香港教育大學
The Education University of Hong Kong

A Project entitled

An Investigation of the Perspectives and Implementation of Variation Theory by Pre-service Primary Mathematics Teachers in Hong Kong

Submitted by

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submitted to the Education University of Hong Kong
for the degree of ***Bachelor of Education (Honours) (Primary) – Mathematics*** in
April 2022

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Declaration

I, *LI Yuhan* , declare that this research report represents my own work under the supervision of *Dr. ZHANG Yuefeng Ellen*, and that it has not been submitted previously for examination to any tertiary institution.

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Abstract

Hong Kong has embarked on a positive road of education reform since the beginning of the 21st century. To facilitate students to cope better with their challenges and needs, the Curriculum Development Council launched “Learning to Learn - The Way Forward in Curriculum Development” in 2001. These reforms promoted student-centered learning and teaching in contrast to the traditional teacher-oriented education. Under the context of education reform, Variation Theory (VT) was one of the most important new theories have tried out. At present, hundreds of primary and secondary schools have implemented VT in different subjects, including mathematics. Previous research showed that VT has been introduced in the course titled “learning study” to the previous pre-service teachers in the Education University of Hong Kong since 2007. Pre-service teachers, to some extent, determine the future of education (Diana, 2019). They had the chance to experience the reform and had the chance for further improvement. However, previous research concentrated on the implementation and perspectives of VT by in-service teachers. These studies did not include the implementation and perspectives of VT by pre-service mathematics teachers in Hong Kong.

This study aimed at investigating the perspectives and implementation of variation theory by pre-service primary mathematics teachers in Hong Kong. Qualitative research methods were adopted. There are 6 pre-service primary mathematics teachers who participated in this study by the Purposive Sampling method. It was found that 1) Perspectives. All six participants preferred to use online resources to understand students’ learning content. They believed the four functions of V3 guide them to use VT to design teaching activities. They considered Contrast as the easiest operated and

the most effective function of V3. Half of the participants missed the importance of critical features. 2) Implementation. Six participants had different topics of teaching experiences related to VT. Two-thirds of the participants did not choose online resources to understand students' learning content. Conversely, half of the participants chose to understand students' learning content by checking the assignments. For critical features, half of the participants did not list them in their lesson plan. Besides, five participants chose contrast as one of the functions to design the learning content which was consistent with the perspectives. What's more, speech intelligibility, organization and presentation skills, and the trade-off between textbooks and VT were the difficulties faced by pre-service teaching found by video observation.

Keywords: Variation Theory, pre-service primary mathematics teachers' perspectives and implementation, Hong Kong education

Acknowledgement

Firstly, I would like to express my sincere gratitude to my supervisor Dr. ZHANG Yuefeng Ellen, who accompanied me through the entire project. When I encountered difficulties, she always inspires me, guides me, encourages me, and supports me. Dr. Ellen Zhang's rigorous attitude towards academic research really inspire me and help me get prepared for future learning. Her spirit of sparing no effort in providing me with suggestions for improvement also taught me how to be a good educator.

Secondly, thanks to rigorous academic training at The Education University of Hong Kong, I laid a solid foundation in education, mathematics, and science. These meaningful courses help me shape the dream of becoming an educator using life to change a life.

Last but not least, thanks a lot to my parents. Thanks for their dedication to me. Thanks for always being my most important support. Thanks for always supporting me in pursuing my dreams and helping me become a better me! I will cherish the present and work hard for a better future!

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1 Introduction

1.1 Research Background and Significance

Hong Kong has embarked on a positive road of education reform since the beginning of the 21st century. To facilitate students to cope better with their challenges and needs, the Curriculum Development Council launched “Learning to Learn - The Way Forward in Curriculum Development” in 2001. These reforms promoted student-centered learning and teaching in contrast to the traditional teacher-oriented education. Teachers passed the knowledge directly to students and the paper-pencil test was the main way that teachers used to access students. Most of the teachers, usually complained that students cannot achieve their suggested objective of learning because they always had the assumption that if they explain the content of the teaching to students, the students will get the same understanding as the teacher (Lo, 2011). However, students are more likely to be considered as an individual with diverse learning needs now. Education becomes student-oriented. Teachers start to concentrate on reflecting and improving their teaching pedagogy. They are more likely to become facilitators to use diverse teaching strategies to cater to the diverse learning needs of students and guide them to develop each ability and potential (Ng et al., 2010). Schools also develops school-based curriculum to better cater to the diverse learning needs of each student. Under the context of orientation change, theories were developed for teachers to enhance their teaching strategies to better facilitate students’ learning. Variation Theory (VT) was one of the most important new theories have tried out.

In learning, people may detect and verify meaningful knowledge through changes in things, and then summarize it into patterns. Therefore, it can be concluded that variation plays an important role during the process of learning (Lo, 2011). In recent

years, the research of VT moved into the level of learning study. A theoretical framework of the variation was developed for educators to guide students to discern the critical features (CFs) of the object of learning through experiencing the variation to finally promote students' study (Ki et al., 2005).

At present, hundreds of primary and secondary schools have implemented VT in different subjects, including mathematics. Previous research showed that VT has been introduced in the course titled "learning study" to the previous pre-service teachers in the Education University of Hong Kong since 2007. In that course, a group of student teachers guided by a course instructor. They mainly learned VT in the first two lessons (9 hours). Then student teachers worked in a group and spent the rest 30 hours of learning on implementing VT by finishing the learning study projects. In the learning study project, three domains of VT, V1 (students' understanding of learning content), V2 (students' understanding of learning content) and V3 (Use Variation Theory to design teaching content) are concentrated by student teachers and the instructor. However, when the student teachers finished this course, they will not gain the guidance of the instructor. It means that they will lose the V2 domain and only focus on V1 and V3 during their future implementation.

