香港教育大學
The Education University of Hong Kong

MTH 4902 Honours Project II (2021-22)

A Project entitled

The effectiveness of using GeoGebra to teach the topic of "speed and travel graph" in Hong Kong

Submitted by

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Declaration

I, Wong Zie Bin declare that this research report represents my own work under the supervision of Dr. Cheng Kell Hiu Fai, and that it has not been submitted previously for examination to any tertiary institution.

Signed

Wong Zie Bin 08th April, 2022

Content

1. Introduction

The topic of speed and travel graph are complicated topics that are related to the relationship between time and distance. According to Mathematics Education Section Curriculum Development Institute (2020), the topic of speed and travel graphs is taught at primary 6 in Hong Kong. It has been discovered that speed is an abstract concept that students often find difficult to experience in real life and understand its meaning. The possible reason is that the relationship between distance and time is not concretely demonstrated during the learning process. On the other hand, under the trend of e-learning recently, technology tools are useful for visualizing and presenting mathematics concept to students. GeoGebra, as a free dynamic mathematics software, is utilized for many mathematics lessons and positively affects students' mathematics learning (柯、程, 2014). Therefore, It is believed that GeoGebra can demonstrate the relationship between distance and time concretely. However, it is hard to find academic papers to discuss the usage of GeoGebra in facilitating the learning of speed and travel graphs. Therefore, this project is going to investigate the effectiveness of using GeoGebra to teach the topic of speed and travel graphs in Hong Kong.

2. Literature review

2.1. The Child's development of the concept of speed

The concept of speed talks about how many distances do the object moves in the unit of time (邱, 2005). Piaget (1970), which makes an in-depth investigation of the child's development of the concept of position, displacement, speed and acceleration, points out there are possibly four stages of child's development of the concept of speed (see Table 1 below). 王、鍾（2004）and Siegler & Richards (1979) share a similar result with Piaget in their research. Under Piaget's theory, it is believed that the primary six students (11-12 years old) should go into stage IVb after learning the topic of speed, which means they can

1) *Calculate the speed*
2) *Understand the relationship among time, distance and speed.*

Therefore, this research will use stage IVb as a standard to assess students' understanding of the concept of speed after the designed teaching sequence.

On the other hand, 王、鍾（2004）also points out that a child's learning and life experience possibly affect the development of the concept of speed. The above point of view may be in line with the view of 馮（1999）, who states that students' development of the concept of speed is easily confused by life experiences. The possible reason is that students will have more experience undergoing the instantaneous speed than the average speed in life. Therefore, he suggests that teachers may clarify the relationship between instantaneous speed and average speed. Therefore, clarifying the relationship between instantaneous speed and averaged speed will incorporate into the teaching content of this research.

Stages	Description
I (5-6 years old)	Judge the speed of the moving body by seeing transcendental phenomenology intuitively.
II (6-7 years old)	Know if the time of moving bodies is the same, and the distance of one of the moving bodies travelled is longer, then the speed of that body is faster.
IIIa (7-9 years old)	Understand to use the distance moved to judge the speed if moving bodies move simultaneously.
IIIb (9-11 years old)	Correctly judge the speed of moving bodies under “same time and different distance” and “same distance and different time”.
IVa (9-11 years old)	During the situation of “different distance and different speed’, able to adjust one variable by ratio and compare the speed of moving bodies by comparing another variable.
IVb (10-12 years old)	Able to calculate the speed and understand the relationship among time, distance and speed.

Table 1: The Child’s development of the concept of speed

2.2. Travel graph

Travel graph describes the relationship between the distance and time of moving bodies, which is also called distance-time graph. Like the number sense or spatial sense, scholars suggested that developing “graph sense” may be necessary for pupils in order to cultivate their understanding of the travel graph (Duijzer et al, 2020). Generally, “graph sense” is a high order thinking skill of mathematics that develops by interpreting the graph and constructing the graph (Friel et al, 2001).

Constructing the travel graph is beneficial to the graph sense of students. Some scholars utilize some motion sensors to allow pupils to produce the travel graph by their body movement and gain a positive effect on developing the graph sense of students. (Gravemeijer, 2009; Deniz and Dulger, 2012; Duijzer et al ,2019; Duijzer et al, 2020; Nemirovsky et al., 1998). They indicated that students' ability of graph interpretation improves and are possibly able to point out the characteristic of the travel graph (e.g., the steepness of the slope can compare the speed of objects) in their research. The possible reason to explain this is that pupils possibly develop the reasoning of the travel graph while constructing the travel graph by themselves (diSessa et al., 1991). Moreover, Duijzer et al (2019) indicated that the learning experience of constructing travel graphs by students themselves may make the concept of speed more apparent and easier to understand. Therefore, constructing a travel graph by students themselves may enhance their understanding of the travel graph and the concept of speed. Under this line of thinking, travel graph construction should be included in teaching the topic of speed and travel graph.

However, under the Hong Kong primary mathematics curriculum, students are only required to interpret travel graphs (Mathematics Education Section Curriculum Development Institute, 2020). This means students may lack chances to construct a travel graph, thus influencing their development of graph sense. Therefore, graph interpretation and construction will be included in the teaching design of this research to foster the graph sense of students.

2.3. GeoGebra and its application in primary mathematics learning

GeoGebra is a free dynamic mathematics software that allow educators to design mathematics learning materials that suit the needs of students (Lilla, 2017). Poon & Wong (2017), Poon (2018) and 柯（2015）use GeoGebra to visualize and simulate the concepts of similar triangles, fraction and 3D shapes. They reveal that using GeoGebra for visualizing the mathematics concepts may enhance their understanding of the concepts. Moreover, using GeoGebra for learning mathematics may enhance the motivation and engagement of students in the mathematics lesson (柯、程, 2014; Poon, 2018）. Students possibly show high engagement in the class discussion and presentation. However, it is hard to find literatures to talk about using GeoGebra to teach speed and travel graph in Hong Kong. Therefore, this research is going to investigate whether students use GeoGebra to visualize and construct the travel graph and learn the concept of speed and travel graph is effective or not.

2.4. Learning difficulties in the topic of speed and travel graph

Several scholars point out different learning difficulties of pupils when they learn the topic of speed and travel graph (王、鍾, 2004；黃, 2003；吳、陳、李、陳, 2014). The following is to summarize the learning difficulties in the topic of speed and travel graph:

1) *Found it difficult to apply speed in normal life;*
   Seems to divorce from reality.
2) *Do not understand the meaning of speed;*
   Just remember the speed formula
3) *Show weak performance in solving the problem about calculating the speed of two moving bodies*

The first difficulty is possibly arisen from lacking enough experience in life. Moreover, speed may not be easy to measure and observe by the body senses (吳、陳、李、陳, 2014). The GeoGebra applets cannot provide a learning experience that is related to body senses. Thus, the first difficulty cannot be alleviated by using GeoGebra to teach. On the other hand, the second and third difficulties are possibly alleviated by using GeoGebra to teach. It is because GeoGebra applets can possibly visualize and simulate the relationship among distance, time and speed through the construction and animation of the travel graph. Therefore, this research will emphasize alleviating the students' limited understanding of the meaning of speed and weak performance in solving the problem of calculating the speed of two moving bodies.

3. Research question /Purpose

Questions:

1) Does the GeoGebra teaching design alleviate students' limited understanding of the meaning of speed?

