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The Education University of Hong Kong

A Project entitled

MTH4902 Honours Project II

Explore the effectiveness of using 'maximizing the area of a region with a fixed perimeter' to eliminate the misconceptions of the relation between perimeter and area

Submitted by

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submitted to The Education University of Hong Kong

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in *8th April, 2022*

Declaration

I, Chan Nok Ting declare that this piece of research is fully conducted on my own der the supervision of Dr Chan Wing Sum, the Senior Lecturer II at The Education University of Hong Kong. The research report has not submitted previously in any one of the tertiary institutions.

Signature

8th April, 2021

Content

1. <u>Abstract</u>

Due to the lack of relation between the concept of perimeter and area in the current primary curriculum in Hong Kong, misconceptions about their relationship are evitable. This research uses the GeoGebra classroom to explore 'maximising the area of a region with fixed perimeter', then figure out the effectiveness of eliminating students' misconceptions on the relationship between perimeter and area. Data are collected mainly through observation, pre-test, and post-test to investigate students' understanding of the relationship between perimeter and area before and after the exploration. Based on the data collected, the result shows students' misconceptions before exploration; it also demonstrates a significant improvement in understanding of the exploration and simultaneously eliminates some of the misconceptions mentioned. The causes for misconceptions and how the exploration helps to eliminate them will also be discussed.

2. Acknowledgment

May I express my greatest gratitude to my supervisor, Dr. Chan Wing Sum, the Senior Lecturer II of the Department of Mathematics and Information Technology at the Education University of Hong Kong. Without her concrete comments, advice, and suggestions, the honors project will not be disclosed.

3. Introduction

This paper will explore the effectiveness of 'maximising the area of a region with a fixed perimeter' to eliminate the misconceptions about the relationship between perimeter and area.

The relationship between perimeter and area in the primary mathematics curriculum is not included, not to mention isoperimetric inequality, whereas misconceptions were commonly seen. According to Curriculum Development Council (2017), the learning targets of primary 4 to 6 mathematics curriculum in measurement strand include recognising the concepts of perimeter and area; using different ways to compare the perimeter and area of 2D shapes; and choosing appropriate standard units to measure and compare the perimeter and area of 2D shapes. The relationship between perimeter and area of a quadrilateral is not included in both primary and secondary mathematics curricula. In tertiary education, the focus of the isoperimetric inequality is to prove circle has the largest area with a fixed length, rather than regular polygons or equilateral polygons have the largest area with a fixed length (Osserman, 1978). As far as we know, no empirical research was done on teaching it in primary school. Although perimeter and area are taught separately, primary students can use prior knowledge of the formula of ‘perimeter of triangle and quadrilaterals’ to find the largest quadrilateral area under a fixed perimeter. Therefore, this research aims at filling up the research gap.

In the ever-changing teaching and learning environment, the demand for e-learning has gradually risen. It is of vital importance to utilise GeoGebra to infer the relationship between perimeter and area of a quadrilateral with the prior knowledge of perimeter and area of triangles and quadrilaterals. Firstly, identify the difficulties of students learning relationships between perimeter and area; then explain the importance of using Geometry software in mathematics lessons; and the relationship between perimeter and area of a quadrilateral by letting students resolve plane figures, such as triangle and quadrilateral, of the largest possible area whose boundary with a specified length. The progress of maximising the quadrilateral area with a specified length will be further explained.

4. <u>Literature Review</u>

4.1 <u>Difficulties of primary students in learning relationship of perimeter and area</u>

Machaba (2016) indicates that the complex relationship between area and perimeter creates many misconceptions among learners. They have to acquire experience in handling the spaces that they are measuring. The mathematics education literature illustrates that many learners assume figures with the same perimeter must have the same area (Outhred & Mitchelmore, 1996). Tirosh and Stavy (1999) stated that some learners predict the increases or decreases of the area associated with the increases or decreases of the perimeter. It is hard for these learners with a misunderstanding of the concepts of area and perimeter to realize that if the perimeter remains constant in a set of quadrilaterals, then the area of those quadrilaterals does not have to be the same; and vice versa.

Piaget's theory on manipulation of compensation to attain conservation explains learners' view on the intuitive rule 'Same perimeter – Same area; and vice versa' (Piaget, 1976). For instance, one side of the square is lengthened while the same amount shortens the other; these learners may claim that both perimeter and area of the new rectangle are equivalent to the original square based on their intuition. Another example is when doubling the length of the sides of a rectangle, learners intuitively had a perception of doubling its area (Kidman, 2001).

The primary mathematics curriculum focuses on the procedure and formula of perimeter and area of quadrilaterals (i.e., square, rectangle, parallelogram, and trapezium) instead of a deep understanding of the relationship between the length of sides and area (Curriculum Development Council, 2017). Learners tend to believe the area remains unchanged when holding 3 points of a quadrilateral and gradually adjust the tacks of the remaining point with a fixed length of the perimeter (Nunes et al., 1994, p.256). Graphically, it is illustrated as fixing the point on quadrilateral A, B, and D and adjusting point C. (refer to Figures 1a and 1b)

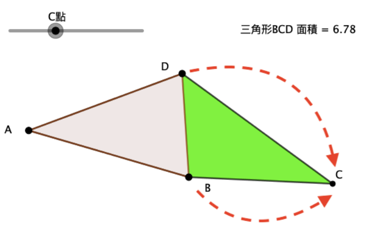


Figure 1a: Holding points A, B, and D and adjusting the tacks of remaining point C with a fixed length of the perimeter

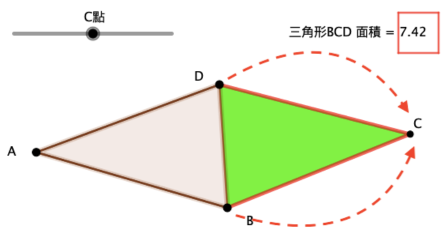


Figure 1b: Holding points A, B, and D and adjusting the tacks of remaining point C with a fixed length of the perimeter

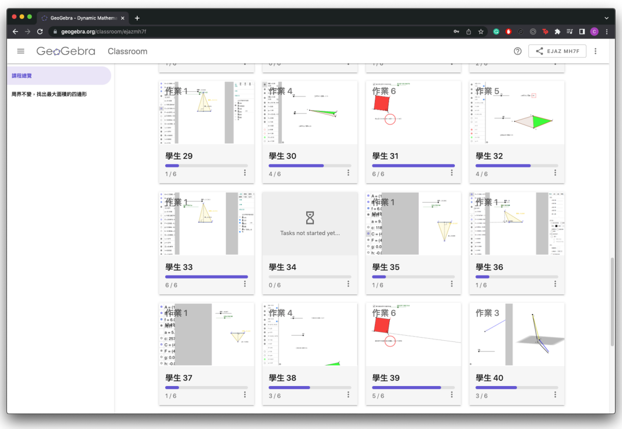
Therefore, this method will be adopted to explore the relationship between the perimeter and area of a quadrilateral.

According to Wong et al. (2002), the committee of mathematics education conducted research that found geometry strand is the most challenging to students, as they prefer learning more about daily life and down-to-earth mathematics. With the assistance of concrete teaching tools, developing and consolidating geometric knowledge is more effective (簡珮華等, 2009).

4.2 GeoGebra and its application in primary mathematics learning

The digital environment motivates students in the teaching and learning of Mathematics (Korenova, 2012). Without using daily life examples to introduce geometric topics is harder for learners to grasp complex concepts (簡珮華等, 2009). Arbain and Shukor (2015) indicated that students using GeoGebra have positive consciousness of enthusiasm, confidence, and motivation. It could be introduced to students for more critical and creative thinking in Mathematics. GeoGebra is a free dynamic mathematics software that enables educators to design learning materials suitable for students' needs (Lilla, 2017). Poon & Wong (2017), Poon (2018), and 柯 (2015) utilise GeoGebra to picture and animate the concepts of geometric concepts and 3D shapes. The use of GeoGebra to visualise mathematical concepts that facilitate understanding and enhances the motivation for learning Geometric topics (柯、程, 2014; Poon, 2018). It also cultivates an open atmosphere for class discussion and presentation. However, we can hardly find GeoGebra teaching materials on 'maximising the area of a region with a fixed perimeter' in Hong Kong. Therefore, this research will determine how students use GeoGebra to explore 'maximising the area of a region with a fixed perimeter.'