Pre-service teachers, to some extent, determine the future of education (Diana, 2019). They had the chance to experience the reform and had the chance for further improvement. According to the informal communication with the course coordinator Dr. Ellen Zhang, in the past 15 years, there were more than 4,600 pre-service teachers received the training course on VT. Previous research focused on the in-service teachers' perspectives and implementation of VT and did not focus on the learning

effectiveness of pre-service Hong Kong teachers who majored in mathematics (Diana, 2019; Royea & Nicol, 2019 & Brante et al., 2015).

As a pre-service teacher, although the author did not take the learning study course, the communication with the supervisor and the positive feedback of VT from references arouse the author's interest to investigate more about VT on pre-service teachers. In this study, the author investigated the perspectives of pre-service teachers towards implementing Variation Theory in mathematics teaching in Hong Kong. This was also the significance of the research.

The thesis included six parts. Firstly, the author reviewed the literature on VT, VT in mathematics education, and VT related to pre-service teachers. Secondly, the design of the purposive sampling method research was introduced. Thirdly, the result of the data analysis and conclusion about suggestions for further improvement was reported.

2 Literature Review

"Perspective" means a particular way of considering something. It is about human's personal thoughts. Different people may have different ways of considering when they face the same thing (Åkerlind, 2015). Although there exists a definition of VT, different people may have different ways of considering and understanding VT (Ahlquist & Gynther, 2020). "Implementation" means the act of starting to use a plan or system. Humans first have thoughts of something, they will start to take actions after consideration (Jankvist et al., 2021). When people shape a structure of VT in their mind, implement, try to use VT will become the next step. Therefore, "Perspective" and "Implementation" are worthwhile elements for this study into VT.

In this section, three domains of the VT (V1, V2, and V3) were reviewed. The Education University of Hong Kong was the first university to introduce VT for pre-service teachers' training. Pre-service teachers learn and use three domains in their learning study project. However, they will become work as individuals after they finish the course. It means that the V2 domain will not be the focus. The Critical Features (CFs) are the decided element of VT. V1 will lead to the identification of CFs, which is one of the most important contributions of VT to the pedagogical development, but the CFs are implicitly integrated into V1. Teachers use CFs to carry out V3. The framework of this study made by V1, V3, and CFs was shown. Besides, previous studies on pre-service teachers' perspectives and implementation were also introduced.

2.1 Variation Theory and Pre-service Teachers in Hong Kong

VT was introduced to pre-service teachers in Hong Kong since 2007. Learning study has become an essential module in the education curriculum in the Education University of Hong Kong for pre-service teachers. The Education University of Hong Kong was the first university to introduce VT for pre-service teachers' training. Pre-service teachers should take a learning study course in year 2 or year 3 study before they go to Field Experience. Pre-service teachers learn and use three domains in their learning study project. Learning study is school-based action research of a university-school partnership developed after the research For Each And Everyone in Hong Kong (Lo et al., 2006). Learning study refers to the professional development process of teachers who collectively prepare lessons, observe teaching, work collaboratively, and systematically reflect on the teaching content of a class to achieve more effective

teaching and learning. Its goal is to help students learn more effectively (Lo et al., 2006). Variation Theory is fundamental to Learning Study (Marton & Pang, 2003).

As a professional teacher, he or she should be able to rationalize their actions of pedagogy with sufficient authority. It is important for them to draw inspiration from learning theories to develop their own teaching strategies to engage in rigorous inquiry and facilitate student learning (Elliott, 2015). However, it is quite challenging for pre-service teachers (PSTs) to provide evidence for their teaching behaviors and to explore the relationship between their beliefs and theories about teaching (Lunenberg and Korthagen, 2009). To help PSTs construct a deep understanding of the relationship between theory and practice, teacher educators (TEs) have devised a significant method -- learning study to integrate theory, learning, and teaching practice (Tan, 2018). According to the informal communication with the course coordinator Dr. Ellen Zhang, in the past 15 years, there are more than 4,600 pre-service teachers received the training course “Learning Study” on VT.

- Variation Theory

Marton and Booth (1997) summarized the research findings which had been investigated over the years on phenomenography. In 1998, Professor Ference Marton raised Variation Theory (VT), illustrating that seeing, perceiving, and experiencing critical aspects of object of learning are three important elements of the learning process. They took the findings to the theoretical level and developed the prototype of the Variation Theory. To understand an object, it is necessary to notice the difference between this object and other objects. As a learning theory, Variation Theory concentrates on how educators can help learners discern critical features (CFs) of the objects of learning in learning content. During the process of teaching, teachers

should use appropriate strategies to enable students to effectively understand and master critical features through experiencing variation (Ng et al., 2010).

According to Lo (2012), "Learning objectives generally refer to the kinds of behavioral changes expected of students because of learning activities. by stating the learning objectives, the teacher is in fact specifying the expected learning outcomes and treating the result of learning as if it can be predetermined" (p. 41). However, the object of learning refers to the content that students need to learn to achieve the expected learning objectives (Lo, 2012). Lo (2012) states that "The internal horizon of the object of learning refers to the critical features or aspects and parts, and their relationships to each other and to the whole" (p. 26).

The variable learning theory emphasizes the relationship between learning content, variation, and learning. Learning Study brings out the above concept with three domains of variation -- V1, V2 , and V3 (Ng et al., 2010).

- V1: Students' understanding of learning content
- V2: Teachers' understanding of learning content
- V3: Use Variation Theory to design teaching content

2.1.1 V1: Students' understanding of learning content

It is impossible to discover the key critical features of learning content simply by research and analysis of subject knowledge. Teachers need to understand students' previous learning experiences, pay attention to what they have done, observe what they have learned, what may lead them to learn, and analyze what learning means to them (Lo, 2011). There are usually five methods for teachers to have a better understanding of students' learning.

