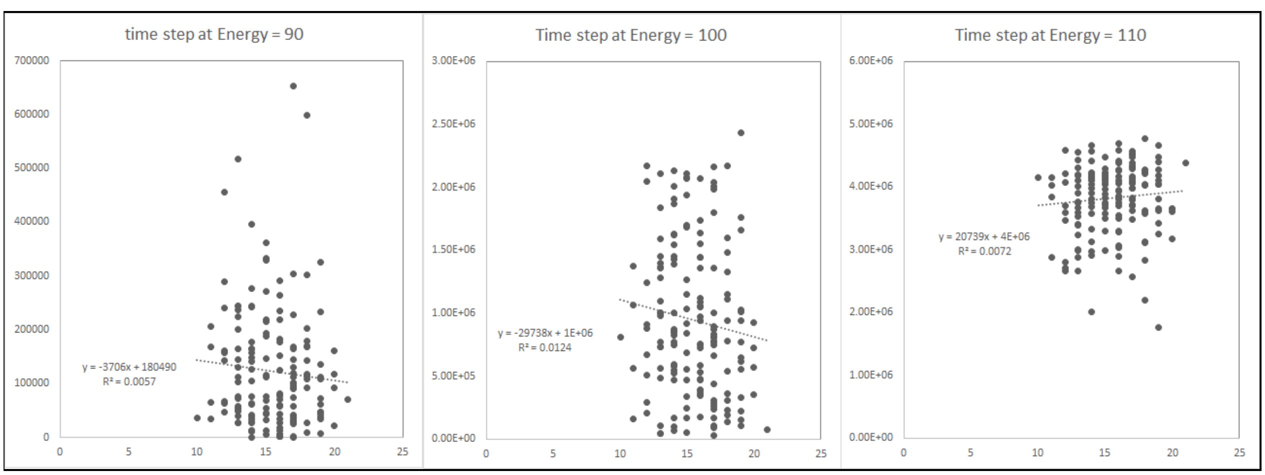

Figure 21. Timestep need to arrive different energy (where E = 90, 100, 110) against number of self-sustained clusters

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After that, another method is applied. For the experiment in which the spin system only finds one energy state at the lowest temperature. The program saves the value at the first trial, then measures the timestep the system arrives at the lowest energy. By this method, it is expected that a more accurate timestep to cool down the system will be observed.

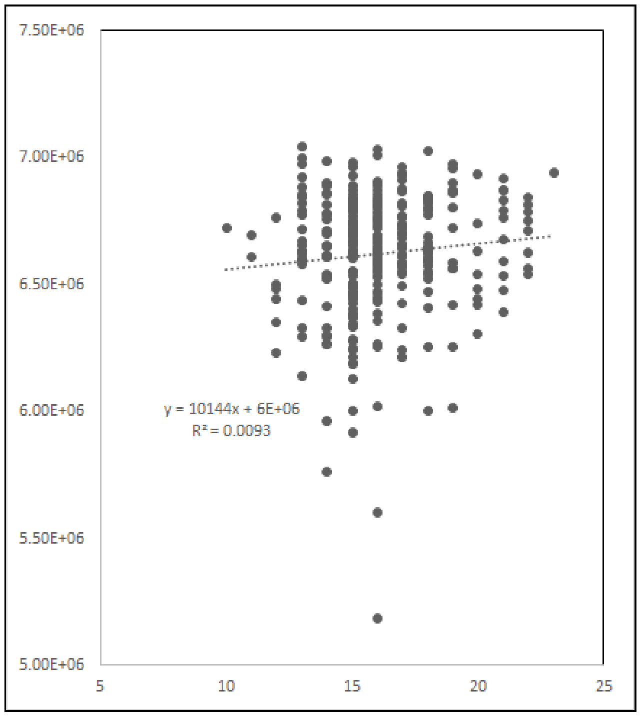

Figure 21. Timestep need to arrive the lowest energy against number of self-sustained clusters

In studying the time needed to cool down the system, the R-squared of all results is about 0.01 which is very small. In that aspect, the result only shows a very weak relationship between the number of self-sustained clusters with the timestep needed to cool down the system. There may be too many uncontrollable variables in the examine method. For example, the randomness of coupling results in the data can not be analyzed.

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7.2 Energy of random coupling configurations

Different from the major part, the project attempts to measure the system energy at the lowest energy with random coupling variables at each sample in this part.