On top of that, GeoGebra classroom is a virtual platform through which teachers can carry out interactive tasks, monitor the updated working progress of students, display students' work, and keep students' work as a record.



4.3 The relationship between perimeter and area of a quadrilateral

The theory of isoperimetric inequality is complex for primary students; part of the theory explains the relationship between the perimeter and area of a quadrilateral. Osserman (1978) demonstrated that when maximising the area of a domain under the constraint that the length of its boundary is fixed, a regular polygon has the largest area. Machaba (2016) also illustrated that given a fixed perimeter, the quadrilateral with the largest area would be the one with the dimensions that are closest together, namely a square. This relationship of perimeter and area will further be discussed.

5. Research Questions

Given such context, this research project will be conducted to the following questions:

- What is the misconception of primary students on the relation between perimeter and area?
- How do students understand the relationship between perimeter and area before and after exploration of ‘maximising the area of a region with a fixed perimeter’?

The purpose of the research questions is to figure out how students use GeoGebra to maximize the area of a domain under the constraint of fixed length step by step with the researcher’s guidance. After the exploration, students’ understanding of the relationship between perimeter and area will be examined.

6. Methodology and research design

The concept and formula of the polygonal perimeter were taught in primary 4; while the concept and formula of the polygonal area were introduced in primary 5 (Curriculum Development Council, 2017). Participants with this prior knowledge are invited to participate in the research. Therefore, 31 Primary 5 students from my block practice school are invited to be research participants.

This research is conducted in a mixed method of qualitative and quantitative research. For data collection, video recording, students' work in worksheets, and GeoGebra, pre-test and post-test will be primary sources and data.

For qualitative data, the whole lesson will be video recorded. Through class observation, it is supposed to figure out the difficulty and how participants respond in the exploration of 'maximising the area of a region with fixed perimeter'. Their working progress and findings will be recorded in class worksheets (see Appendix 3) serving as written records and those in the GeoGebra classroom serving as video records.

For quantitative data, a pre-test and post-test (see Appendix 1) with the same content are designed to test students' understanding of the topic and the relationship between the perimeter and area of a quadrilateral. To increase reliability, the tests will be conducted under the researcher's supervision and with 15 minutes of time limitations during face-to-face lessons. The number of correct answers are used to examine students' strengths and weaknesses in learning this topic. Questions in the test are mainly divided into two parts, first is to examine their understanding of the exploration activity (questions 1 to 2, 6 to 8); another part is to examine students' understanding of the relationship between perimeter and area (questions 3 to 5). For instance, 'Is it true that two quadrilaterals with the same perimeter must have the same area?', 'Does the increases or decreases of perimeter associate with the increases or decreases of the area?'

The research is designed to firstly do a pre-test during a face-to-face lesson, followed by a 45 minute online (via ZOOM and GeoGebra classroom) exploration, at last, a post-test to examine students' understanding of the exploration and relation perimeter and area.

Research design

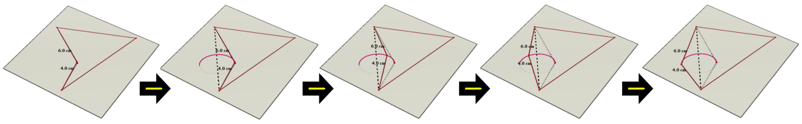
1. Revise prior knowledge required for the exploration, including the names, characteristics, and area formulas of triangles and quadrilaterals learned in primary 3 to 5; methods and formulas to calculate the area of polygons; as well as introducing some unlearnt terms of quadrilaterals, such as kite shapes, concave, convex quadrilaterals).
2. Use GeoGebra classroom to carry out the exploration. Through the exploration, participants will fix 2 points on a triangle and 3 points on a quadrilateral, then adjust the other point to maximise the area of the region with a fixed perimeter. During the progress, they will be able to understand the change of area does not necessarily correlate with the perimeter, so as to eliminate the misconception of the relationship between perimeter and area. The teaching plan (see Appendix 2) and the exploration progress is as follows:

The interference progress is to start from an irregular quadrilateral, gradually narrow it to some specific quadrilateral that we are familiar with. The inference form is 'For any quadrilateral X, a quadrilateral Y must be found with the same perimeter.

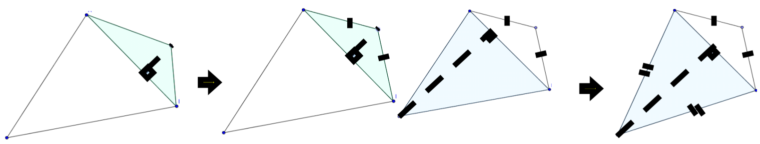
While the area of quadrilateral Y must not be smaller than that of quadrilateral X. Therefore, we will no longer consider quadrilateral X, but consider quadrilateral Y.’

The operation progress is to ensure the length of the perimeter remains consistent and change quadrilateral from X to Y. Firstly, fix the length of each side but change the interior angle to restructure the shape. Secondly, change the length of the two sides if the area remains unchanged. The procedure would be best if one side is lengthened while another side is shortened by the same amount, so the length of the changed two remains unchanged. Finally, if the above progress is not working, then change more than two sides.

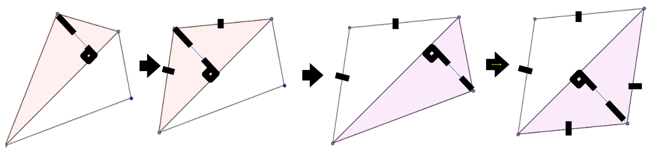
In other words, the progress of the investigation should start from scalene triangle to isosceles triangle, to equilateral triangle, with the conclusion of polygons with equal sides have the largest area with a fixed perimeter. Then start again from concave quadrilateral to convex quadrilateral, to kite, to rhombus, and a square at last. For any concave quadrilateral with the same perimeter as a convex quadrilateral, the area of a convex quadrilateral must be larger. This part will use geometric sticks or GeoGebra.



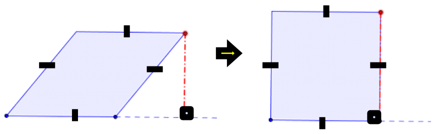
To find a quadrilateral with an area not smaller than the convex quadrilateral, we have to divide the convex quadrilateral into two triangles and find the largest area of the two triangles respectively. In a triangle with a fixed base, the increases of its height increases in the area. Therefore, a kite will be resulted as having the largest area if only two vertices are changed.



Find a quadrilateral with an area not smaller than a kite. Repeat the previous steps by fixing another two vertices and moving the previous moved vertices. A rhombus will be generated as having an area larger than a kite.



Lastly, the rhombus can be seen as a parallelogram. Adjusting the angles of the rhombus area of the square will have resulted in the quadrilateral having the largest area. This part will also use geometric sticks or GeoGebra.



7. **Result**

The result of the research is divided into two parts, respondents' misconceptions of 'relation between perimeter and area' and overall performance on pre-test and post-test will be described.

7.1 Result 1 – The misconception of 'relation between perimeter and area'.

Questions 3 to 5 in pre-test and post-test examine students' performance on the relation between perimeter and area. The data collected from the pre-test are as follows:

Q3: The perimeter of a quadrilateral increase, the area ______.	Pre-test	Post-test
Increase	22	14
Decrease	0	2
Unchange	2	1
May not change	5	14
No answer	2	0

Table 1: Respondents' distribution of answers in question 3 in pre-test and post-test

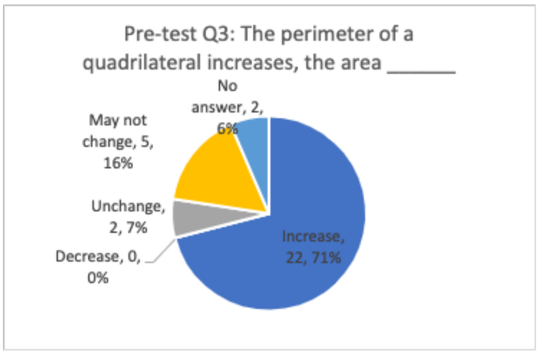


Figure 2: Respondents' distribution of answers in question 3 in post-test

Question 3 is asking once the perimeter of a quadrilateral increase, its effect on the area. According to table 1 and figure 2, it was shown that 71% of respondents believe the increase in perimeter of a quadrilateral will lead to an increase in area in the pre-test. Only 16% of respondents got the correct answer that an increase in the perimeter of a quadrilateral does not necessarily affect the change in the area.