1. **Pre-interview before lessons**

Teachers can conduct interviews with students, or we can find some students who have already learned the target learning content to interview. In this way, teachers can try to find out the difficulties they encountered in learning (Lo, 2011).

2. Pre-test and post test

Interviewing students can only help teachers know the views of some students. If a teacher aims to know if other students have the same views or difficulties, he or she can design analytical pre-test and post-tests to test students and analyze their answers. Questions must be analytical and include open-ended sections (Lo, 2011).

3. Observation in lessons

Sometimes, critical features are highlighted when the teacher interacts with the students during the lessons. It is impossible to predict a hundred correct of critical features before each lesson. It should be an ongoing process. Teachers can understand better of students through dialogues with students in class (Lo, 2011).

4. Post-interview after lessons

Learning is different for every student. Students' understanding of learning content depends on their identification and focus on the characteristics of the situation. It is impossible to find all critical features through one study. Teachers must carry the spirit of action research, learn from actions, reflect on, and feedback into teaching, and finally make continuous progress (Lo, 2011).

5. Checking of assignment

Teachers can find students' problems and learning difficulties through assignments including homework and classwork. It provides teachers a direct way to analyze each student's situation and identity the critical features through students' answers and steps of the assignment (Ng et al., 2010).

2.1.2 V2: Teachers' understanding of learning content (the role of experts)

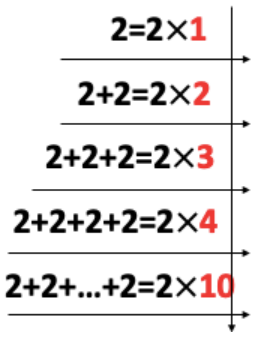
V2 focuses more on the cooperation of various teachers to understand the learning content (Lo, 2011). Teachers collaborate and reflect together, to better facilitate students to learn. In this study, the author main focused on investigating individual pre-service teachers' perspectives and implementation of VT.

2.1.3 V3: Use Variation Theory to design teaching content

There are four functions of variation that can be used to analyze critical features of the learning content. They distinguish the different levels of cognition brought about by different simultaneous concentrations during the variation process (Leung, 2012).

1. Contrast

Contrast is to distinguish between different and dissimilar things. In mathematics, the invariant nature of concepts can be discerned with the help of contrasts. Contrast, therefore, is a sort of change that seeks constant traces among differences (Ng et al., 2010). For example, the addition will be involved in teaching multiplication. A shop sells candies, each candy is $2. We use the following expression to express the total amount of money we spend on different numbers of candies.



There are two contrasts horizontally. The first one is the contract between addition and multiplication. The second one is the contract between 5 multiplications. Through the two contrasts, students may obviously find the ways of basic expression of multiplication. Students may more clearly understand the difference between

multiplier and multiplicand. There is another contrast vertically. The red-colored numbers stand for the number of candies which is the multiplier in multiplication. Students can find the changing number and the unchanging number to clearly distinguish the difference between multiplier and multiplicand.

2. Generalization

Generalization is High-level of contrast, which focuses on the unchanged part. It can be regarded as a mathematical induction activity. Generalization is the comparison after the distinction, and it is a change activity of verification to check whether the mathematical rules identified are universal (Ng et al., 2010). If we change the price of each candy to $5, we will have the below formulas.

$$5=5\times1$$

$$5+5=5\times2$$

$$5+5+5=5\times3$$

$$5+5+5+5=5\times4$$

$$5+5+\ldots+5=5\times10$$

According to the ideas of generalization, we focus on the unchanged part, which is the price of each candy. Students will recognize the meaning of multiplicand through this function of variation.

3. Separation

Separation is the driving force of change, and its main function is to reveal the dimensions and critical features of change. When one aspect of an object is fixed and there is a change in other aspects, some qualities of the thing will be revealed. Separation is an important changeable activity for discovering the relationship between the whole and the parts. For example, when teaching the topic of multiplication, there may exist two difficulties of learning which can be regarded as critical features. Separation is one of the best ways to scaffold students to understand.

Critical Features 1: **2+2=2×2**

The price of each candy is $2, and the price of buying two candies can be expressed as 2+2=2×2. Although the number 2 are the same, by involving separation, the meanings of the two same numbers 2 of the multiplicand and the multiplier are different will be highlighted.

Critical Features 2:

3+3+3+3=3×4

4+4+4=4×3

Distinguishing the difference between 3×4 and 4×3 might be a challenge for students. When teachers involve the separation function of variation and put the formulas together, students can discern the changed and unchanged things. The meaning of multiplier and multiplicand will be understood clearly.

4. Fusion

Merging different critical features of things into a whole after synchronic co-variation is the main idea of fusion. Teachers solve students' learning difficulties to help students to integrate and complete a whole learning unit (Ki et al., 2005). For the above Critical Features 2, it is also an example of fusion. Students can discern the meaning of multiplier and multiplicand and finally construct a firm knowledge system of multiplication.

2.1.4 The importance of Critical Features

According to the literature on V1, V2, and V3, CFs were not mentioned. However, critical Features is something necessary but missing in students' mind (Lo, 2011). According to Lo (2011), "Critical aspect refers to a dimension of variation, whereas the critical feature of an object of learning is a value of that dimension of variation"

(p. 65). For example, in the content of multiplication **2×2**, students may feel confused about the meaning of the two same numbers 2 of the multiplicand and the multiplier. Thus, distinguishing the meaning of two same numbers of the multiplicand and the multiplier is a critical feature.