2) Does the GeoGebra teaching design alleviate the weak performance in solving the problem with two moving bodies?

3) After using GeoGebra to learn travel graphs, what are students' levels of reasoning about travel graph interpretation and construction?

Purpose:

There are three purposes in this research. Firstly, we may study the possibility whether the two learning difficulties will be alleviated or not by using GeoGebra to teach. We would also like to try to involve travel graph construction into teaching the topic of speed and travel graph. Moreover, we may show the effectiveness of using GeoGebra to learn speed and travel graphs.

4. Methodology

4.1. Participants

32 primary 6 students will be invited to be the participants of this research during 7th March to 24th March of 2022. Participants come from different primary school and have not learnt the topic speed and travel graph before. They will be separated into five groups and follow the teaching sequence designed by this research. This research was conducted according to the ethical review of The Education University of Hong Kong.

4.2. Procedure

There are two online lessons for a teaching sequence in two consecutive days on Zoom. The whole teaching sequence will be taught by the author of this research and each lesson will last for 90 minutes. The details teaching design, content and trajectory will be discussed in the session 5 of this paper (page 12). After each lesson, a test will be conducted to all participants (also called Test 1 and Test 2). The test 1 will include the question about travel graph interpretation and construction (see appendix 2). The test 2 will include the question about speed, travel graph interpretation and construction (see appendix 3). Moreover, after the whole teaching sequence, an interview will be conducted which contains travel graph interpretation and construction tasks. 12 interviewees will be randomly selected in the participants.

The flow chart of the research procedure

4.3. Research instrument

This research implements qualitative method to investigate students' levels of reasoning about travel graph interpretation and construction after undergoing the teaching sequence in this research. With reference to the coding scheme for students' level of reasoning about graphs which is designed by Duijzer et al (2020), an assessment rubric is designed (see Table 2) for assessing students' levels of reasoning about travel graph interpretation and construction. This assessment rubric will be used in the interview to answer the research question 3.

Table 2

Assessment Rubrics for students' levels of reasoning about travel graph interpretation and construction

Level of reasoning	Grade	Description of students' reasoning	
		Travel graph interpretation	Travel graph construction
		Students interpret travel graphs	Students constructs travel graphs
Unrelated reasoning	G1	without making any reference to the representation of the travel graph	without considering the description of the movement
Perfunctory reasoning	G2	based on the appearance of the travel graph or the surface characteristics of the movement	based on the surface characteristics of the description of the movement
Single variable reasoning	G3	based on a single variable (distance or time or speed) of the travel graph	by considering a single variable (distance or time or speed)
Multiple variable reasoning	G4	based on multiple variables (distance and/or time and/or speed) of the travel graph	by considering multiple variables (distance and/or time and/or speed)

Table 2: The assessment rubric for students' levels of reasoning about travel graph interpretation and construction

4.4. Data collection

Both qualitative and quantitative data will be collected in this research. Two tests, interviews, lesson materials and videotaped lessons will be the methods for collecting data. The following will describe how do the data is used and analyzed:

4.4.1. Videotaped lessons

To observe the learning process of the participants and collect data to answer three research questions, the lessons situation will be recorded by the Zoom recording function. The recording videos are the justification of participants' learning performance and students' responses. Moreover, it allows the author can review the videos to analyze the learning process of the participants and the reason why students have misconception or make mistakes. The recording has first gotten the consent of students and parents before the research starts.

4.4.2. Interview

The aim of the interview is to assess students' levels of reasoning about travel graph interpretation and construction. As mentioned, the interview will be conducted after the teaching sequence which contains travel graph interpretation and construction tasks (see Fig 1a & 1b). To minimize the mistakes leaded by technical issue, students will use the same function of GeoGebra which they have already learnt in the lessons. Students will be asked to explain their answer to assess their level of reasoning more accurate. The assessment rubric mentioned in 4.3 will be used to assess students' performance. The participation of the interview has first gotten the consent of students and parents before the research starts.

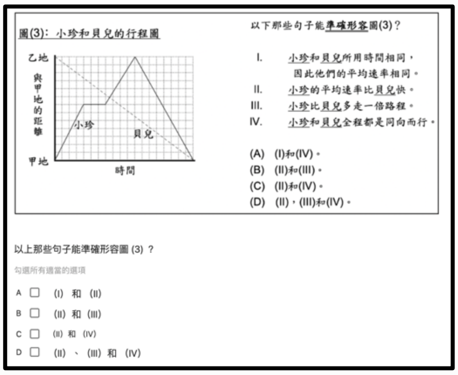


Fig 1a: Interpretation task

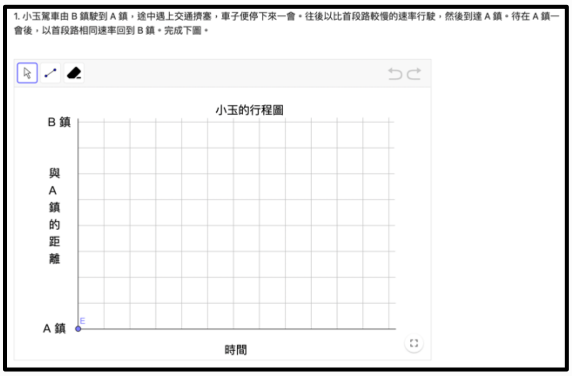


Fig 1b: Construction tasks

(Whole task: https://www.geogebra.org/m/mn9uwent)

4.4.3. Lesson materials and tests

Lesson materials and tests will be collected to analyze the participants' learning process and understanding. Furthermore, the score and performance of the test will be used for analyzing the effectiveness in alleviating students' learning difficulties in the topic of speed and travel graph (research question 1 & 2).

All documents and lesson videos will only be use for the research and be kept in my locker and will not be shown to the public. Only participant numbers will be used to record to compare the results and response to the research questions. No personal information is needed in this research. All participation in this research have gotten the consent of students and their parents.

5. Design

5.1. GeoGebra classroom

This research will use the GeoGebra classroom as the whole teaching sequence is conducted in an online learning mode. GeoGebra classroom is an online platform that can interact with students online and facilitate learning by using GeoGebra applets. This research allows students to operate the designed GeoGebra applets in the GeoGebra classroom and monitor students' learning process in real-time (see Fig 2) to allow the author to analyze.

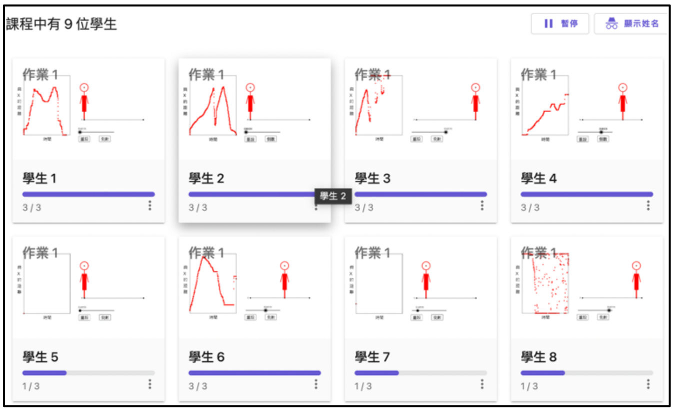


Fig 2: Example of students' learning process in GeoGebra classroom

5.2. Teaching sequence

The teaching trajectory of this research will be teaching travel graph first and the concept of speed follows. This may be different from the traditional way of teaching the topics of speed and travel graph (speed→travel graph). The key to the topics of speed and travel graph is that students understand the relationship between distance and time. If we use the animation function of GeoGebra, the relationship between distance and time in the travel graph can be concretely visualized. This may also allow the students to have more experience undergoing the relationship between distance and time and make it easier to understand the concept of speed. It is assumed that the prior knowledge of students will be the concept of "distance", "time" and "the construction of line graphs".