Q4: The perimeter of a quadrilateral decreases, the area ______	Pre-test	Post-test
Increase	1	1
Decrease	23	13
Unchange	2	2
May not change	3	15
No answer	2	0

Table 2: Respondents' distribution of answers in question 4 in pre-test and post-test

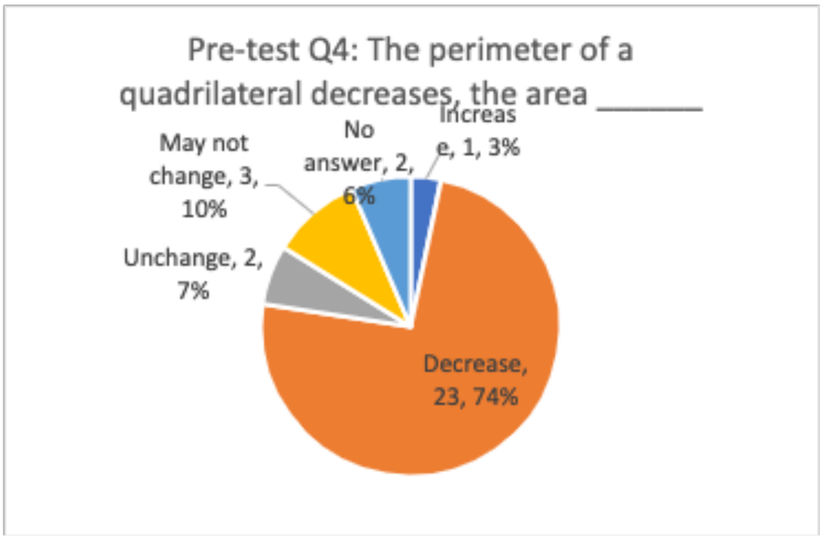


Figure 3: Respondents' distribution of answers to question 4 in post-test

Similar to question 3, question 4 asked about the decrease in the perimeter of a quadrilateral and its effect on the area. Table 2 and figure 3 show that 74% of respondents believe reducing the perimeter of a quadrilateral will lead to a decrease in area in the pre-test. Only 10% of respondents got the correct answer that decreases in the perimeter of a quadrilateral do not necessarily affect the change in the area.

Q5: If two quadrilateral has the same perimeter, their area is ____	Pre-test	Post-test
The same	16	8
Not necessarily the same	12	23
No answer	3	0

Table 3: Respondents' distribution of responses in question 5 in pre-test and post-test

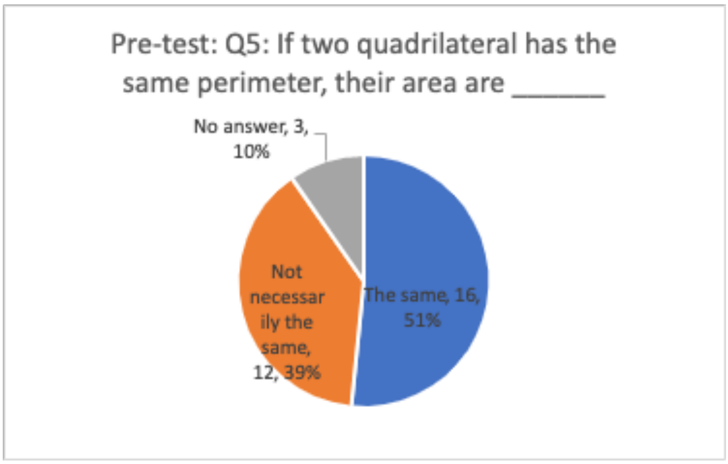


Figure 4: Respondents' distribution of answers to question 5 in post-test

Question 5 asked whether two quadrilaterals with the same perimeter will have the same area. According to table 3 and figure 4, over 51% of respondents believe two quadrilaterals with the same perimeter will have the same area. Only 39% of them think perimeter remains constant; the area of those quadrilateral does not have to be the same.

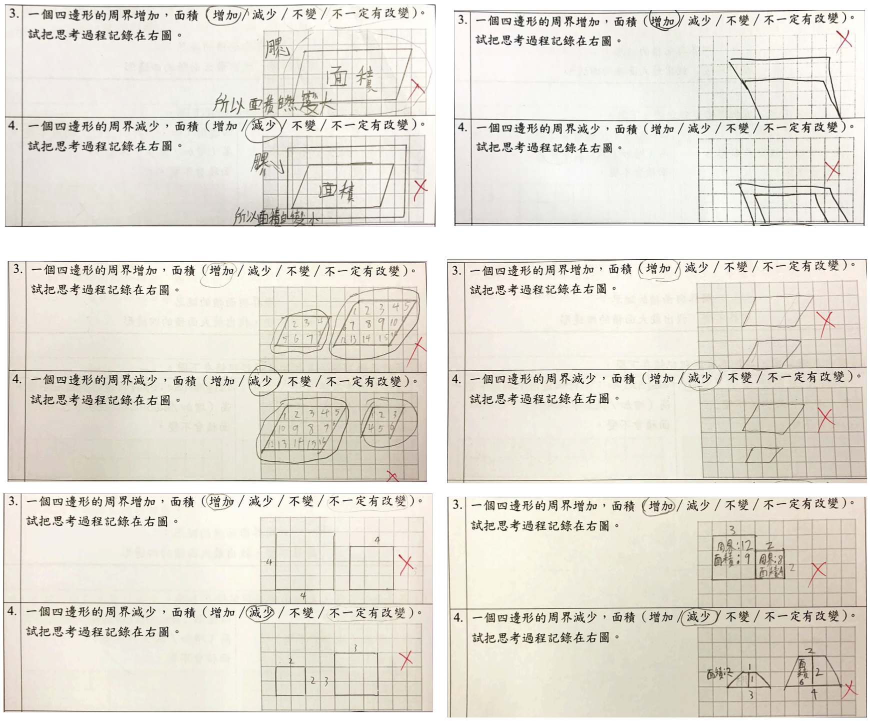


Figure 5: Respondents' thinking progress on questions 3 and 4

Apart from data collected from the pre-test showing respondents' misconceptions about the relationship between perimeter and area, respondents' working progress is shown above. They tended to draw the same type of two quadrilaterals (i.e., square, trapezium, and parallelogram) with different sizes to compare how the area changes by the increase or decrease in the perimeter. Some calculate area by counting the number of squares or applying formulas. They concluded area of a quadrilateral is related to the increase or decrease of the perimeter.

7.2 Result 2 – The performance of understanding the exploration 'maximising the area of a region with a fixed perimeter and 'relation between perimeter and area.'

This part will compare the performance in pre-test and post-test on an understanding of the 'relation between perimeter and area' after the exploration of 'maximising the area of a region with a fixed perimeter'. Comparing the performance of 3 questions related to the 'relation between perimeter and area' can reflect the effectiveness of eliminating the misconception by the exploration 'maximising the area of a region with a fixed perimeter.'

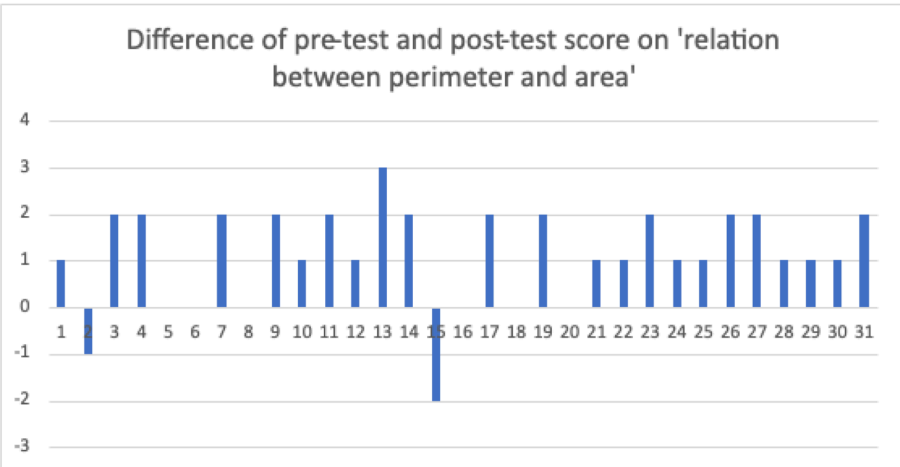


Figure 6: Respondents' score difference in pre-test and post-test on questions related to 'relation between perimeter and area.'