To conclude the relationship between Critical Features, V1 (students' understanding of learning content), and V3 (use Variation Theory to design teaching content), Lo (2011) states that V1 is the pathway of identifying critical features and teachers use critical features to carry out V3. There can be no discernment without experiencing variation, and no learning can be taken place without discernment. The quality of learning content is determined by the critical features that are simultaneously focused. The critical features of the learning content must be developed before the teacher decides to deal with a selected learning content (Lo, 2011). There are three important reasons to identify critical features.

1. Help teachers revise expected learning content

The expected learning content contains the teacher's insight into factors such as what students should learn and what is the key to learning and reflects the teacher's understanding of the learning content. Often there is a discrepancy between what students experience and what they expect to learn. Guiding students to focus on specific critical features can the learning effect expected by the teacher be obtained, thereby improving teaching effectiveness (Lo, 2011).

2. Change into learner-orientated

Phonomyography research shows that teachers and students have very different perspectives of the same learning content. Previously in Hong Kong, teachers only passed the knowledge in the textbook directly to students. Most teachers usually questioned students' ability when they cannot achieve their suggested objective of

learning (Berry & Lee, 2014). Identifying the critical features of the object of learning provides teachers chances to consider and make judgments on students' perspectives which can improve the effectiveness of both teaching and learning (Leung, 2012).

3. Cater to students' learning diversity

The reason why students have learning diversity is that students fail to identify critical features of certain learning content due to a misunderstanding of prior knowledge or latent cognition. Different key features have different levels of difficulty. Teachers can deal with individual differences of students in a targeted manner by identifying the critical features (Voon et al., 2020).

2.2 The theoretical framework for this study

- **Figure 1 Theoretical framework of the research**

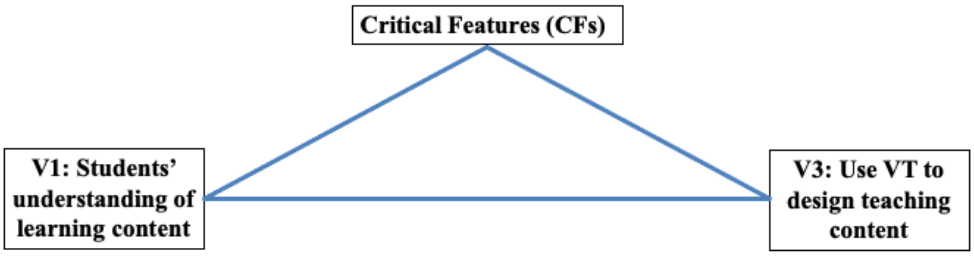


Figure 2.2.1: research framework

	V1 students' understanding of	**V3 use VT to design teaching content**	**Critical Features**

	learning content		
PERSPECTIVE			
IMPLEMENTATION			

Table 2.2.2: research structure

2.3 Previous studies on pre-service teachers' perspectives and implementation of VT

Previous research showed that for PST, they believe that Variation Theory can enhance their theory-guided lesson planning capabilities, pedagogical knowledge, and understanding of more advanced pedagogical concepts (Diana et al., 2019). VT is the best choice to overcome the gap between theory and practice. It can provide PST chances to connect ideas about teaching, teaching practice, and articulating theory. For the implementation of pre-service teachers, according to Yuk (2011), "a research on pre-service Chinese showed that the LS module appears to be able to help pre-service teachers learn productively through close scrutiny of their instructional content and process, and through collaboration and reflection, as identified by studies on other pre-service lesson/learning study programs" (p. 61). However, each coin has two sides, certain challenges in engaging in VT have led to concerns about its applicability to PST. According to Diana (2019), "Learning Study's substantial time requirements, PSTs' difficulties learning to use Variation Theory, and PSTs' readiness to intensely focus on learning and teaching given their limited teaching experiences" (p. 565). Besides, Yuk (2011) states that "one major challenge for large-scale implementation of the model is the availability of experienced tutors as facilitators, as the quality of the student teachers' inquiry depends very much on the

quality of the input and feedback from the tutors" (p. 62). It can be concluded that implementing VT in teaching is relatively challenging for PSTs.

3 Research Design

3.1 Research Questions

This research study developed, on the one hand, investigates the perspectives of pre-service teachers on the implementation of variation theory in mathematics teaching in primary schools in Hong Kong. On the other hand, by understanding the situation of pre-service teachers' implementation of variation theory in teaching, making suggestions for pre-service teachers' education, and helping pre-service teachers overcome challenges through implementation.

- There are two research questions:

1. What were the perspectives of pre-service teachers on VT?
2. How did pre-service teachers implement VT in their mathematics teaching?

3.2 Research Design

This study adopted a qualitative research approach. Qualitative research is a process of natural inquiry that aims to gain insight into social phenomena in the natural environment (Burgess & Bryman, 1999). In Education, qualitative research collects the results of educational research conducted in different aspects, disciplines, and methodological methods (Hatch, 2002). The goal of these investigations is to improve the development of the educational process or environment. The sampling method of this research was Purposive Sampling.

3.3 Research Participants

The qualitative research was conducted between March and April 2022. During the whole research, 6 pre-service teachers who were currently studying at the Education University of Hong Kong and majoring in primary mathematics were chosen by purposive sampling method. The 6 pre-service teachers learned and used the Variation Theory. Below is the table of participants' profiles.