At the beginning of the teaching sequence, in order to teach, there are activities to allow students to use GeoGebra applets to construct travel graphs based on the given data (distance and time). Two essential GeoGebra applets will then be used to teach the characteristics of the travel graph (see session 5.3). After that, the concept of average speed will be introduced in the following path:

Part 1: Compare speed by comparing time spent/ distance travelled in equal distance/ same period

Part 2: Compare speed by converting time/distance on the ratio (or use common multiple) to the condition in part 1.

Part 3a: Give five pairs of data to students to cause inconvenience in the calculation

Part 3b: Guide students to convert the time into 1 second and "reinvent" the formula of average speed.

Part 4: Distinguish the difference between average speed and constant speed, and their characteristics on the travel graph

The whole lesson materials are attached in appendix 1. The overview of the teaching sequence, teaching objectives and key activities are shown in Table 3.

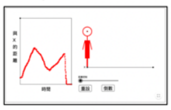
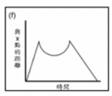
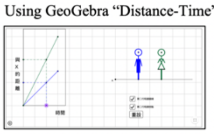
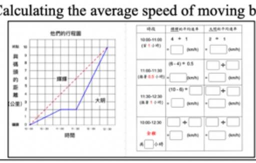
Lesson	Main objectives	Key activities
1	- Recognizing travel graph	- Travel graph construction based on given data (on GeoGebra)
	- Recognizing the relationship between a moving body and a travel graph - Justify the possibility of different travel graph	- Guess the possibility of different travel graph (without GeoGebra) - Using GeoGebra "Free-Move" to simulate the given travel graph  
	- Recognizing travel graph with multiple journey - Comparing which moving bodies move faster based on the travel graph (equal time length)	- Using GeoGebra "Distance-Time" to simulate the given travel graph 
2	- Comparing which moving bodies move faster based on the travel graph (equal distance travelled) - Discovering the steepness of the lines can compare the speed of moving bodies	- Using GeoGebra "Distance-Time" to simulate the given travel graph
	- Recognizing the concept of average speed - Applying the speed formula	- Compare the speed based on same time length and distance travelled - Compare the speed based on different time and distance
	- Distinguishing the difference between average speed and speed - Calculating the average speed of moving bodies based on travel graph	- Calculating the average speed of moving bodies based on travel graph 

Table 3: Overview of the teaching sequence

5.3. Key GeoGebra applets

Two key GeoGebra applets are developed to allow users to simulate the given travel graph and provide immediate feedback to them by visualization and animation of the moving bodies.

The first GeoGebra applets will be “Free move” (see Fig 3a). It allows users to control the moving body by the slider and the corresponding travel graph will be generated immediately. The immediate feedback can help students to explore the characteristics of travel graph. For instance, when the moving body does not move, the travel graph line will move horizontally. Next, students are provided six travel graph (see examples from Fig 3b) and asked to guess the possibility of each graph. After that, they will use the “Free move” to simulate the given graph and investigate the possibility. Students will evaluate and revise their guesses based on the immediate feedback from the GeoGebra applet.

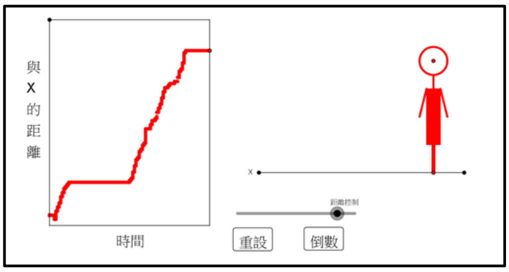


Fig 3a: GeoGebra applet “Free move”

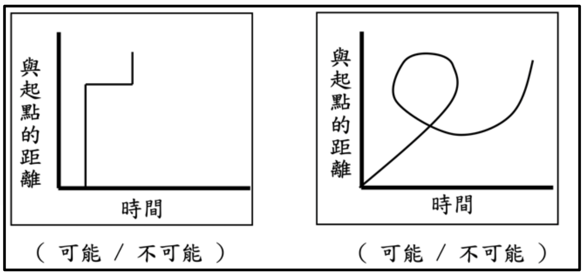


Fig 3b: Examples on given travel graphs

(Whole applet: https://www.geogebra.org/m/vrbqvuzc#material/kmkr9ghw)

The second GeoGebra applet will be “Distance-Time” (see Fig 4a). This applet is mainly used to animate the journey of two moving bodies to compare which moving bodies move faster. Students can adjust the travel graphs and simulate two journeys by using the animation function of two moving bodies. Travel graphs with two moving bodies under the condition of equal distance/ same period are provided to students (see an example from Fig 4b). They have to guess which moving bodies move faster. After that, students will use this GeoGebra applet to simulate the situation and judge which moving bodies move faster.

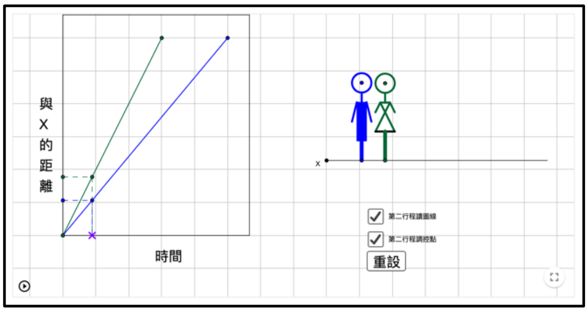


Fig 4a: GeoGebra applet "Distance-Time"

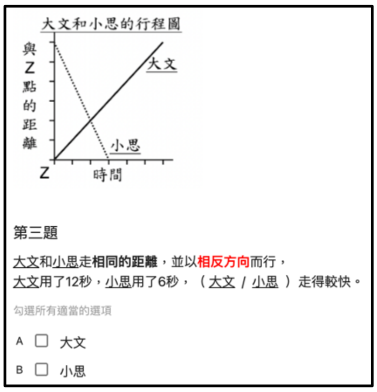


Fig 4b: Examples on given travel graph in equal distance

(Whole applets: https://www.geogebra.org/m/bkku5gvr ; https://www.geogebra.org/m/w7zueybd)

6. Result and Discussion

This session will show and discuss the result of research questions 1-3. Moreover, a case study about outstanding progression on the levels of reasoning about travel graph interpretation and construction will be discussed.

6.1. The three research questions

Question 1: Alleviate students' limited understanding of the meaning of speed

As mentioned in session 2.1, students should achieve stage IVb after learning the topic of speed (Piaget, 1970). The stage IVb includes:

a. Speed calculation

b. The relationship among distance, time and speed

Therefore, in this research, specific questions are designed in the test to assess students' performance in the above perspectives.

a. Speed calculation

Participants' performance in calculating average speed is excellent after the learning in this research. There is one question (question 1 in test 2) to assess students' ability in speed calculation (see Fig 5). Students need to apply the formula of speed to find the average speed of the character in the question. Figure 6 demonstrates that the correct percentage of answering question 1 is 75%. This indicates that most students can apply the formula of speed and calculate the average speed successfully.