Figure 6 compares the pre-test and post-test scores on three questions related to the 'relation between perimeter and area,' 25 respondents recorded an improvement of

1 to 3 scores. In contrast, 4 and 2 respondents' scores remained unchanged and decreased, respectively. Overall, a steady increase in score for this part can be concluded.

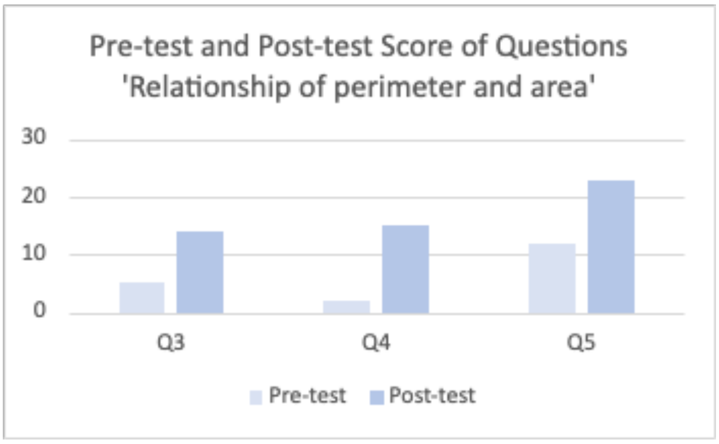


Figure 7: Respondents' scores on pre-test and post-test on questions related to 'the relation of perimeter and area'

From figure 7, the performance of all three questions asking 'the relation of perimeter and area' for all 31 respondents has improved results in the post-test, recording an increase of 9, 13, and 11 scores for questions 3 to 5, respectively. The rapid growth in score in these three questions is the most significant in the post-test.

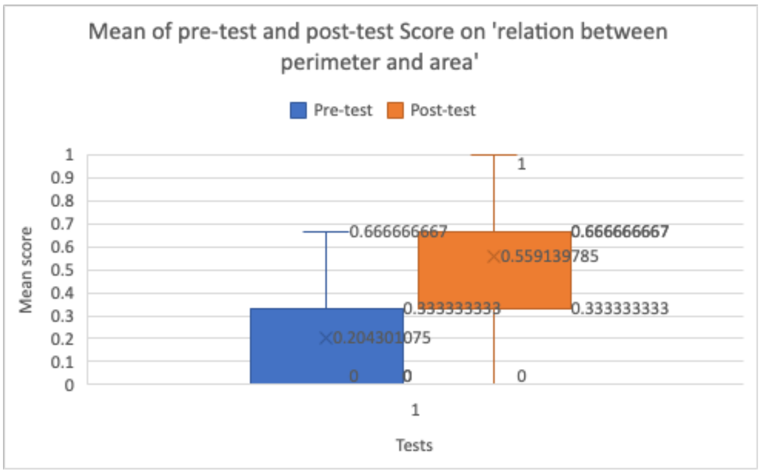


Figure 8: Measure of dispersion of scores on questions related to 'relation between perimeter and area'

Figure 8 shows the measure of the dispersion of 3 questions about 'relation between perimeter and area,' with a maximum score of 1 in the pre-test. The lowest score is 0, the lower quarter is 0, the mean is 0.2043, the upper quarter is 0.3333, and the highest score is 0.6667. Comparatively, in the post-test, the lowest score is 0, the lower quarter is 0.3333, the mean is 0.5591, the upper quarter is 0.6667, and the highest score is 0.1. Compared to the pre-test result, the post-test score nearly tripled by 0.3548, which grew strikingly by 173.4%. It can be concluded that the overall performance of 'relation between perimeter and area' in post-test improved.

Apart from the performance of ‘relation between perimeter and area,’ the performance on the exploration of ‘maximising the area of a region with a fixed perimeter’ will also be examined.

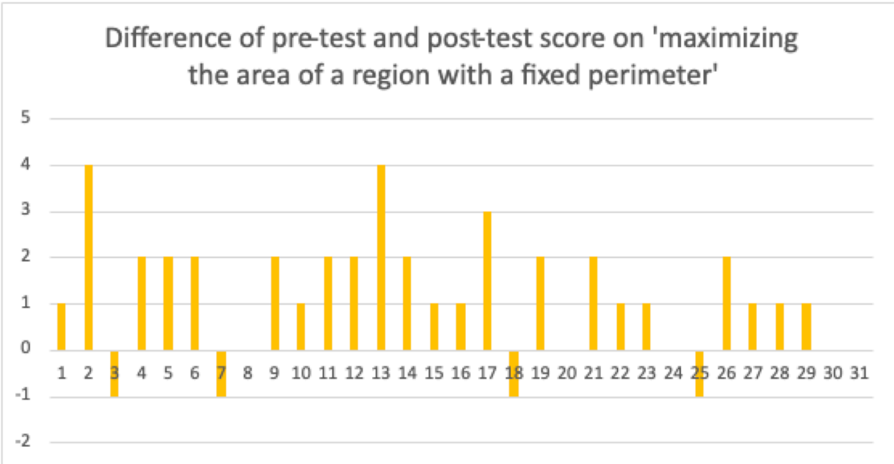


Figure 9: Respondent’s score difference in pre-test and post-test on questions related to the exploration of ‘maximising the area of a region with a fixed perimeter’

According to figure 9, comparing the pre-test and post-test scores on eight questions related to the exploration of ‘maximising the area of a region with a fixed perimeter,’ 25 respondents recorded an improvement of 1 to 4 scores. In contrast, 2 and 4 respondents’ scores remained unchanged and decreased. Overall, it can be concluded with an increase in score for this part.

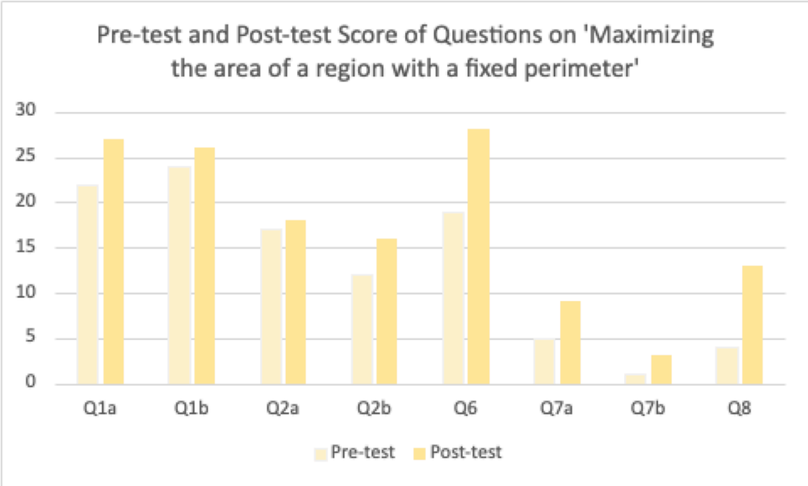


Figure 10: Respondents' scores on pre-test and post-test on questions related to the exploration of 'maximising the area of a region with a fixed perimeter'

Referring to figure 10, the performance of all eight questions asking the exploration of 'maximising the area of a region with a fixed perimeter' among all 31 respondents improved results in the post-test, recording various increases of 1 to 9 scores for the eight questions. The most significant changes are questions 6 and 8. Asking 'If perimeter remains unchanged, a convex quadrilateral's area is (larger/ smaller) than that of a concave quadrilateral.' and 'If perimeter remains unchanged, the quadrilateral with the largest area is ______.'

The overall performance of respondents in pre-test and post-test on an understanding of 'relation between perimeter and area' after the exploration of 'maximising the area of a region with a fixed perimeter' is shown in Figures 10 and 11.