Participant	Gender	Age	Years have learned VT	Learned outside the course
N1	Female	22	1	No
N2	Female	22	3	Yes, supervisor
N3	Female	23	2	No
N4	Female	22	3	No
N5	Male	24	3	No
N6	Female	23	3	Yes, workshop

Table 3.3.1: research participants' profiles

3.4 Data Collection Method

To investigate the perspective of pre-service teachers, the interview was one of the efficient ways. However, if we want to discover the implementation of VT by pre-service teachers, the sharing from their conversation is not convincing enough. Document analysis and observation helped the author understand the participants' implementation comprehensively. Therefore, multiple research methods were adopted to better investigate the motioned research questions:

Research Questions	Interview	Document Analysis	Observation
RQ1	√		
RQ2	√	√	√

Table 3.2.1: research questions and methods

According to Legrand et al. (2003), “In-depth interviews are one of the main methods of data collection used in qualitative research” (p. 139). Interview is the most obvious and direct way to understand participants’ perspectives of VT (Legrand et al., 2003). Through in-depth conversation, the meaningful data will be collected. As for the implementation of VT, only adopting interview is not enough. To gain more objective data of implementation, author collected 6 lesson plan and 2 recorded videos from the participants to analyze their use of VT to design teaching content.

1. One-to-one interview

There was a one-hour interview for each participant. The author held a one-to-one interview with each participant. The participants were interviewed individually and the whole interview session was recorded for review. According to the latest pandemic developments, the interviews were conducted via ZOOM.

During the process of the interview the author collected data by asking interview questions, 1) Perspectives: the background of learning Variation Theory, personal understanding of the VT (CFs, V1 and V3), the evaluation of VT. 2) Implementation: the details of experience using Variation Theory, the difficulties on implementation. After the interview, the perspective, and methods of implementation of pre-service teachers were analyzed. The challenges faced by pre-service teachers were highlighted and explained while some suggestions for pre-service teachers’ education on Variation Theory deal with those defects and challenges.

2. Documentary analysis (For RQ2)

6 teaching plans was collected from participants. 1 teaching plan was collected from each participant. The lesson plan should be related to VT. The learning activities, teaching strategies, patterns of variation and learning materials designed by pre-service teachers related to Variation Theory were analyzed.

3. Observation (For RQ2)

2 recording videos (N1 and N3) of participants' teaching during the Block Practice was collected. I planned to collect at least one recording video from each participant. Two videos were about using VT to teach primary mathematics. However, only 2 of them shared one of their lessons with me. The teaching strategies, presentation skills of pre-service teachers were analyzed.

3.5 Data Analysis

The theoretical framework Figure 2.2.1 and Table 2.2.2 were mainly used to analyze data. For RQ1, the content of participants' sharing from the interview on the perspectives of VT was first collected. The author divided the data into three groups (V1, V3, and CFs) and then filled the data in Table 2.2.2 for findings. For RQ2, the content of participants' sharing (V1, V3, and CFs) from the interview on the implementation of VT was first collected and filled in Table 2.2.2. Lesson plans will then be analyzed by V1, V3, and CFs separately. There was also a comparison to investigate differences between the ideas in sharing and in lesson plans. Last, 2 videos shared by the participants (N1 and N3) were analyzed. The author analyzed the identification of CFs, methods used to understand V1, the function used for V3, and difficulties met during implementation.

4 Findings

4.1 Perspectives of Per-service Teachers on Variation Theory

According to the sharing of the participants on the perspectives of VT, the author firstly summarized the main content that participants mentioned in Table 4.1.1.

Participants	V1	CF	V3

N1	√		√
N2	√	√	√
N3	√		√
N4	√		√
N5	√	√	√
N6	√	√	√

Table 4.1.1: Participants' overview of perspectives

- V1

Participants	Main methods to understand V1
N1	1st Online resources and reference, 2nd pre-interview, 3rd checking of assignment
N2	1st Online resources, 2nd ask for supporting teachers, 3rd checking of assignment
N3	1st Online resources, 2nd pre-interview
N4	1st Online resources, 2nd checking of assignment
N5	1st Online resources, 2nd observation in class
N6	1st Online resources, 2nd checking of assignment

Table 4.1.2: Main methods to understand V1

Firstly, it was found that every participant mentioned the preference of using online resources to understand students' learning content. They stated the convenience of using online resources to find lesson plans, recoding videos, and references when preparing to design learning activities by VT. However, two of the participants pointed out the limitation of online resources related to VT.

"The online recourse is very limited. I cannot always find the topics I want" (N4).

"It is challenging to predict the difficulties of teaching from the online resources" (N6).

For a pre-service preparing a lesson during Field Experience, the convenient and effective ways of searching for knowledge is online. However, the limitation of time and sorts of resources restrict them to predict the difficulties accurately. They believe that they may prepare better if they can get more examples on different topics.

Secondly, four of the participants stated the importance of checking the assignment of students can help teacher understand students' learning content.

"When I correct students' assignment, I can identity their learning difficulties through their steps of answers and mistakes" (N1).

Mathematics is a subject of logic. During analyzing students' mistakes and steps of the answer, teachers can accurately identity the critical features to help students learning. However, there exist an interesting phenomenon motioned by N2.

"I met difficulties when checking students' homework to identify Critical Features. Lots of students got 100% correct on their homework. However, I found nearly half of my students had the mark of correction from parents on their homework which the handwriting is very different from the students. This behavior of correction influences my judgement a lot" (N2).

Guiding and teaching children to do home has become a trend in China (Wong et al., 2018). The word "Monster Parents" has been used to describe those parents who force their hard to gain high marks. They cannot stand their children to perform not perfectly even in family homework (Wong et al., 2018). Therefore, this may become the possible reason why parents involve in correcting students' homework. When a teacher wants to know more about students' learning difficulties to identify the critical features of their performance of the homework, they usually cannot get the accurate result because some of the students' mistakes are corrected before they hand to the teachers. To conclude, if a teacher would like to judge the critical features of students' learning accurately, the involvement of parents guiding homework should be avoided.