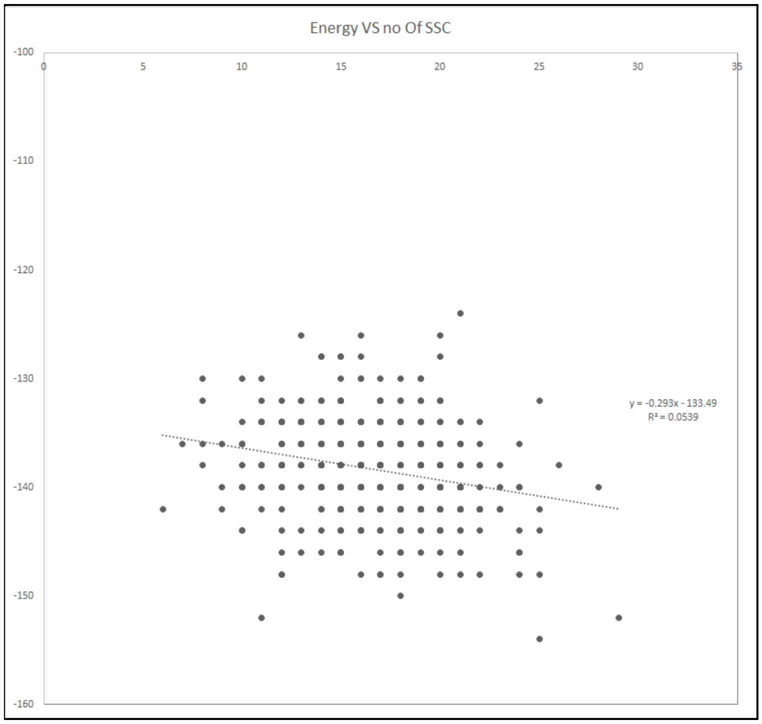

Figure 22. Timestep need to arrive the lowest energy against number of self-sustained clusters

By the method, the spin system finds many energy states after the simulated annealing process. Yet, the R-squared of many energy states is very small at about 0.05. Therefore the relation on the number of SSC with Energy is weak if the spin system has random coupling configurations.

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Chapter 8. Summary

By applying the method of simulated annealing into the SK spin glass model, the project achieves research outcomes. First of all, the project finds that 2×2 Self-Sustained Clusters exist in SK spin glass under special condition. *(See 5.1)* Secondly, multiple self-sustained clusters measured in the SK spin glass model that almost one-third of area exist of 2×2 self-sustained clusters on the square lattice. *(see 5.2)* It is evidence to show that self-sustained cluster is not a rare event in SK spin glass. By the measurement, the project also successfully visualised the arrangement of self-sustained clusters on the spin glass system on the square lattice. *(see Figure. 15)*

Thirdly, it comes to the major finding of the project. The relation of self-sustained clusters to system energy at the lowest temperature is observed. The result supports that, **in SK spin glass, higher the number of self-sustained clusters causes the system energy at lowest temperature to be higher** and an energy graph is summarised to present the relation. *(See Figure. 20)* After that, the finding combines with the theory of self-sustained clusters to connect to the existing theory to provide a possible explanation to the energy landscape of spin glass.

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Chapter 9. Future Work

Overall, the research outcome is as expected. Yet, there are still limitations and room for improvement. In the future, the research can be optimized in terms of those three directions - periodic boundary conditions, size of self-sustained clusters and lattice size of spin glass model.