	Q1a	Q1b	Q2a	Q2b	Q6	Q7a	Q7b	Q8	Q3	Q4	Q5	Overall
Pre-test Score	22	24	17	12	19	5	1	4	5	2	12	123
Post-test Score	27	26	18	16	28	9	3	13	14	15	23	192
Difference	5	2	1	4	9	4	2	9	9	13	11	69
Percentage change	22.7%	8.3%	5.9%	33.3%	47.4%	80%	200%	225%	180%	650%	91.7%	60%

Table 4: Data analysis of score of 31 respondents in pre-test and post-test

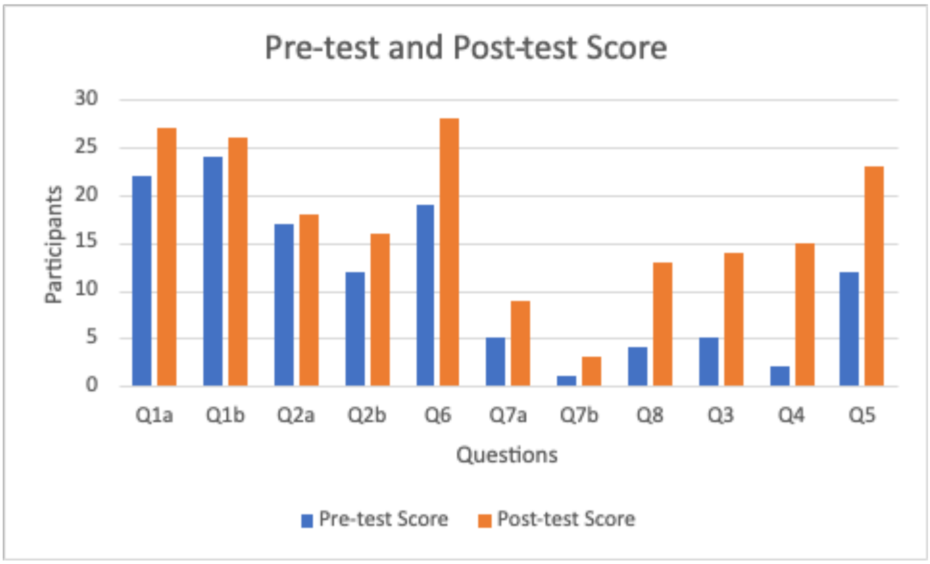


Figure 11: Respondents' scores on all questions in pre-test and post-test

According to table 4 and figure 11, it is shown that all 11 questions resulted in an increase in scores in the post-test, from 8.33 % to 650%. When comparing pre-test and post-test percentage changes, the most significant growth is in question 4, with a dramatic increase of 650%. This question tested the respondent's understanding of the misconception of perimeter and area, asking about the effect on the area of a quadrilateral if its perimeter decreases.

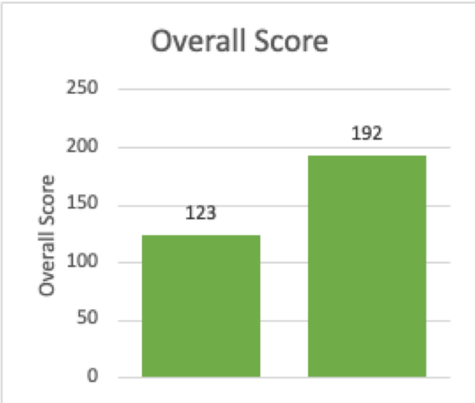


Figure 12: Overall score among 31 respondents in pre-test and post-test

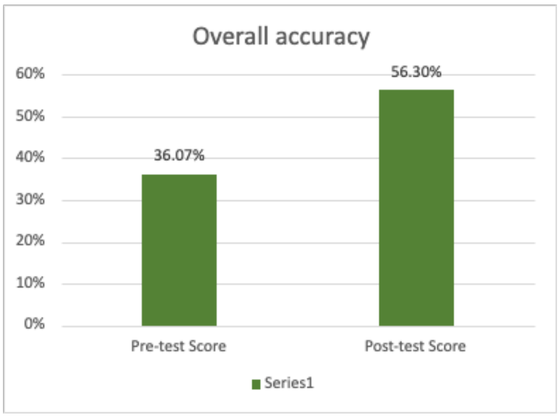


Figure 13: Overall percentage of accuracy among 31 respondents in pre-test and post-test

According to figures 12 and 13, the overall score and percentage of accuracy in pre-test and post-test. 31 respondents resulted with 123 out of 341 scores in total and a 36% accuracy in the pre-test, while 192 out of 341 scores in total and a 56.3% accuracy in the post-test. Compared to the pre-test, the overall score increased by 69, which reached up 56.1% in the post-test.

8. **Discussion**

In this section, respondents' results will be discussed, with the implications of findings and reflected on them, including the reason for having misconceptions about the relationship between perimeter and area; and how the exploration affects them to understand the relationship between perimeter and area.

8.1 The reason for the misconception of the 'relation between perimeter and area.'

According to table 1 to 2 and figure 2 to 3, it is shown that most of the respondents believe the change of area is associated with the change of perimeter, such that the increases or decreases of the area are associated with the increases or decreases of the perimeter. In addition, table 3 and figure 4 display that respondents believe two quadrilaterals with the same perimeter will have the same area. Olivier (1989, p. 12) stated that these misconceptions are inevitable symptoms of underlying conceptual structures that are causes of error.

Inadequate prior knowledge of perimeter (the distance around a region) and area (amount of surface) is the first root cause of these misconceptions. Respondents tend to generalise previous knowledge of perimeter and area as related concepts. Learners usually construct new knowledge or skills based on prior knowledge. According to Curriculum Development Council (2017), Hong Kong students have been taught the concept of perimeter and its formula to calculate surrounded length of polygons. Also, the concept of area and its formula apply to calculate the area of squares and rectangles in primary 4. Afterwards, they will build new knowledge of the area of other polygons' formulas (i.e., parallelogram, triangle, and trapezium) in primary 5. Learners who have learned the concept of the perimeter in their early stages of learning overgeneralise these concepts, extending them to the idea of the area. They may not be able to add additional knowledge to the existing knowledge by organising, structuring and restructuring. Olivier (1989) argued that inadequate preparation in the early stages always results in confusion between perimeter and area in the cognitive structure of learners. It can explain why some respondents responded to the word 'area of the rectangle' by saying '(length plus width) times 2' instead of 'length times width' before the exploration.

In contrast, another finding is that they perform better when the question relates to their current learning. From figure 11, it is shown that respondents scored highest in questions 1a and 1b in the pre-test, whereas they learned the topic of the area of the triangle a week before the research. It illustrates how inadequate prior knowledge leads to the occurrence of misconception.

Piaget's theory of cognitive development is another explanation of the cause of these misconceptions. According to Piaget (1976), respondents are at the concrete operational stage from 7 to 11, struggle mentally representing objects but make strides in logical thinking. The misconceptions mentioned above could be attributed to their use of compensation to attain conservation and make decisions based on intuition. They applied the intuitive rule of ' Increase perimeter – increase area; decrease perimeter – decrease area; and same perimeter – the same area' because it is a small universal ruleset, or maybe their previous experiences with overgeneralisation are successful (Tirosh and Starvy, 1999). Understandably, learners at this age generalise their experiences into a universal maxim: 'Same A – Same B.'

Misconceptions are an essential part of learning and are unavoidable. However, they can be dealt with appropriately. Misconceptions will quickly arise in new knowledge construction, reconstruction, reorganisation, and prior knowledge. Misconception can hardly be corrected by simply saying it is inaccurate, but indeed, experiences will facilitate learners to reorganise their thinking.

8.2 How exploration affects understanding the 'relation between perimeter and area.'

Concerning Figures 6 to 8, the individual and overall performance on the three questions about 'relation between perimeter and area' show significant improvements after exploration. By explanation, respondents' understanding of the relationship has been enhanced; the misconception is eliminated simultaneously. In addition, refer to Figures 9 to 10. The individual and overall performance on the eight questions about the exploration of 'maximizing the area of the region with fixed perimeter' has also shown progression, specifically on the conclusion of exploration. To conclude, the overall scores of all questions in the post-test have increased, which could be seen in figures 12 to 13.

The exploration progress facilitates the understanding of the relationship between perimeter and area. In the exploration, respondents have to fix the perimeter and base of the triangle or quadrilateral, then adjust another point or the length of the other two sides to adjust the triangle's height or quadrilateral's height to maximise the area. By repeating this progress, respondents can draw to a conclusion that regular polygons (i.e., equilateral triangles and squares) have the most significant size if the perimeter is fixed. During the exploration, respondents should understand that perimeter is the constant factor, while the area is a variable that changes according to the change in height. Respondents should be capable of summarising that the change in the region has no relationship with the perimeter. In other words, perimeter and area have no direct correlation.