- CFs

The table is the description of CFs by the six participants.

Participant	Main Ideas of CFs
N2	*"Teachers need to consider students' learning difficulties to judge **Critical Features**. Then carry out teaching activities to guide them find **unchanged truth through changing**"*
N5	*"**Critical Features** are the most important for teachers to decide. Then according to the **Critical Features**, teachers should help students understand the **unchanging by changing**"*
N6	*"Firstly, teachers should justify **Critical Features**. Secondly, teachers teach students to fix the difficulty through **changing and unchanging**"*

Table 4.1.3: Main Ideas of CFs

According to the interview, six participants can use their own words to describe their personal understanding of VT. All the participants stated the discern of changing and unchanging. According to the literature review, a Critical Feature is the decided role of VT. Without identifying Critical Features of learning content, we cannot carry out any activities of variation. From their sharing, we found that only three of the participants (N1, N3, and N4) mentioned the concepts of Critical Features, while others considered the more important role was discerning changes. It means that three of the participants failed to understand the VT accurately and in-depth. Only N2, N5, and N6 illustrate the importance of identifying the Critical Features and then using the Critical Features to design teaching activities.

- V3

Participants	Perspectives of using VT to design teaching
N1	*"Four functions are the guidance of design."*
	Prefer Contrast, basic and easy operated
N2	*"When I identity the critical features, I check the four functions to decide how to plan a lesson."*
	Prefer Contrast, sometimes use the fusion in a learning unit
N3	Prefer Contrast and Separation
N4	Less attention to Generalization, because used to focus on changing part
N5	Prefer Separation
N6	Prefer Contrast

Table 4.1.3: Main Ideas of CFs

All participants showed their understanding of the four functions of V3. They believed that considering functions can help them decide the ways of designing teaching content through variation. Five of the participants preferred using contrast to teach because it was both easy to design the teaching activities and easy for students to understand. However, Generalization and Fusion were not the preference of the participants.

4.2 Implementation of Per-service Teachers on Variation Theory

According to the sharing of the participants on the implementation of VT in one lesson during the interview, the author firstly summarized the main content that participants mentioned in Table 4.2.1.

Participants	V1	CFs	V3
N1	Online resources	Listed	Separation
N2	Checking of assignments	Listed	Contrast
N3	Checking of assignments	Listed	Contrast, generalization, fusion
N4	Checking of assignments	No mentioned	Contrast
N5	Online resources	No mentioned	Contrast
N6	Pre-test	No mentioned	Contrast

Table 4.2.1: Participants' overview of implementation

- V1

The following Table 4.2.2 is the comparison of the method that participants used during their shared lesson plan with the preference of the methods they shared in the interview of the perspectives on V1.

Participants	Implementation	Priority ranked in perspectives
N1	Online resources,	1st
N2	Checking of assignments	3rd
N3	Checking of assignments	Not mentioned
N4	Checking of assignments	2nd
N5	Online resources	1st
N6	Pre-test	Not mentioned

Table 4.2.2: Comparison of implementation and perspectives on V1

According to Table 4.2.2, the method that two of the participants (N3 and N6) choose to understand students' learning content is not mentioned during their sharing of the perspectives of V1 in the interview. The other two participants (N2 and N4) did not choose the first preference which is online resources during their implementation. Only two participants (N1 and N5) adopted the same methods (online resources) that they shared during the interview. To conclude, although students consider the online resources as their first preference to understand students learning content. However, in their implementation, two-thirds of the participants did not choose online resources to understand students' learning content. Conversely, half of the participants chose to understand students' learning content by checking the assignments.

- CFs

According to Table 4.2.1, half of the participants listed the CFs in their lesson plan, while the other half of the participants did not list the CFs. Table 4.2.3 showed the critical features that participants listed.

Participants	CFs
N1	Distinguishing the meaning of two same numbers of the multiplicand and the multiplier
N2	The area of the triangles is the same if they have the same base and height
N6	In problem related to dates, at least two known elements are required to calculate additional unknown elements.

Table 4.2.3: CFs from N1, N2 and N3

In Table 4.1.1, N2, N5, and N6 mentioned the importance of CFs in VT. However, N5 did not list the critical features in his lesson plan. N1 did not mention CFs when she described VT but listed the CFs in her lesson plan. For the three who did not list the CFs, although their teaching activities are related to VT, the author could not find the patterns of variation in their lesson plan. To conclude, CFs are the deciding factors of VT, from the implementation of participants, the importance of CFs was needed to be strengthened.

- V3

Table 4.2.1 showed the function that participants used in their lesson plan. Although six participants taught different topics of mathematics, 5 of participants chose contrast as one of functions to design the learning content. They consider contrast is one of the most effective and easiest methods of teaching. For N1, the learning topic she taught was using addition to recognize multiplication. There can only find one pattern of variation in her lesson plan.

Invariant	Varied	What is to be discerned
Different numbers	The number of multiplicand and the multiplier are same	Distinguishing the meaning of two same numbers of the multiplicand and the multiplier

Table 4.2.3: Patterns of Variation by N1

N1 used the generalization function to teach using addition to recognize multiplication. However, she reflected that there exists another critical feature (Distinguishing the difference between multiplicand and the multiplier e.g., 3×4 and 4×3) but she did not use VT to teach. The reason why she did not use was that she considered she lack of knowledge of both subject and VT. When the author investigated the situation of other participants, the author found that all 6 participants state a lack of confidence when using VT in teaching. One possible reason might be because they lack subject knowledge. The other possible reason because they only have one face-to-face Field Experience because of the pandemic, which means their experience may be relatively less. 4 participants stated that the course offered by the university is less, and they do not gain enough training on VT.