1. 小莉家到圖書館的路程長 500 米。小莉從家步行到圖書館，需要 400 秒，

她步行的平均速率是______米每秒。（列式作答）

Fig 5: Question about speed calculation

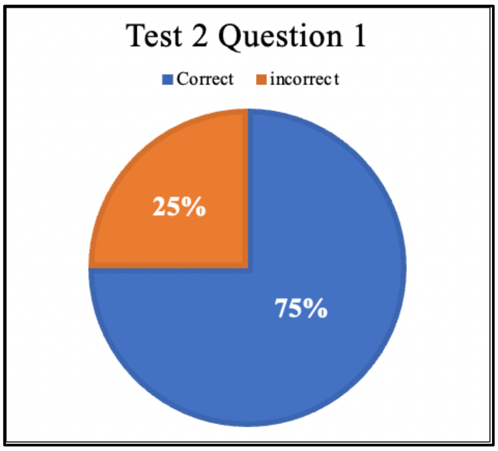


Fig 6: The correct percentage of answering question 1

The possible reason why some students answer question 1 incorrectly can be divided into three aspects:

(1) Calculation mistakes (5 participants; see Fig 7a)

(2) Wrong understanding of speed (2 participants; see Fig 7b)

(3) Careless mistake (1 participant; see Fig 7c)

After asking the students, it is founded that students who answer the question incorrectly due to (1) and (3) can correctly apply the speed formula. If we count them as correctly applying the formula of speed, more than 90% of participants could correctly apply the formula of speed to calculate the speed of moving bodies.

$$\frac{500}{400} = 0.65$$

Fig 7a: Calculation mistakes

$$500 - 400 = 100$$

Fig 7b: Wrong understanding in speed

$$\frac{400}{500} = 0.8$$

Fig 7c: Careless mistake

b. The relationship among distance, time and speed

Participants show adequate understanding of the relationship among distance, time and speed after using GeoGebra to learn the topic of speed. There is one question (question 6 in test 2) to assess students' understanding of the relationship among distance, time and speed. (See Fig 8). To solve this question, students will often be taught to use the formula to find the distance (speed × time). However, we would not teach the formula in this research to allow students to only use their understanding of the relationship among distance, time and speed to solve the question. Figure 9 demonstrates that the correct percentage of answering question 6 is 62%. This illustrates that several students could comprehend the meaning of speed.

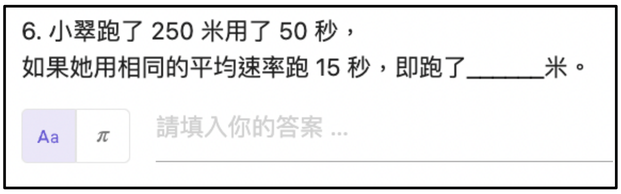


Fig 8: Question about the relationship among distance, time and speed

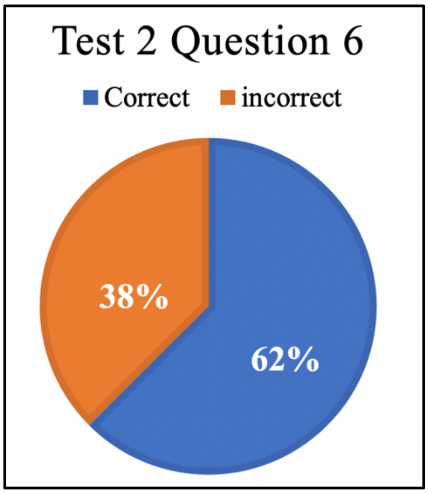


Fig 9: The correct percentage of answering question 6

One of the participants gives an outstanding response to explain the question relationship among distance, time and speed.

Teacher: *Why your answer is* $\frac{250}{50} \times 15$ *?*

Participants 18:

"I regard average speed as a grid as it talks about how many meters do the person travel in every second and the time is the repeating number of the "grid", so, the distance is "grid" × "time". "

This illustrates that the student could use metaphor to explain the meaning of speed and show his clear understanding of the relationship among distance, time and speed under the GeoGebra teaching design in this research.

Question 2: Alleviate the weak performance in solving the problem with two moving bodies

In this research, problems with two moving bodies are divided into:

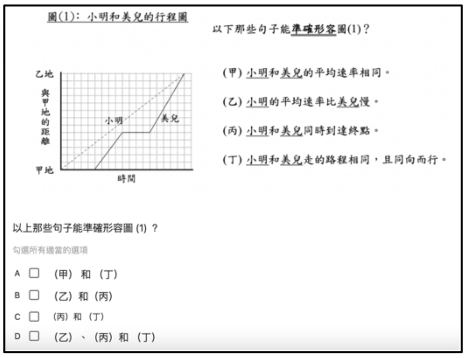
a. Travel graph interpretation

b. Calculating the speed of moving bodies through the graph

Therefore, the test will include these problems to evaluate students' performance.

a. Travel graph interpretation

Participants' performance in interpreting travel graph is excellent after using GeoGebra to learn the topic of speed and travel graphs in this research. Questions 3 and 4 are designed in test 2 to assess participants' performance in travel graph interpretation (see Fig 10 and 11). Students are required to justify the correctness of some statements (question 3) and find related information in the travel graph to answer the question (question 4). More than 70% of students answered each question correctly (see Fig 12 and 13). This shows that most of the students can solve the problems with two moving bodies after using GeoGebra to learn the topic of speed and travel graph. The possible reason to explain this is that the teaching sequence offers students many opportunities to verbally describe or interpret the travel graph. Moreover, by using the GeoGebra applets, students can easily verify their interpretation and fix their understanding of travel graph interpretation. This may share a similar effect with Duijzer et al (2020). They used a motion sensor to let students generate the travel graph and found that the quick response from the motion sensor assisted students in learning travel graph interpretation. Therefore, the immediate feedback provided by GeoGebra applets in this research can help students acquire the ability of travel graph interpretation.



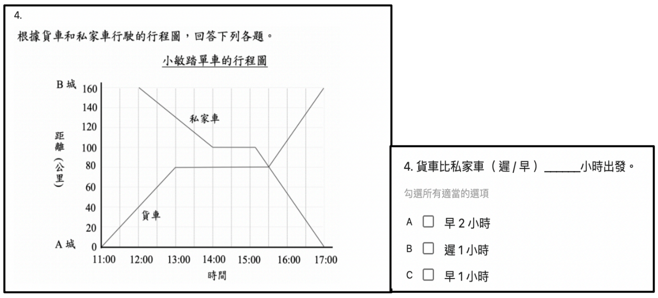
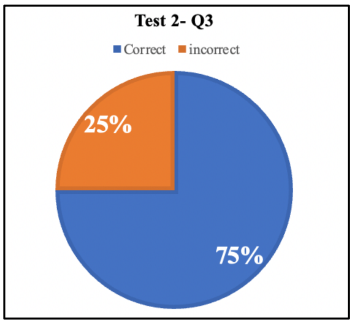


Fig 10 and 11: Question about the travel graph interpretation



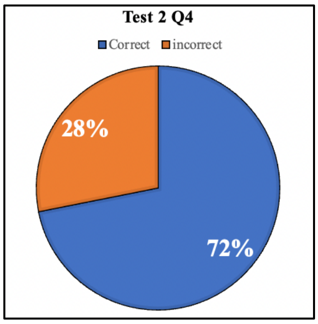


Fig 12 and 13: The correct percentage of answering question 3 and 4

b. Calculating the speed of moving bodies through the graph

Participants' performance in calculating the speed of moving bodies is satisfactory. Question 5 is designed in test 2 to test the performance of calculating the speed of moving bodies in the travel graph (see Fig 14). Figure 15 shows that 62% of students can answer this question correctly. More than 60% of students can calculate the speed of moving bodies based on the travel graph, including the ability to select related information in the graph and apply the speed formula.