9.1 Periodic boundary conditions

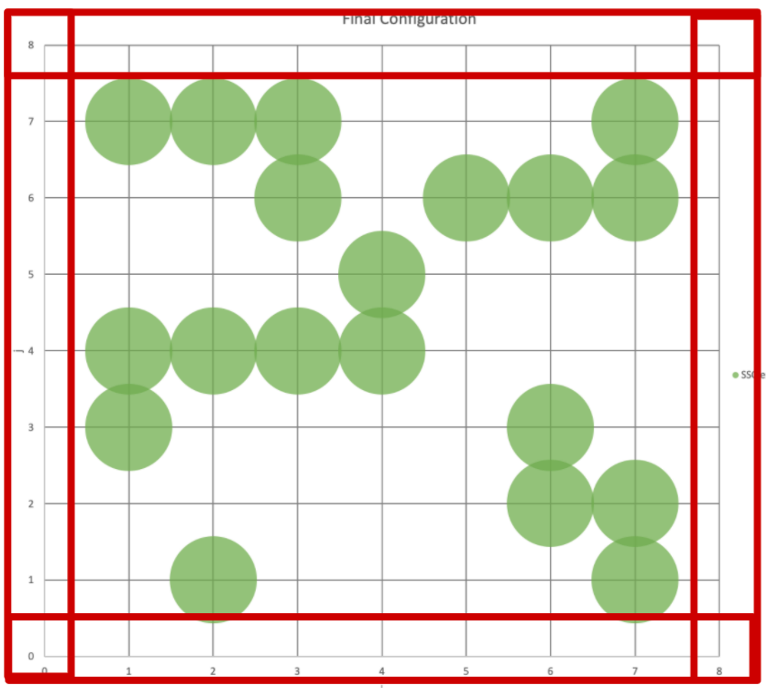

Figure 23. Measuring self-sustained clusters on SK spin glass on square lattice, periodic boundary conditions

Measuring the Self-Sustained Clusters (SSC) with considering the periodic boundary conditions (PBCs). It will increase the available area to detect self-sustained clusters.

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9.2 Detected Size of Self-Sustained Cluster

The project examines the existence and explores the properties 2×2 self-sustained clusters in spin system on square lattice. Self-sustained cluster has different shape apart from 2×2 if all individual spins in the cluster set C to fulfil the condition in equation (9). It is worth exploring the other shape of self-sustained clusters on spin systems.