Piaget (1976) states that learners can apply logical operations at the concrete operational stage, but only to physical objects. To explain the minor improvement on post-test, such as questions 3, 4, 5, 7, and 8 with a relatively low accuracy rate among all questions. It can be attributed to intuition in solving geometric problems, especially those without graphs as visual reminders, like questions 3, 4, 5, and 8, or misleading graphics in question 7. Respondents may find it challenging to understand abstract concepts and ideas if no pictures are given. Therefore, some may apply logical operations or insert visual or physical objects to help self-understanding. Using figure 5 as an example of how respondents use physical objects as a cognitive thinking process, some may draw two quadrilaterals of the same type by different sizes to compare how the area changes by the increase or decrease in the perimeter. Some may use intuition, some may apply counting squares to calculate area, and some may use logical operations like applying formulas. Another example reflects that respondents have better performance in post-test questions 6 and 8 as they have experienced using physical objects in exploration.

During exploration, only two parts required geometry sticks, including the process from a concave quadrilateral to a convex quadrilateral and the last part from rhombus to square. It can be applied to questions 6 and 8 in the post-test. Hence, with the assistance of physical teaching tools and graphics printed on the pre-test and post-test, these two questions have better results. It illustrates the importance of physical objects or resources like bricks and cuttings, which can fit, fold, match and count to work concretely to develop a conceptual understanding of perimeter and area (Machaba, 2016). Therefore, using GeoGebra and geometry sticks as multi-sensory stimulation in exploration, respondents have eliminated some misconceptions about the relationship between perimeter and area.

Besides, other findings of respondents exploring the classroom and online have a slight difference. Since four respondents explore in a real-time face-to-face setting, others do it online. The former receive immediate responses from the researcher and may communicate with other participants during the exploration, which they comes to conclude faster than the latter. Also, the majority of respondents preferred to use geometric sticks instead of GeoGebra in the part of a concave quadrilateral to convex quadrilateral and the last part from rhombus to square since the image is visualised and tangible so as to consolidate memory and foster understanding. It is observed that physical teaching tools and face-to-face teaching may be more effective for hands-on exploration.

9. <u>Limitations and suggestions</u>

First of all, the sample population has a similar educational background. They are primary five students from my block practice school taught by two different teachers, whereas similar learned prior knowledge may lead to similar answers to questions. It may not represent all of the primary students in Hong Kong. It is suggested that participants could be recruited from different educational backgrounds, and the sample size could also be enlarged if similar research is to be done in the future.

The research was carried out during the covid-19 pandemic, in which data collection was more challenging. For instance, face-to-face observation of respondents, thinking and working progress, peer and teacher-students discussion in a face-to-face class are restricted. It is hard to get all respondents' instant responses and reactions only through a few digital devices and one researcher, not even mentioning answering all respondents' questions. Their performance can only be reflected in pre-test and post-test written format. The researcher can hardly give immediate feedback or adjust the teaching pace throughout the research. So for further study, face-to-face and one-on-one exploration could be considered for more detailed and accurate data collection.

Moreover, much more time is spent on solving technical issues, such as the how-to-use GeoGebra, ZOOM (i.e., video conferencing platform), and tackling poor internet connection. Respondents are not familiar with iPad use, like switching from ZOOM to GeoGebra, or how to adjust one point of the triangle by not moving the other two points in GeoGebra. The researcher takes up ¼ time to deal with these problems, which lowers teaching efficiency. In that case, better preparation for participants and researcher is suggested, such as briefing participants on the use of technological tools before exploration; meanwhile, the researcher may practice using them as better preparation. Furthermore, respondents may find it demanding to handle many teaching tools simultaneously, including seeing the researcher's demonstration in ZOOM, controlling GeoGebra and geometric sticks, and filling in in-class worksheets. It is proposed to minimise the number of teaching tools or switch teaching tools to tangible ones, for example, strings or geometric sticks as fixed perimeter, magnets as fixed or adjustable points of polygons. Questions on in-class worksheets might be reduced or replaced by verbal questioning as they are a minor part of the exploration.

Questioning skills on the pre-test, post-test, and exploration could be improved. For the pre-test and post-test, students find it demanding to understand questions without graphics, or they find too much information in one question. It is suggested to insert more pictures, break up questions into sub-parts, and add hints for vocabulary that they have not learned. The researcher could scaffold participants with guiding questions to facilitate thinking as well.

As mentioned in the discussion, respondents have inadequate prior knowledge of the concept of perimeter and area. Thus, it causes misconceptions about their relation. It is proposed that teachers may put more emphasis on the concept of perimeter and area instead of applying their formulas when teaching related topics. The relationship between perimeter, area, and even volume could also be added to the current curriculum as an enrichment chapter.

10. Conclusion

This research aims at exploring the effectiveness of using ‘maximising the area of a region with a fixed perimeter’ to eliminate the misconceptions of the relation between perimeter and area. To summarise data mainly collected from in-class worksheets, pre-test, and post-test, it is observed that the misconceptions of ‘Same A – Same B’ could be attributed to their inadequate prior knowledge and application of the intuitive rule to all circumstances. The exploration progress facilitates a better understanding of the exploration and elimination of misconceptions. With the assistance of visualisation through GeoGebra and physical objects, students’ improved in solving problems related to the relation of perimeter and area. It is also realised that face-to-face teaching may be more effective for hands-on exploration. Despite the limitations of pandemics, further study could be done if face-to-face exploration is allowed. It is concluded that the exploration of using ‘maximising the area of a region with a fixed perimeter’ is effective to help students eliminate misconceptions on the relation between perimeter and area.

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12. Appendix

12.1 Appendix 1 – Pre-test and Post-test

班別:五年級______班　　　　學號：_________

周界與面積的謎思：

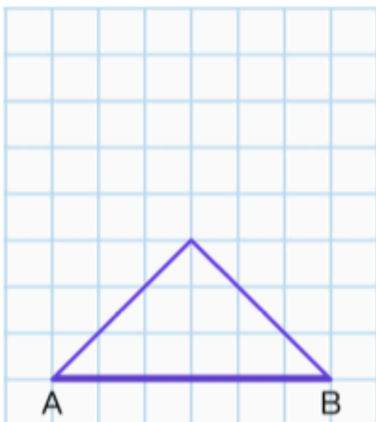
周界不變，找出最大面積的四邊形

1. 圈出正確答案，並把思考過程記錄在下圖。

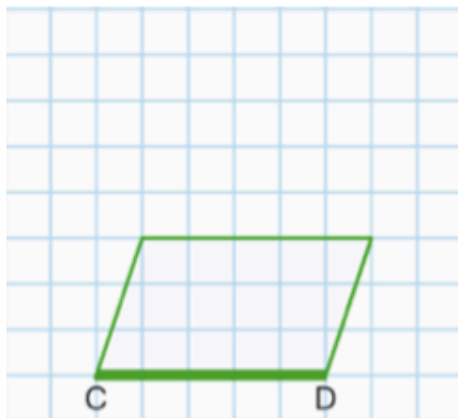
三角形的底不變，
高（增加 / 減少 / 不變），
面積會不變。

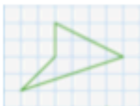
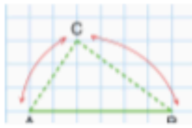
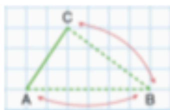
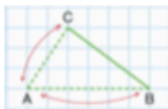
四邊形（如：平行四邊形）的底不變，
高（增加 / 減少 / 不變），
面積會不變。

2. 以線 AB 為底，畫出
一個比下圖大的三角形。



以線 CD 為底，畫出
一個比下圖大的平行四邊形。



3.	一個四邊形的周界增加，面積（增加／減少／不變／不一定有改變）。 試把思考過程記錄在右圖。
4.	一個四邊形的周界減少，面積（增加／減少／不變／不一定有改變）。 試把思考過程記錄在右圖。
5.	如果兩個四邊形的周界相同，他們的面積（相同／不一定相同）。
6.	如果周界不變，凸四邊形的面積比凹四邊形的面積（大／小）。  
7.	如果三角形的周界不變， (1) 先固定其中一條底 AB， 把面積擴至最大時， 會得出一個__________三角形。  (2) 重覆以上步驟， 分別固定另外兩條底，AC 和 BC， 把面積擴至最大時， 會得出一個__________三角形。  
8.	如果周界不變，最大面積的四邊形是__________形。