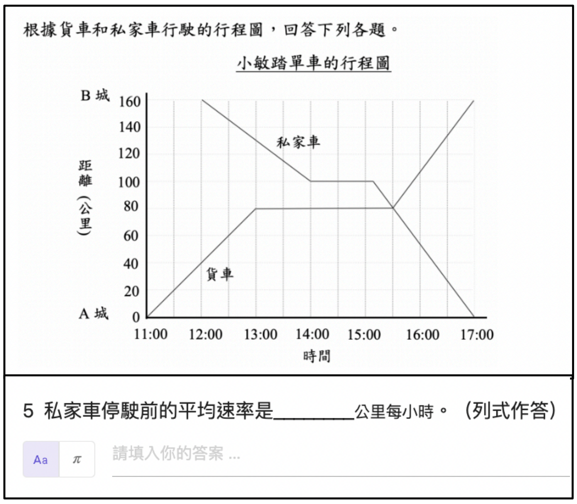


Fig 14: Question about calculating the speed of moving bodies

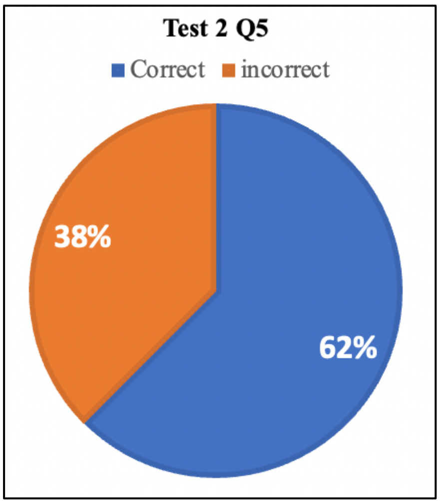


Fig 15: The correct percentage of answering question 5

Overall, students give an exceptional performance in solving problems with two moving bodies after learning the topic of speed and travel graphs in this research. Over 60% of students answered both designed questions correctly in the problem with two moving bodies. On the other hand, when we compare the results of the two above questions, it is discovered that students show better performance in travel graph interpretation than in calculating the speed of moving bodies through travel graph. The possible reason for this is that there are more graph interpretation tasks (three) than calculating the speed through graph task (one) in the teaching sequence. Therefore, students have more experience in interpreting travel graphs.

Question 3: Students' levels of reasoning about travel graph interpretation and construction

As mentioned above, an assessment rubric (see session 4.3 or below table) is designed as a research instrument to assess students' levels of reasoning about travel graph interpretation and construction. 12 participants will be randomly selected and attend the interview to do the assessment about travel graph interpretation and construction. The following part will separate travel graph interpretation and construction to discuss students' performance.

Table 2

Assessment Rubrics for students' levels of reasoning about travel graph interpretation and construction

Level of reasoning	Grade	Description of students' reasoning	
		Travel graph interpretation	Travel graph construction
		Students interpret travel graphs	Students constructs travel graphs
Unrelated reasoning	G1	without making any reference to the representation of the travel graph	without considering the description of the movement
Perfunctory reasoning	G2	based on the appearance of the travel graph or the surface characteristics of the movement	based on the surface characteristics of the description of the movement
Single variable reasoning	G3	based on a single variable (distance or time or speed) of the travel graph	by considering a single variable (distance or time or speed)
Multiple variable reasoning	G4	based on multiple variables (distance and/or time and/or speed) of the travel graph	by considering multiple variables (distance and/or time and/or speed)

Table 2: The assessment rubric for students' levels of reasoning about travel graph interpretation and construction

a. Travel graph interpretation

The majority of interviewees perform multiple variable reasoning about travel graph interpretation after learning about the topic of speed and travel graphs in this research. Interviewees are required to justify the given statement based on the information of the travel graph in the travel graph interpretation task (see Fig 16). Almost 60% of students perform multiple variable reasoning about travel graph interpretation (see Fig

17). This illustrates that most of the students can focus on the correct information on the travel graph and interpret the travel graph based on the distance, time and speed. The possible reason for this level of reasoning may be same as question 2a. The sufficient chances for students to interpret the travel graph verbally and the prompt visualization from the GeoGebra applets possibly assist students in establishing the proper understanding of travel graph interpretation.

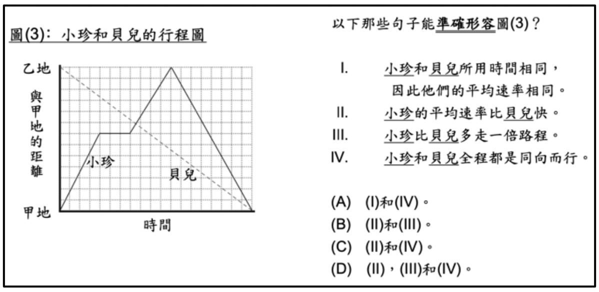


Fig 16: Task about the travel graph interpretation

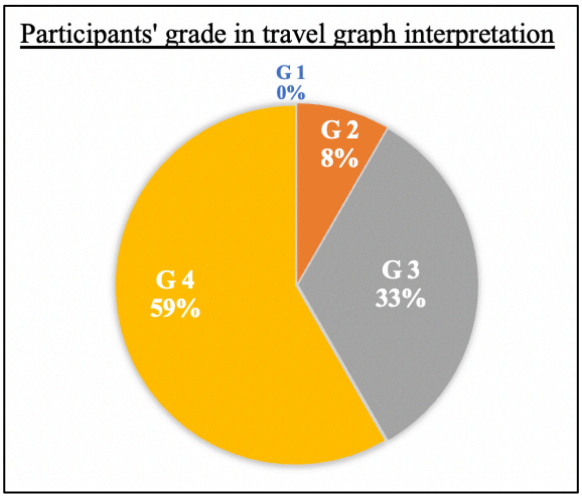


Fig 17: The percentage of participants' grade in travel graph interpretation

b. Travel graph construction

Most interviewees perform multiple variable reasoning (G4) about travel graph construction after learning the topic of speed and travel graphs in this research. Interviewees are required to construct the travel graph based on the given conditions in the travel graph construction task (see Fig 18). Fig 19 shows that 75% of students perform multiple variable reasoning about travel graph interpretation. This indicates that most students will consider the distance, time and speed when constructing a travel graph.

The possible reason for students performing G4 level of reasoning is that the animation function of the GeoGebra applets can concretely present the actual situation of the travel graph and help students construct the travel graph accurately. In the teaching sequence, students need to construct or duplicate the travel graph based on the given conditions in the lessons. After that, they can play and watch the animation of their graph by using the designed GeoGebra applets and fix their travel graph again. Therefore, students can easily discover the characteristics of travel graph in this process and thus perform well in travel graph construction.