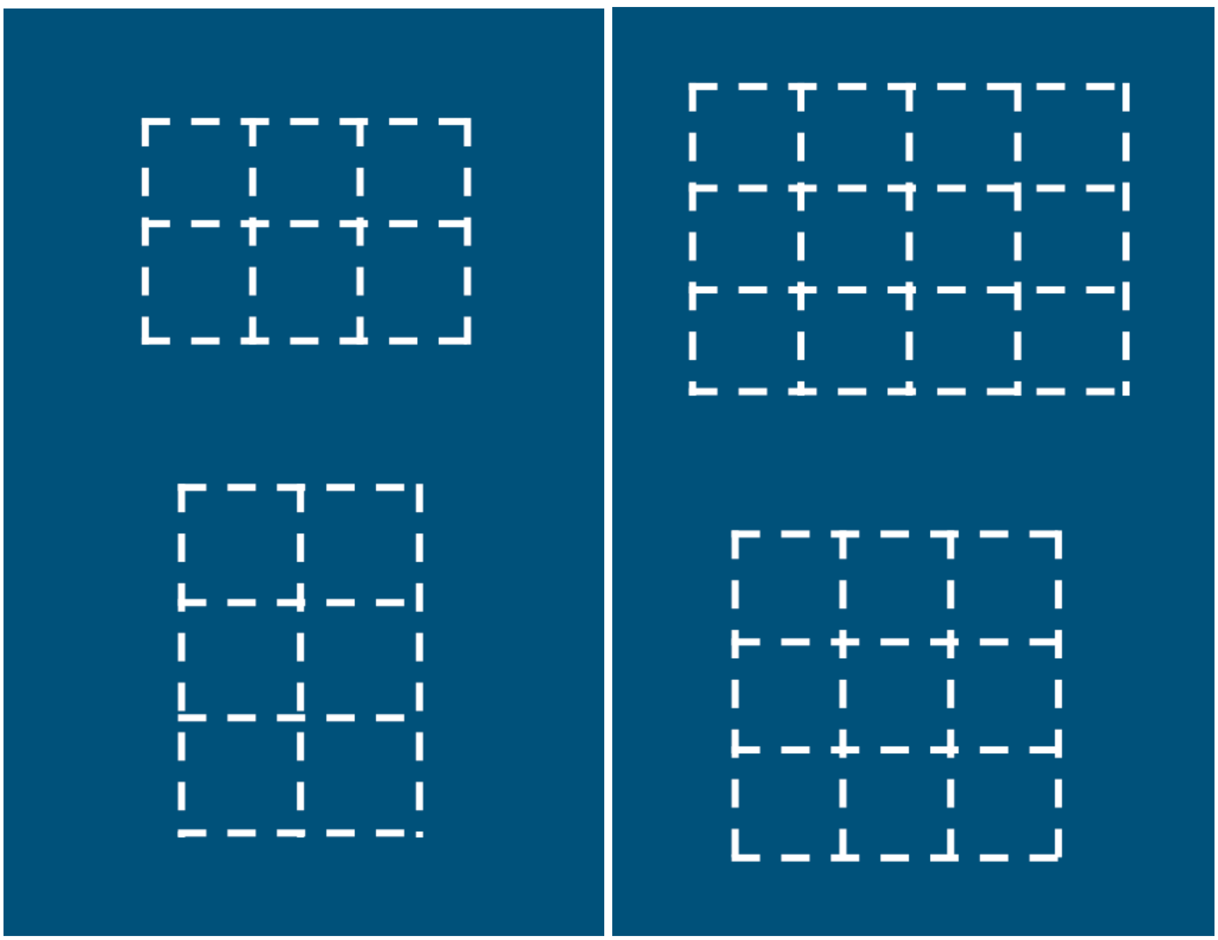

Figure 23 Different size of Self-Sustained Clusters (2×3, 3×2, 3×3, 4×3)

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9.3 Lattice Size of Spin Glass Model

In the project, the spin glass on square lattice size is 10×10. For further work, the lattice can be enlarged to 20×20 or even 50×50. Large size the spin model is likely to give results with more accuracy.

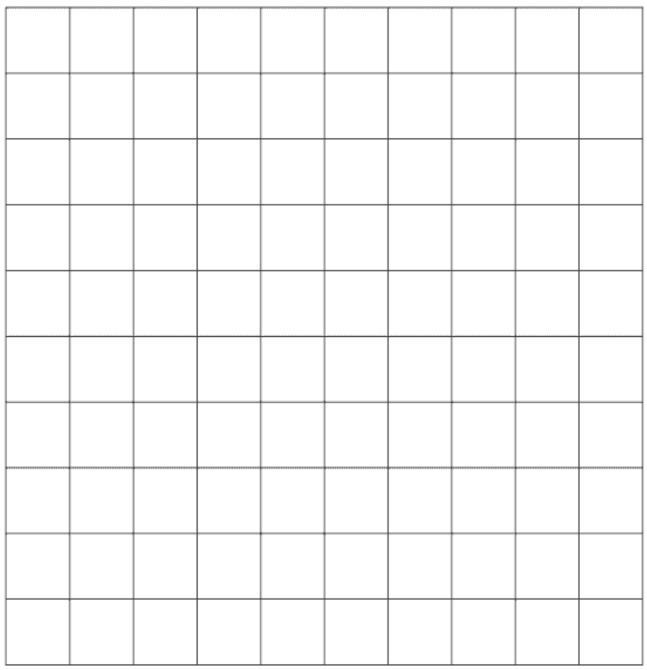

Figure 24. a 10×10 square lattice

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Appendix

Appendix 1

Data of Measurement of SSC at specific set C

magnetization	energy	SSCexist (True = 1/ False = 0)			
0.058795	-138	0	0.093663	-142	0
0.05396	-140	1	0.097984	-142	1
0.081385	-128	0	0.053437	-134	1
0.060745	-138	0	0.069222	-144	0
0.073265	-138	0	0.070161	-136	0
0.081598	-142	0	0.086618	-142	1
0.117251	-148	1	0.083396	-146	0
0.093559	-140	1	0.099727	-142	1
0.067913	-138	0	0.100175	-148	0
0.088349	-138	0	0.064597	-136	0
0.099465	-136	0	0.084239	-142	1
0.073168	-138	0	0.101505	-136	1
0.073334	-138	0	0.11262	-142	1
0.082179	-136	0	0.077138	-136	0
0.083003	-138	1	0.090252	-142	1
0.080044	-134	0	0.078316	-142	0
0.078072	-148	1	0.069843	-134	0