12.2 Appendix 2 – Lesson Plan

探究教案 （12/11/2021）

日 期： 12/11/2021　　教 師： 陳諾葶
時 間： 45 分鐘　　課 題： 等周問題
年 級： 5B（＋5C）　　教 節： 1/1
學生人數： 22 + 10跨境生 (Zoom)

教學目的：

1. 三角形和四邊形在固定底的前提下，高愈大，面積愈大。
2. 在固定周界的前提下，找出最大面積的三角形（等邊三角形）。
3. 透過觀察圖形面積，得出凸四邊形的面積比凹四邊形的大。
4. 在固定周界的前提下，找出最大面積的四邊形（正方形）。
5. 周界與面積沒有固定關係。

已有知識：

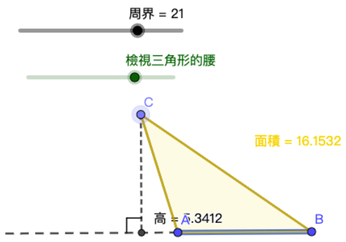
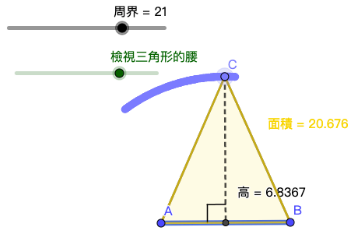
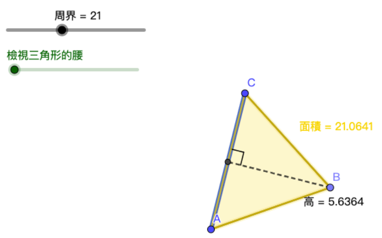
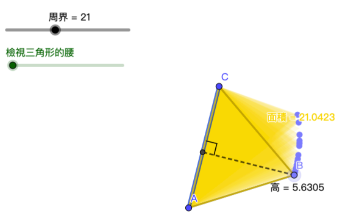
1. 學生能分辨各個三角形及四邊形的種類。
2. 學生能運用三角形及四邊形面積公式。
3. 學生能以分割法求多邊形面積。
4. 學生能解涉及周界和面積的應用題。

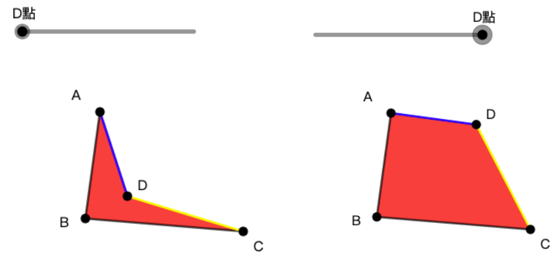
資源、器材

1. iPad
2. GeoGebra
3. 幾何條
4. 面積公式字卡

黑板運用：

正方形面積：	邊長 × 邊長
長方形面積：	長 × 闊
平行四邊形面積：	底 × 高
三角形面積：	底 × 高 ÷ 2
梯形面積：	（上底＋下底）× 高 ÷ 2

		教具	時間	目的
重溫		教具	時間	目的
1	重溫及張貼大紙條於展示板： （1）三角形的種類； （2）四邊形的種類，並介紹凹四邊形及凸四邊形； （3）三角形及四邊形面積公式。 教師介紹今日課堂探究活動： 在固定周界的前提下，找出最大面積的四邊形。	面積公式字卡 堂課	5	/
發展		教具	時間	目的
2	教師先從三角形引入： <u>任務一：</u> 固定周界，使得三角形面積增加。教師引導學生說出**如果三角形有固定的底，高愈大，面積愈大**。（可根據三角形面積公式：被乘數不變，乘數愈大，積愈大）。先固定的其中一條底（AB），同時改變兩邊的邊長（AC & BC），調整C點，會得出一個**等腰三角形。**  依照相同的方法，分別固定另外兩條底，把三角形面積擴至最大，會得出：**固定周界，面積最大的三角形是一個等邊三角形**。 	GeoGebra 堂課	8	1、2
3	<u>任務二：</u> 固定周界，找出面積不比凹四邊形小的四邊形。 教師引導學生固定各邊的邊長，改變內角來重組圖形。學生會發現把四邊形的反角改為銳角，即從凹四邊形改為凸四邊形。學生會得出：	幾何條 GeoGebra	5	3

<table>
<tr>
<td></td>
<td>固定周界，面積不比凹四邊形小的是凸四邊形。

</td>
<td>堂課</td>
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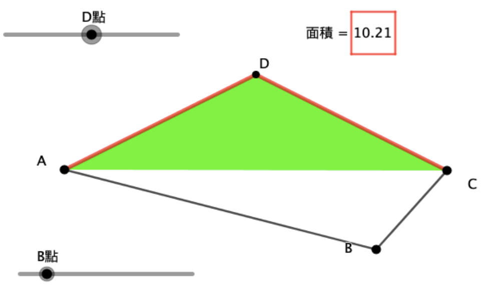
<td>4</td>
<td><u>任務三：</u>
固定周界，分割四邊形，增加它的面積。利用分割法把圖形分割成可用面積公式計算的圖形，即是兩個三角形。
先檢視其中一個三角形（ACD），固定對角線AC作的底，調整D點，增加三角形面積，會得出一個等腰三角形。
依照相同的方法，固定對角線AC作的底，調整B點，增加另一個三角形（ABD）面積，會得出一個等腰三角形。兩個不相同的等腰三角形拼砌成一個鷂形。

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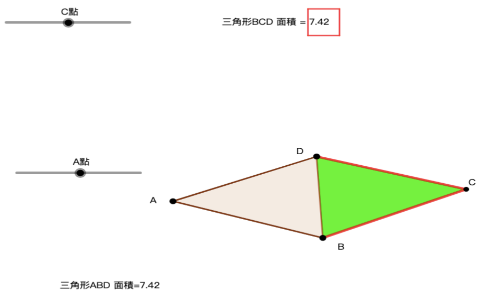
<tr>
<td>5</td>
<td>任務四：
連上另一條對角線，重施故技，增加四邊形的面積。以另一對角線為底，同時改變兩邊的邊長，使得三角形面積增加。會得出兩個完全相同的等腰三角形，拼砌成一個菱形。

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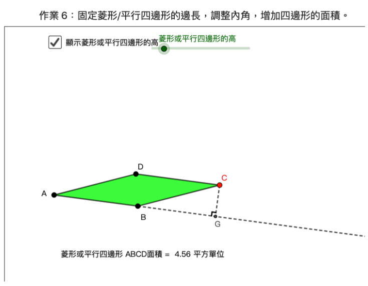
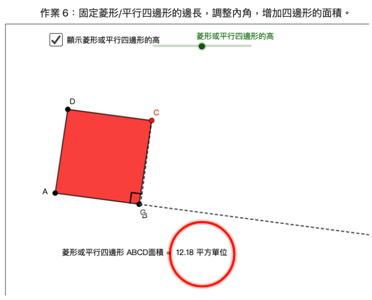
<table>
<tr><td>6</td><td>任務五：
固定菱形的邊長，調整內角，增加四邊形的面積。
將菱形看成平行四邊形，固定一邊長為底，調整內角。利用平行四邊形面積公式思考。當高愈大，菱形的面積最大，即一個正方形。



</td><td>幾何條GeoGebra堂課</td><td>8</td><td></td></tr>
<tr><td colspan="2">總結</td><td></td><td>時間</td><td>目的</td></tr>
<tr><td>7</td><td>教師總結：
• 固定周界，最大面積的四邊形之推論過程：
凹四邊形 → 凸四邊形 → 鷂形 → 菱形 → 正方形
• 周界不變，面積也會改變。即周界與面積沒有固定關係。</td><td></td><td>5</td><td></td></tr>
<tr><td colspan="2">課後延續</td><td></td><td>時間</td><td>目的</td></tr>
<tr><td>8</td><td>教師佈置後測</td><td></td><td>2</td><td>/</td></tr>
</table>

12.3 Appendix 3 – In-class worksheet

教學工作紙（一）

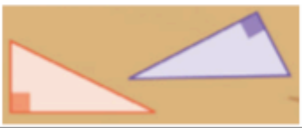
班別:五年級____班　　　　學號：________

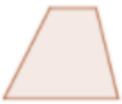
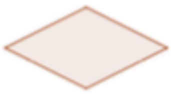
到 www.geogebra.org/classroom 輸入以下代碼　或