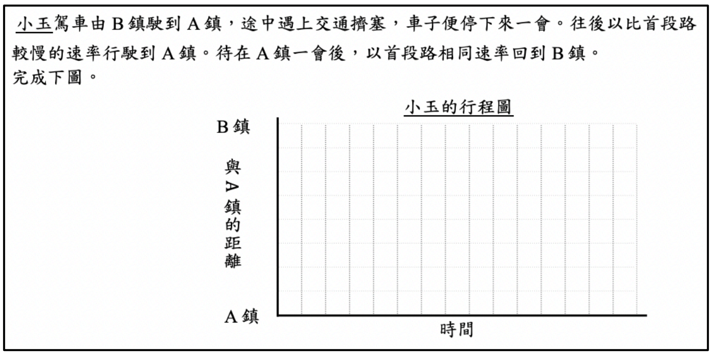


Fig 18: Task about the travel graph construction

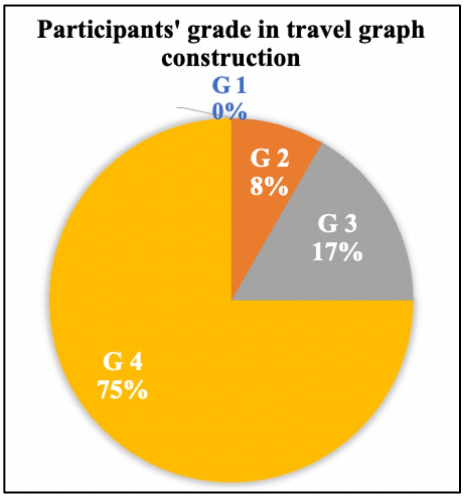


Fig 19: The percentage of participants' grade in travel graph construction

Overall, almost 60% of interviewees' levels of reasoning about travel graph interpretation and construction are in G4. This means they can consider the distance, time and speed when interpreting and constructing travel graphs. During the learning activities of using two key GeoGebra applets (see session 5.3), students are undergoing the following learning stage:

Stage 1: Making a guess on the possibility of travel graph ("Free Move") or which moving bodies move faster ("Distance-Time")

Stage 2: Investigate the correctness of their guess by using GeoGebra applets

Stage 3: Evaluate and revise their guess

Students will develop an adequate level of ability in travel graph interpretation and construction during the above learning stages and eventually cultivate the "graph sense". This explains why most of the interviewees perform multiple variables reasoning (G4) about travel graph interpretation and construction after the teaching sequence in this research.

On the other hand, when we compare the results of travel graph interpretation and construction, more percentage of the interviewees performed G4 level of reasoning in travel graph construction than travel graph interpretation. This reveals that students are capable of constructing travel graph after using GeoGebra to learn and proper learning activities. Considering the advantages of travel graph construction together, travel graph construction should be included in the mathematics curriculum.

G2 and G3 performance in travel graph interpretation and construction

It is also crucial to investigate why some students will perform G2 and G3 levels of reasoning to understand their learning difficulties.

Students who perform perfunctory reasoning (G2) about travel graph interpretation and construction will have the following performance (see Fig 20 and 21).

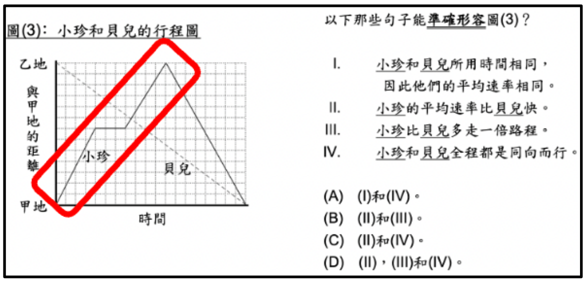


Fig 20: G2 performance in travel graph interpretation

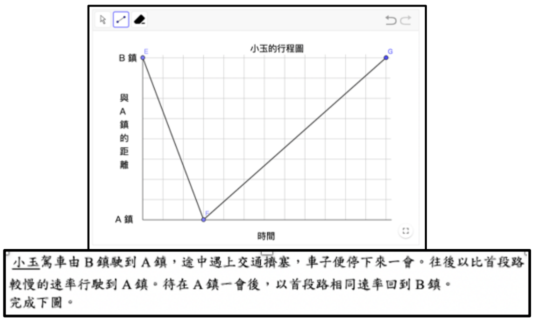


Fig 21: G2 performance in travel graph construction

For travel graph interpretation, a student is asked to describe the journey of the red circle part (see Fig 20). His description is as follow:

" 小珍 moves upward in the beginning, and then moves forward half-way through, and then moves upward at the end"

This shows that this student describes the travel by the appearance of travel graphs, so he is in perfunctory reasoning (G2).

For travel graph construction, the student only constructs a return journey in the graph in figure 21. This means he only follows the essential characteristics of the description of the travel graph to construct.

Students who perform single variable reasoning (G3) about travel graph interpretation and construction will have following performance (see Fig 22 and 23).

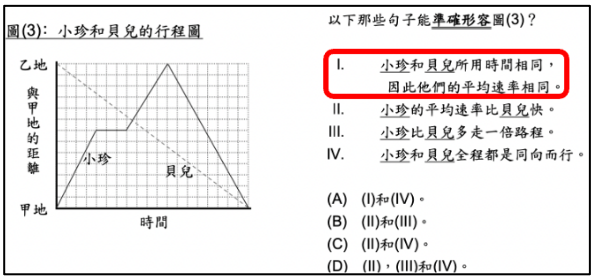


Fig 22: G3 performance in travel graph interpretation

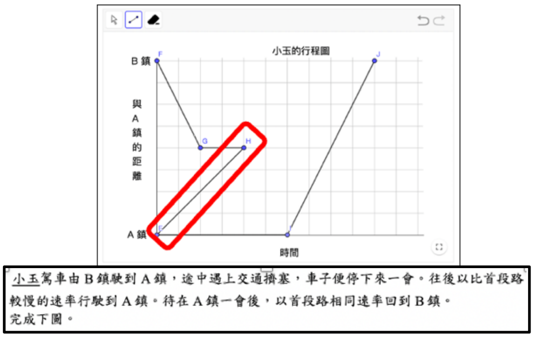


Fig 23: G3 performance in travel graph construction

For travel graph interpretation, a student thinks the statement with the red circle in figure 22 is correct. His explanation is as follows:

"The starting time and ending time of two people are same, they are using same time duration to walk, so their speed is same."

This shows that this student just considered the variable time and ignored another variable.

For travel graph construction, the student constructs a journey which turns back the clock (the red circle part in figure 23). He explains that he thinks that the graph has to turn the point of town A when the moving body goes to town A. This means he ignored the variable time.

6.2. A case studies

A case study about the outstanding progression of a student after the learning in this research will be highlighted in this session. The student, called Edward, demonstrated an upward progression from unrelated reasoning (G1) to multiple variable reasoning (G4) about travel graph interpretation and construction.

At the beginning of the teaching sequence, all students will be assumed that they are in G1 as they have not learnt the topic of travel graph. After learning the skills of travel graph construction, Edward was asked to guess and describe the journey of the travel graph (see Fig 24). He only the journey based on the appearance of the travel graph:

"This cat moves upward until 6 seconds, move forward and move upward again."