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0.092156	-140	0	0.062474	-140	0
0.06509	-136	1	0.042012	-132	0
0.089998	-138	1	0.088321	-144	0
0.103906	-142	1	0.102085	-142	1
0.06751	-134	1	0.110567	-146	0
0.079403	-132	1	0.080525	-140	0
0.072602	-140	0	0.069041	-140	0
0.085492	-140	1	0.091599	-142	0
0.065553	-132	1	0.0514	-144	0
0.086929	-144	1	0.072637	-142	0
0.113041	-140	0	0.067677	-134	1
0.036865	-130	0	0.09442	-140	1
0.104877	-136	1	0.115396	-148	1
0.114963	-142	1	0.067762	-134	0
0.086137	-132	1	0.085264	-142	0
0.054187	-140	0	0.058188	-138	0
0.094205	-138	1	0.075678	-144	1
0.072058	-132	0	0.094669	-144	1
0.058356	-138	0	0.107145	-142	1
0.062421	-136	1	0.066003	-140	0
0.107582	-144	1	0.066532	-134	0
0.068609	-138	0	0.083517	-142	1
0.109518	-138	1	0.084743	-138	0

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0.106371	-144	1	0.09726	-142	1
0.084143	-136	0	0.061524	-132	0
0.068294	-136	0	0.077545	-138	0
0.081581	-144	0	0.061435	-136	0
0.080709	-138	0	0.090445	-142	1
0.073042	-140	0	0.099047	-138	0
0.080474	-138	0	0.078548	-144	0
0.0704	-140	1	0.05702	-144	0
0.077417	-144	0	0.117441	-150	1
0.06719	-134	0	0.097068	-136	1
0.063439	-140	0	0.077929	-138	0
0.102721	-138	0	0.052663	-126	0
0.069372	-134	0	0.078195	-148	1
0.091719	-138	1	0.073654	-136	1
0.075465	-136	1	0.071539	-132	1
0.070291	-140	0	0.068359	-128	1
0.094762	-146	1	0.073581	-146	0
0.058772	-134	0	0.076644	-140	1
0.099442	-142	1	0.050314	-140	1
0.080527	-144	1	0.095237	-140	1
0.093757	-146	1	0.059941	-136	0
0.061164	-132	0	0.060704	-136	1
0.044095	-132	1	0.091105	-144	0

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0.061653	-136	1	0.093002	-140	1
0.082123	-138	0	0.054442	-132	0
0.132069	-144	1	0.089527	-140	1
0.057649	-136	0	0.083233	-134	1
0.05377	-138	1	0.063874	-148	0
0.08906	-130	1	0.053603	-130	0
0.060013	-136	0	0.064888	-138	1
0.056364	-136	1	0.073435	-136	0
0.099444	-144	0	0.070546	-134	0
0.107868	-142	1	0.101035	-140	1
0.070834	-136	1	0.065369	-136	1
0.063563	-136	1	0.113059	-144	1
0.081516	-150	0	0.058843	-132	0
0.081417	-136	1	0.102799	-138	1
0.111907	-146	0	0.079241	-140	1
0.094768	-134	0	0.104849	-148	0
0.094677	-140	0	0.076966	-136	0
0.092056	-148	1	0.104065	-142	1
0.074602	-134	0	0.061118	-136	1
0.081792	-146	0	0.054294	-130	1
0.105993	-138	1	0.082186	-144	0
0.101671	-146	1	0.09873	-138	1
0.058474	-134	0	0.076925	-142	1

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0.090303	-146	0	0.080704	-138	0
0.048141	-132	0	0.079013	-134	1
0.080025	-134	1	0.109433	-140	1
0.060899	-136	0	0.080368	-140	0
0.077015	-142	0	0.118764	-142	1
0.061765	-136	0	0.04531	-132	0
0.083868	-132	1	0.094071	-142	1
0.092616	-144	1	0.102046	-134	1
0.073897	-144	0	0.064873	-134	1
0.059821	-138	0	0.097197	-132	0
0.070776	-136	0	0.078448	-132	1
0.072774	-138	0	0.056895	-134	0
0.069069	-136	1	0.071482	-132	0
0.098738	-140	1	0.067855	-138	0

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Appendix 2

Data of Self-Sustained Clusters (SSC) configuration

magnetization	energy	no of SSC
0.072922	-138	19

Measuring SSC in at all Spin, s(i, j)

i	j	True = 1 / False = 0	4	4	1
1	1	0	4	5	1
1	2	0	4	6	0
1	3	1	4	7	0
1	4	1	5	1	0
1	5	0	5	2	0
1	6	0	5	3	0
1	7	1	5	4	0
2	1	1	5	5	0
2	2	0	5	6	1
2	3	0	5	7	0
2	4	1	6	1	0
2	5	0	6	2	1
2	6	0	6	3	1
2	7	1	6	4	0
3	1	0	6	5	0
3	2	0	6	6	1
3	3	0	6	7	0