"I always ask my supporting teachers to help me with my lesson plan. When I tried identifying the critical features of my students, I do not have confidence" (N2).

"I only took one course related to VT. When I tried to use VT in teaching, I always feel I made mistakes and did not express clearly" (N4).

Because of the lack of knowledge and confidence. All participants state the need for experts in VT to guide them when implementing VT in mathematics. They believe in the importance of scaffolds from experts. In the beginning, the author considers the irrelevant of V2 in this study because V2 concentrate on the cooperation between teachers, and this study is mainly focused on individual teachers. However, it is because of the lack of scaffold from expert teachers, that those pre-service teachers may realize the importance of cooperation between teachers, which can be concluded as V2 is also an important element of VT.

- Findings from videos

1. Speech intelligibility

According to the observation on the recoded videos provided by N1, speech intelligibility has become a problem faced by pre-service teachers. The teaching topic of N1 was using addition to recognize multiplication. Table 4.2.3 was the pattern of variation she designed. In the video, N1 stammered and repeated the irrelevant content more than three times when teaching by implementing VT. The instruction provided by N1 was not clear. When she expressed “5+5+5+5+5+5”, she said, “five plus five plus five plus five plus five plus five” instead of “there are 6 five add together”, students may confuse and ignore the number of 5, which is the important part of the variation. Clear instruction, accurate mathematics expressions, and confident and fluent speech skills should pay more attention to pre-service teachers.

2. Organization and Presentation

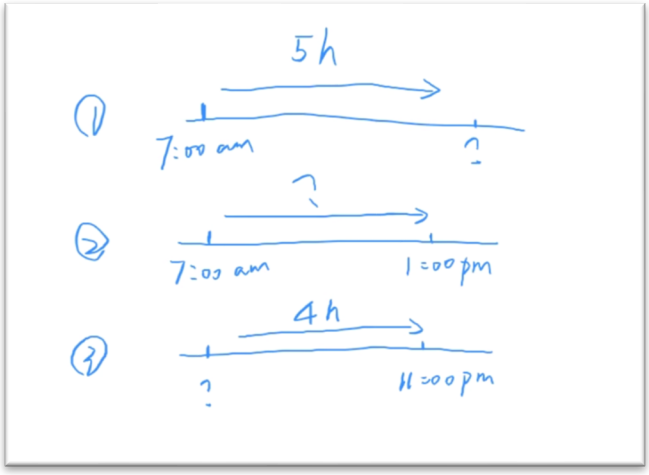


Figure 4.2.1: Figure 1

This is a capture of "handwriting on the blackboard" by N3 when she taught "Problems Related to Time". She considered the following pattern of variation.

Invariant	Varied	What is to be discerned
Different scenarios.	Three elements: Starting Time, Time Passes and Finishing Time.	In each question, at least two known elements are required to calculate additional unknown elements.

Table 4.2.3: Findings 5

It is clearer when N3 organized the content of variation together for students. Students can experience the changes by analyzing each scenario of the question. Putting together can help students contrast, separation, and fusion better. However, it would be more effective when labeling "Starting Time, Time Passes and Finishing Time" on a specific place in the diagram. Students will obviously discern the changes and unchanged patterns easily. What's more, teachers' handwriting should be clear and neat. For primary students, N3's handwriting may lead misunderstanding to students. For the accuracy of a mathematical expression, when we write 7 o'clock in the

morning, it should be 7 “a.m.”. However, N3 forgot to add “.”. To conclude, for pre-service teachers, the organization and presentation of the teaching content should be accurate and clear. It is the best and most obvious way for students to present the content of variation in one slide.

3. The trade-off between textbooks and VT

In some situations, there exists a contradiction between the arrangement of the textbook and the design of VT. According to the above N3’s example, she chose to arrange the three scenarios together. However, the arrangement of the textbook is quite different. The textbook divides the three topics separately into two teaching topics. The first topic is finding time passes or finishing time. The second topic is finding the starting time. If students follow the arrangement of the textbook, the contract and fusion will not be highlighted. In this situation, N3 decided to follow the design of variation instead of the arrangement of the textbook. According to N3’s sharing, the feedback of students is relatively positive. As a pre-service teacher, it is challenging to decide the trade-off between textbooks and VT. However, standing in learners’ position to identify the critical features and carry out suitable activities with variation might be one of the solutions when facing trade-off between textbooks and VT.

5 Discussion

Critical Features is the decided role of VT. Identifying the critical feature is the key concept of variation. Without knowing the critical feature means the teacher dose not understand the necessary knowledge that missing in students’ mind and the variation of teaching design is unsuccessful (Lo, 2011). However, the interview data from pre-service teachers show that some of the pre-service teachers missed the importance of

the Critical Features. Therefore, it is necessary to strengthen pre-service teachers' commiment of critical features through the education in university.

V1 is understanding students' learning content. All participants firstly chose to use online resources to understand their students' learning content because it is more convenient compared to other methods. However, limitation of the online resources may become one of difficulties pre-service teachers face. It provides the public an opportunity to construct an online VT education community to share experiences, resources or question and answer.

V2 concentrate on teacher's cooperation to understand teaching content (Lo, 2011). In the beginning, the author considers this is not the focus of this study since this study concentrate more on individual's perspectives and implementation. However, after the interview with participant, the need of experts to scaffold was highlighted. Because lack of confidence and subject knowledge, pre-service teachers need someone who is professional offering them helps. Therefore, V2 also plays an important role in variation theory for pre-service teachers.