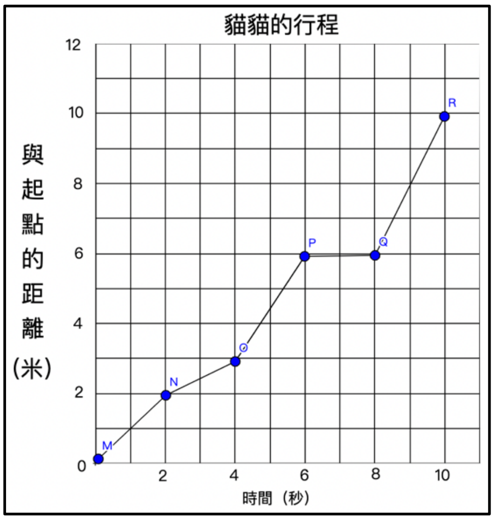


Fig 24: The travel graph of a cat in the lesson

This explanation shows that his level of reasoning about travel graph interpretation at this moment is in G1 before using the two key GeoGebra applets.

After introducing the first GeoGebra applet "Free Move" (Fig 3a), students can use it freely for around 3 minutes. Edward was asked how to construct his travel graph (see Fig 25). He describes the construction correctly.

Edward: *"I repeatedly move the character to the left and the right."*

Teacher: *"When will the graph go upward and downward?"*

Edward: *"When I move the character to the left,*

which means it moves far from the origin,

the travel graph will go upward.

When I move closer and closer to

the origin, the travel graph will go downward."

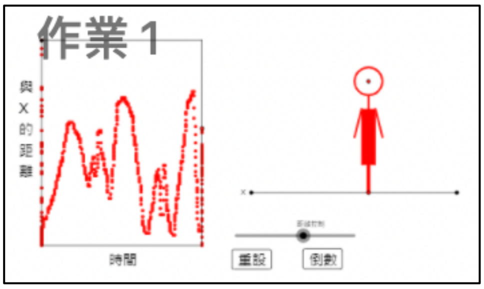


Fig 25: Edward's travel graph in "Free move"

The above description shows that Edward can interpret the travel graph without the appearance. Furthermore, when he guessed the possibility of the given travel graph (see Fig. 26), he could point out that the travel graph is impossible as time cannot be returned without using the "Free-Move". This shows that the experience of using "Free-Move" let him establish a mental simulation and provide reasonable assumptions. After he used the "Free-Move" to simulate the given travel graph, he can accurately describe the motion of the graph (see example Fig 27).

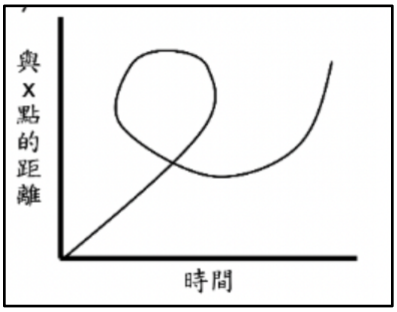


Fig 26

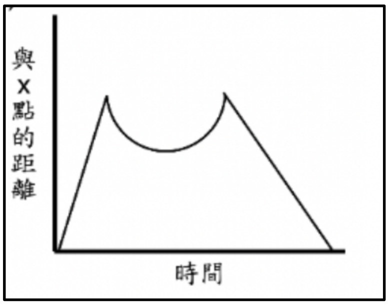


Fig 27

After the first lesson in this research, Edward performed a single variable resonating (G3) in test 1 (see Fig 28). The reason for this is similar to the G3 example in session 6.1. He thought that touching the exact origin point was equal to going to the origin.

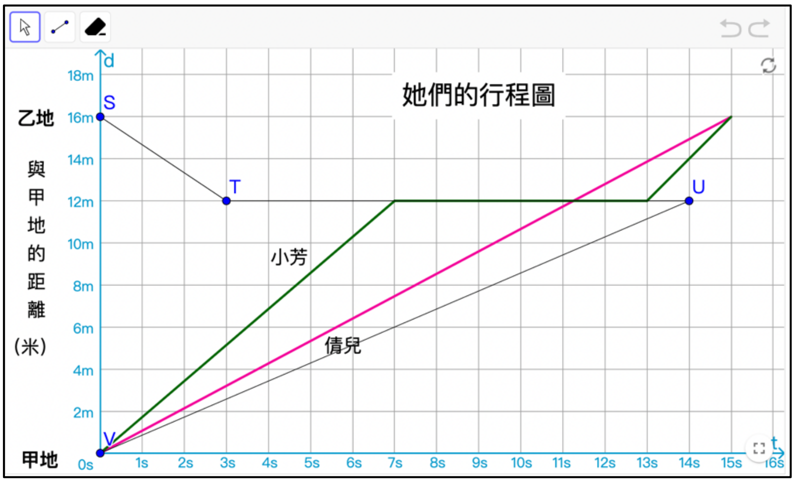


Fig 28: Edward's performance in travel graph construction in test 1

While using the second key GeoGebra applets, Edward shows another significant change in the learning travel graph. Students are given some travel graphs about two moving bodies and asked to guess which moving bodies move faster. When Edward was asked to explain his guess, he pointed out a misunderstanding:

"When the travel graph of the moving body is longer, the moving body moves faster."

Edward thought this guess was correct as it fit the situation in figure 29. Although he made a wrong guess, he tried to observe the characteristics of the travel graph to compare the speed. After that, other travel graphs occur a contradiction with his guess. Edward revised his guess that *when the travel graph is higher, the moving body moves faster.* The misunderstanding comes from figure 30 this time.

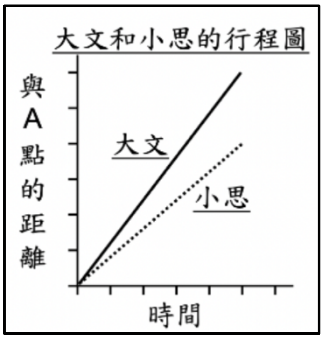


Fig 29

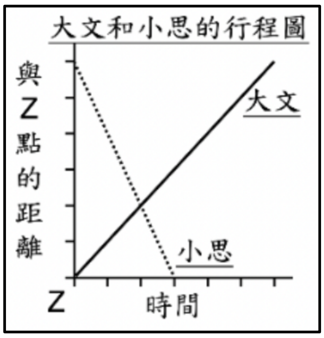


Fig 30

However, some students use the figure 31 as justification to refute Edward's guess. At this time, the teacher put all travel graph together to allow students to compare and figure out the common characteristics of each graph. Edward and other students start to observe the steepness of the travel graph. Edward proposes a guess that *when the travel graph is steeper, the moving body moves faster.* They checked that this guess was correct after using the GeoGebra applet "Distance-Time" (Fig 4a).

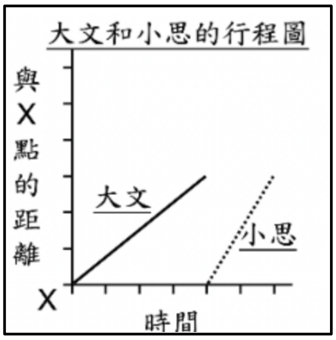


Fig 31

Lastly, Edward performs multiple variable reasoning in travel graph interpretation and construction. After using the two key GeoGebra applets and undergoing the learning process in session 6.1 (guess→investigate→revise), Edward shows a complete understanding of travel graph interpretation and construction. Although the investigation part takes much time, the progression is valuable. Moreover, the visualization and immediate response of the GeoGebra applets can stimulate students' thinking and thus establish an understanding of the relationship among distance, time and speed.