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3	4	1	7	1	1
3	5	0	7	2	1
3	6	1	7	3	0
3	7	1	7	4	0
4	1	0	7	5	0
4	2	0	7	6	1
4	3	0	7	7	1

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Appendix 3

Data of system energy and number of Self-Sustained Clusters

System energy at lowest temperature	Number of Self-Sustained Clusters(SSC)				
-140	12	-140	16	-140	17
-136	13	-140	16	-136	17
-140	16	-140	14	-140	12
-140	15	-136	17	-136	15
-140	19	-140	16	-136	13
-136	17	-136	16	-140	15
-140	13	-136	20	-140	13
-136	14	-140	16	-140	15
-140	16	-136	19	-140	19
-136	20	-136	17	-140	16
-140	14	-140	14	-140	15
-136	18	-140	13	-136	18
-140	15	-136	18	-136	17
-140	19	-136	16	-140	16
-136	18	-136	25	-136	20
-140	13	-136	17	-140	16
-136	14	-136	14	-136	15
-140	15	-140	17	-140	17

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-140	13	-136	18	-140	15
-136	21	-140	20	-140	17
-136	19	-140	14	-136	16
-140	16	-140	14	-140	17
-140	17	-136	17	-136	16
-140	13	-140	15	-136	18
-136	17	-136	17	-140	15
-136	18	-136	20	-136	14
-140	13	-140	13	-140	16
-136	18	-136	16	-140	15
-140	15	-140	16	-140	16
-136	18	-136	15	-136	16
-136	17	-140	16	-136	12
-140	17	-140	17	-140	17
-140	16	-140	14	-136	16
-136	17	-140	16	-136	18
-136	20	-140	16	-140	16
-136	20	-136	18	-140	19
-136	17	-140	13	-136	20
-136	20	-140	16	-136	16
-136	19	-136	17	-136	16
-136	18	-136	16	-136	19
-140	17	-140	16	-140	16

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-136	19	-140	13	-140	16
-140	17	-136	21	-136	19
-136	17	-140	16	-136	17
-140	16	-136	19	-140	16
-140	13	-136	19	-136	16
-140	16	-140	19	-136	20
-140	13	-140	15	-140	16
-140	12	-140	13	-140	19
-136	17	-136	13	-136	13
-140	15	-136	18	-140	16
-140	17	-136	18	-136	20
-140	16	-140	16	-140	14
-136	21	-136	15	-136	19
-140	17	-136	15	-140	16
-140	17	-140	16	-140	17
-140	16	-136	15	-140	15
-140	17	-136	19	-140	16
-140	19	-136	15	-136	16
-136	18	-136	16	-136	20
-136	17	-140	14	-140	16
-140	17	-136	17	-136	12
-140	16	-136	20	-140	19
-136	18	-136	15	-140	16

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-136	21	-136	18	-140	17
-136	14	-136	21	-136	17
-136	17	-140	13	-136	19
-140	19	-140	12	-140	13
-140	16	-140	16	-140	13
-136	16	-140	14	-136	13
-136	17	-136	15	-136	17
-136	19	-136	14	-140	16
-136	18	-136	14	-136	19
-136	15	-136	18	-136	23
-136	15	-136	16	-136	15
-136	10	-136	16	-140	15
-140	16	-140	19	-136	16
-136	18	-136	17	-136	15
-140	16	-140	15	-140	16
-140	16	-140	16	-136	16
-140	15	-136	15	-136	18
-140	16	-140	15	-136	18
-140	16	-136	15	-136	20
-140	16	-136	18	-140	16
-140	16	-136	19	-136	16
-136	22	-140	16	-140	17
-140	13	-136	13	-140	16

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-136	17	-140	14	-136	20
-140	18	-140	15	-136	18
-136	15	-136	20	-136	21
-140	14	-136	17	-140	17
-140	15	-140	17	-140	15
-140	17	-136	17	-140	16
-140	17	-140	13	-136	17
-140	15	-136	15	-136	18
-140	16	-136	18	-140	17
-136	21	-136	18	-136	19
-140	20	-136	21	-140	16
-136	18	-140	17	-140	16
-136	14	-140	17	-136	19

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