V3 is using VT design teaching content (Lo, 2011). Teachers can find students' problems and learning difficulties through assignments including homework and classwork. It provides teachers a direct way to analyze each student's situation and identity the critical features through students' answers and steps of the assignment (Ng et al., 2010). However, from the interview, one participant states the existence of interference from parents. Parents may help correct their children's mistakes which will very influence teachers' judgement of students' critical feature. The effectiveness of VT teaching activities will decrease. Therefore, the author hopes teachers to pay

attention to this phenomenon and identify the critical features through more comprehensive ways.

6 Limitations

Firstly, the author did not attend the Learning Study courses or had the experience of using Variation Theory in teaching before. When the author communicated with her supervisor, her interest in VT was aroused by the supervisor's inspiration. Although the author large number of references, the gap was still existing because of the lack of personal experiences.

Secondly, the size of the research objects is relatively small. The research aims to investigate the perspective and implementation of pre-service teachers. Because of the limitation of time, 6 participants are invited. The research objects are quite limited, and their perspectives and implementation only stand for a relatively small group which cannot represent the large pre-service teachers' group.

Thirdly, only two of the participants would like to share their recorded videos for further observation. Therefore, the Finding 5: the difficulties faced by pre-service teachers from observation may be not enough.

7 Conclusion

This qualitative research concentrates on pre-service teachers' perspectives and implementation of VT. Per-service teachers are the determination of future education development. It shows the significant benefit of VT on both teaching and students' learning. As the only university offering the Learning Study course, the author chose

6 participants who were majoring in primary mathematics and had learned and used VT before. From their sharing during the interview, the author firstly found the missing the mainly important concept “Critical Feature” in some pre-service teachers’ minds. Secondly, the author found pre-service teachers need experts to scaffold them because of the lack of knowledge and the limitation of the online resources. Then, to the author’s surprise, interference from parents was found. This made a contribution to further studying in this field. Lastly, the difficulties faced by pre-service teachers were concluded. The steps of researching and making improvements on VT will never end. Taking the meaningful findings and reflection, the author will keep improving her ability to do research to make more contributions to the development of Variation Theory.

8 Contribution to the research related to Variation Theory

Since there does not exist much research focus on pre-service teachers’ perspectives and implementation of VT, the author paid attention to this object group and found the missing the important role – Critical Features in some of the participants’ minds. It may reflect the weak of teaching in the Learning Study course at the Education University of Hong Kong. The important role of Critical Features should be highlighted. Besides, the difficulties faced by pre-service teachers may make contributions to the design of the Learning Study course. To some extent, these difficulties might become the critical features of the Learning Study course. The designer of the course may take the difficulties as a reference to facilitate the pre-service teachers’ understanding and use of VT. The illustration of the limitation of the online resources and the need for experts may become an idea for developing an online website. On this website, both in-service and pre-service are welcome to share

their cases or offer help to each other. What's more, the findings of parents' interference might become a significant contribution to the VT research development. Parents may possibly have the trend of correcting students' homework which will influence teachers' identification of Critical Features. This will bring a negative influence on the progress of implementing VT. The author considers the interference of parents might become another meaningful research topic to investigate.

9 Recommendation

9.1 Recommendations on improving the research

- The improvement of the research type diversity

To investigate the perspective and implementation of the pre-service teachers and decrease the limitation of research objects, it would be better to design a mixed research method. From the quantity research, more pre-service teachers' perspectives on implementing VT will be found. From the qualitative research, detailed sharing on implementation will be gained. In this way, the result might have been able to represent a relatively large number of pre-service teachers.

9.2 Improvement of Learning Study course

The author found although pre-service teachers who took LS courses can use VT in their design of teaching, the importance of CFs should still be highlighted. Besides, the need for experts for the scaffold was found. Participants stated they lack confidence and knowledge of VT and subjects. The course instructor may construct a chat group online. Students and instructors may share their questions and answers related to VT in this group. What's more, the presentation skills of pre-service

teachers should also be concentrated on. Instructors may provide pre-service teachers more chances to simulate teaching and provide them more guidance on speech.

9.3 For Pre-service teachers' implementation of VT

First, there are diverse ways to understand students' learning content. Pre-service teachers can try more and communicate with others more to improve the understanding of students. Second, the pre-service teacher should concentrate on the important role of critical features in VT. V1 is the pathway of identifying critical features and teachers use critical features to carry out V3. Third, when using VT to design teaching, pre-service teachers should have a comprehensive consideration of using functions. Specific functions are used for specific critical features. Last, pre-service teachers should try their best to train themselves to become teaching professionals. They can attend workshops related to VT or read more case sharing and references related to VT in their spare time.

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11 Appendix

Interview questions:

Name of Participant: _____

Code: _____

Nationality: _____

Year of Study: _____

Perspective:

1. Have you joined any lectures related to Variation Theory?
    I. When? Where?
2. Could you please use your own words to introduce what is VT?
    I. Why can you remember so clearly?
    II. Did you remember critical features?
3. How would you understand your students' learning content (V1)?
4. What functions do you prefer when you use VT to design teaching (V3)?
5. Why do we need VT in mathematics teaching? (The meaning of using VT in mathematics teaching content, V3)

Implementation:

6. Have you ever used/applied VT in your teaching?
    I. Did you finish any project (presentation, essay…) related to VT?
    II. Did you apply during your Field Experience?
7. How did you use VT?
    I. Any Examples?
    II. Did you use it in the whole learning unit or a specific learning topic?
    III. Which pattern (Contrast, Generalization, Separation, and Fusion) did you involve? Why?
    IV. How did you teach the topic by using the pattern?

    The reason why you used or did not use VT in your topic of teaching?
8. Have you ever met any difficulties during implementing VT in your teaching?
    I. Schools' belief
    II. Diverse learning needs
    III. Time arrangement
    IV. Textbook
9. What are the facilitating factors that benefit teachers' implementation?