EJAZ MH7F

周界與面積的謎思：周界不變，找出最大面積的四邊形

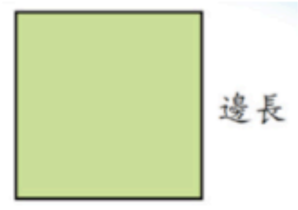
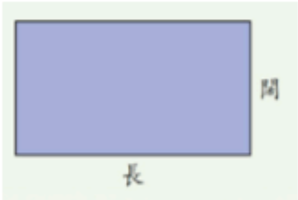
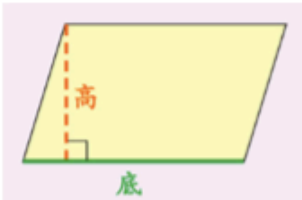
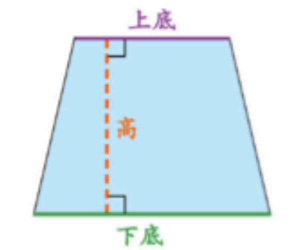
重溫（三角形及四邊形名稱）:

等邊三角形	等腰三角形	不等邊三角形
		
直角三角形	等腰直角三角形	
		

正方形	長方形	平行四邊形
		
梯形	菱形	鷂形
		

凹四邊形	凸四邊形
	

重溫（面積公式）:

	正方形面積公式： 邊長×邊長
	長方形面積公式： 長×闊
	平行四邊形面積公式： 底×高
	三角形面積公式： 底×高÷2
	梯形面積公式： （上底＋下底）×高÷2

任務一：

增加三角形的面積。

思考過程：

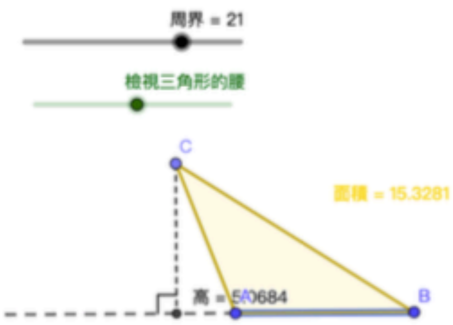
（1）三角形的高愈 ____大_____ ，面積愈大。

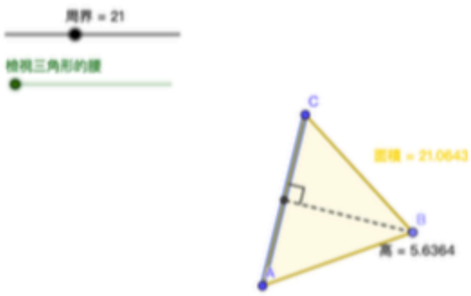
（2）先固定的其中一條底（AB），同時改變兩邊的邊長（AC & BC），

調整 C 點，會得出一個 ____等腰_____ 三角形。

（3）依照相同方法，固定 AC，調整 B 點；及固定 BC，調整 A 點，

會得出一個 _____等邊_____ 三角形。





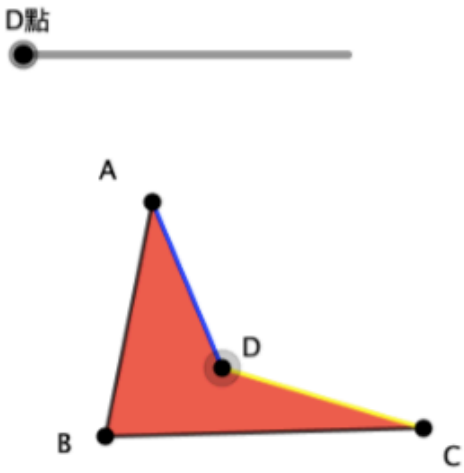
<u>任務二：</u>

找出面積不比凹四邊形小的四邊形。

思考過程：

固定各邊的邊長，改變內角來重組圖形。

面積不比凹四邊形小的是 __________ 四邊形。



任務三：

分割四邊形，增加它的面積。

思考過程：

(1) 利用分割法把圖形分割成可用面積公式計算的圖形。

即是兩個 ____三角______ 形。

(2) 先檢視其中一個三角形 (ACD)，固定對角線 AC 作的底，

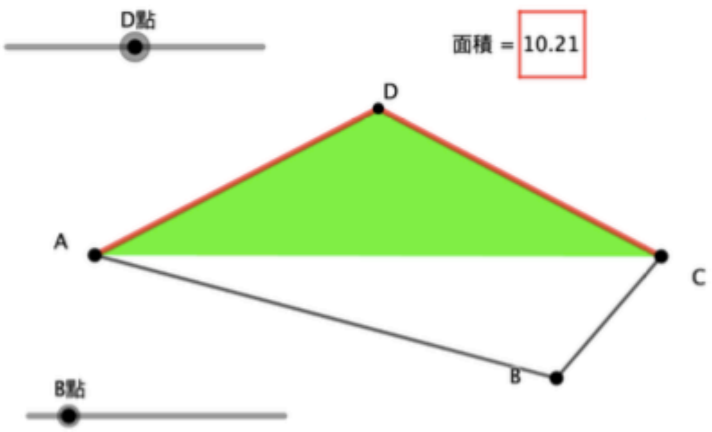
調整 D 點，增加三角形面積，會得出一個 _____等腰______ 三角形。

(3) 依照相同的方法，固定對角線 AC 作的底，

調整 B 點，增加另一個三角形 (ABD) 面積，

會得出一個 _____等腰______ 三角形。

(4) 兩個不相同的 _____等腰______ 三角形拼砌成一個 ___鷂形__ 形。



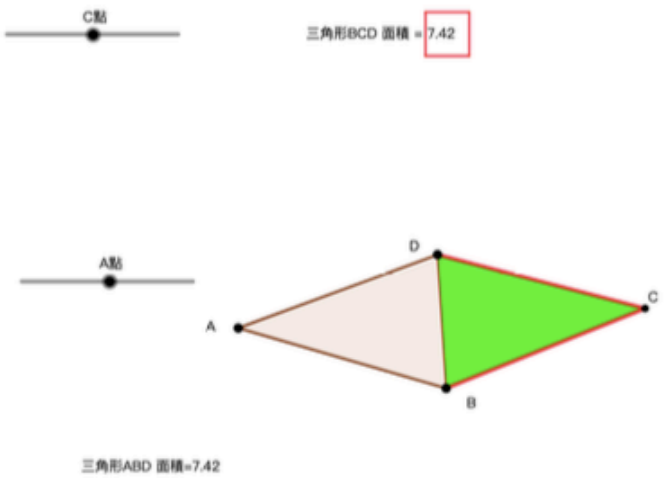
任務四：

連上另一條對角線，重施故技，增加四邊形的面積。

思考過程：

以另一對角線為底，同時改變兩邊的邊長，使得三角形面積增加。

會得出兩個完全相同的 _____等腰______ 三角形，拼砌成一個 __菱__ 形。



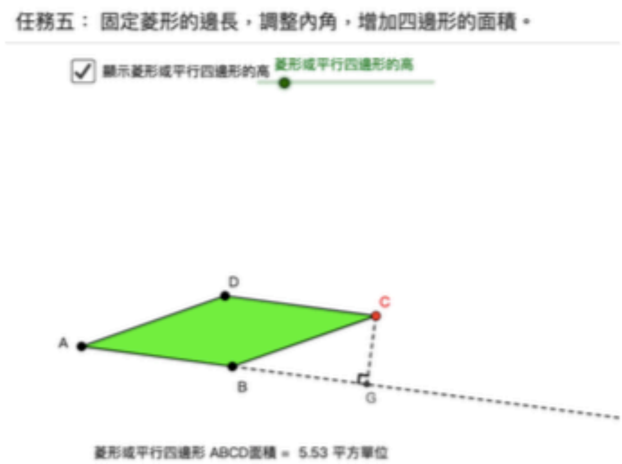
任務五：

固定菱形的邊長，調整內角，增加四邊形的面積。

思考過程：

（1）將菱形看成平行四邊形，固定一邊長為底，調整內角。

（2）利用平行四邊形面積公式思考。當 ______高_____ 愈大，
菱形的面積最大， 即一個 ______正方______ 形。



小總結

在固定周界的前提下，最大面積的四邊形之推論過程：

凹四邊形 → 凸四邊形 → 鷂形 → 菱形 → 正方形