7. Limitation

The result of this research may not apply to the regular classes. It is because this research implements small group teaching and each group only contains 5-8 students due to limited time and resources in this research. Each regular class often has 20-30 students and 1 teacher for public school. The difference in the teacher-student ratio in the lesson possibly affects students' learning performance.

Moreover, the result of this research may not apply to the face-to-face teaching mode. Under the COVID-19 pandemic, the teaching mode of this research can only be conducted in online teaching mode. Group activities or discussions between students are difficult to design due to the restraint of technical issues. Some physical activities can also be implemented to cultivate the body sense of speed of students. These are the possible factors to affect students' learning performance. However, it is believed that students may benefit from the two key GeoGebra applets although the class size is larger or the lessons are in the face-to-face mode.

Apart from the above, this research may not provide enough learning experience due to the limited lesson time. According to Mathematics Education Section Curriculum Development Institute (2020), the suggested time to teach the topics of speed and travel graph is 300 minutes (for 30 minutes a lesson). The teaching time of this research is only 180 minutes. Therefore, students may not have sufficient learning experience and thus affecting their learning.

8. Conclusion

It is believed that GeoGebra is an effective dynamics software to teach the topics of speed and travel graph. The visualization of the travel graph provided by the animation function makes the relationship between distance and time concrete. The concept of speed becomes more apparent and easier to understand. Moreover, the GeoGebra applets can offer immediate feedback to students to investigate the characteristics of the travel graph. All of these can help students enhance their reasoning level about travel interpretation and construction. The “graph sense” of students can be cultivated and benefit their learning.

Furthermore, utilizing the GeoGebra for learn and teach may help students “reinvent” the concept of speed and travel graph. During the learning cycle (guess→investigate→revise), the GeoGebra applets provide a more significant possibility to students to explore and investigate the creating process of mathematics concepts. The visualization of the relationship between distance and time is an excellent example to illustrates it. It is hoped that GeoGebra will be utilized more in mathematics teaching in the future to provide a meaningful learning experience to students.

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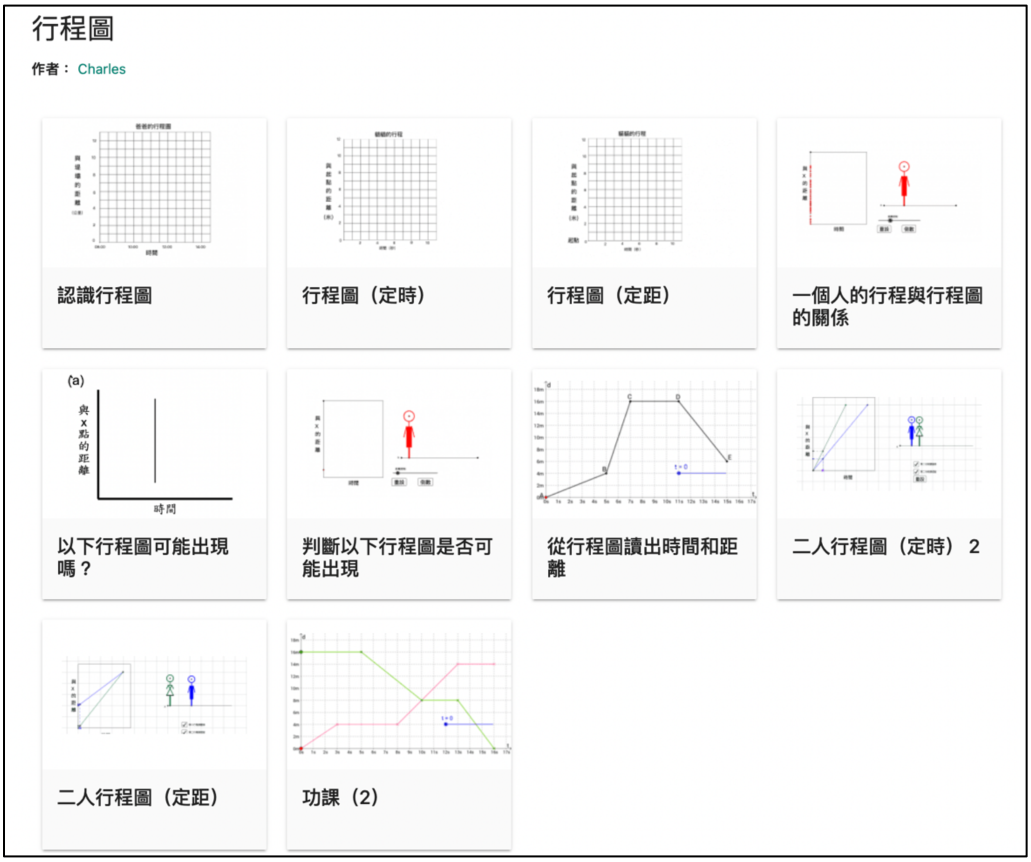
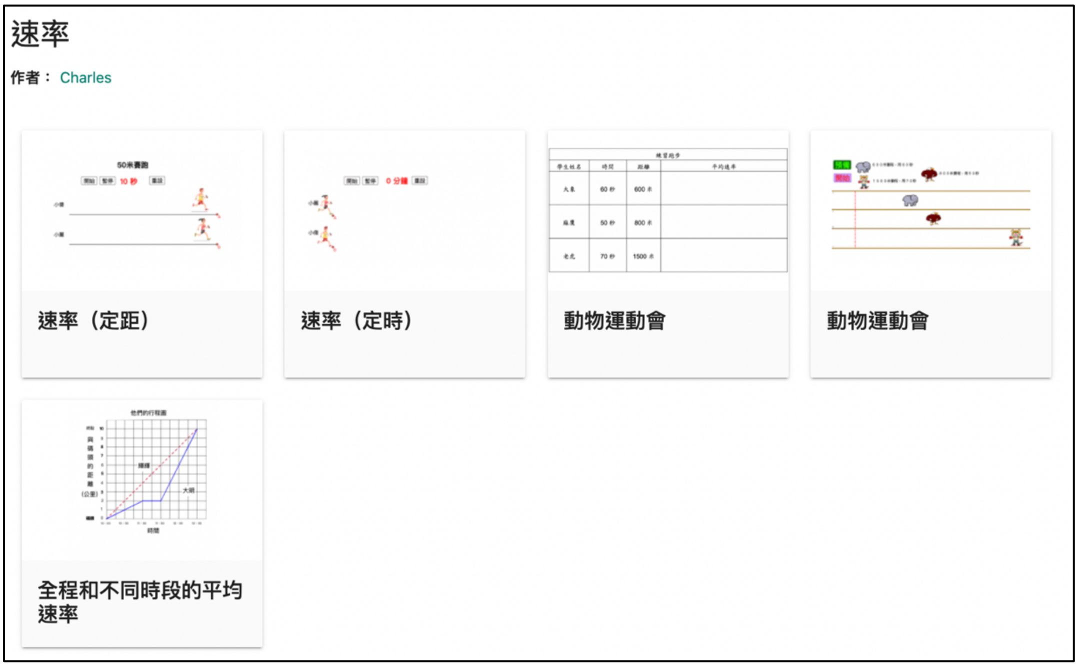
Appendix 1 : Learning materials designed in GeoGebra books



https://www.geogebra.org/m/vrbqvuzc



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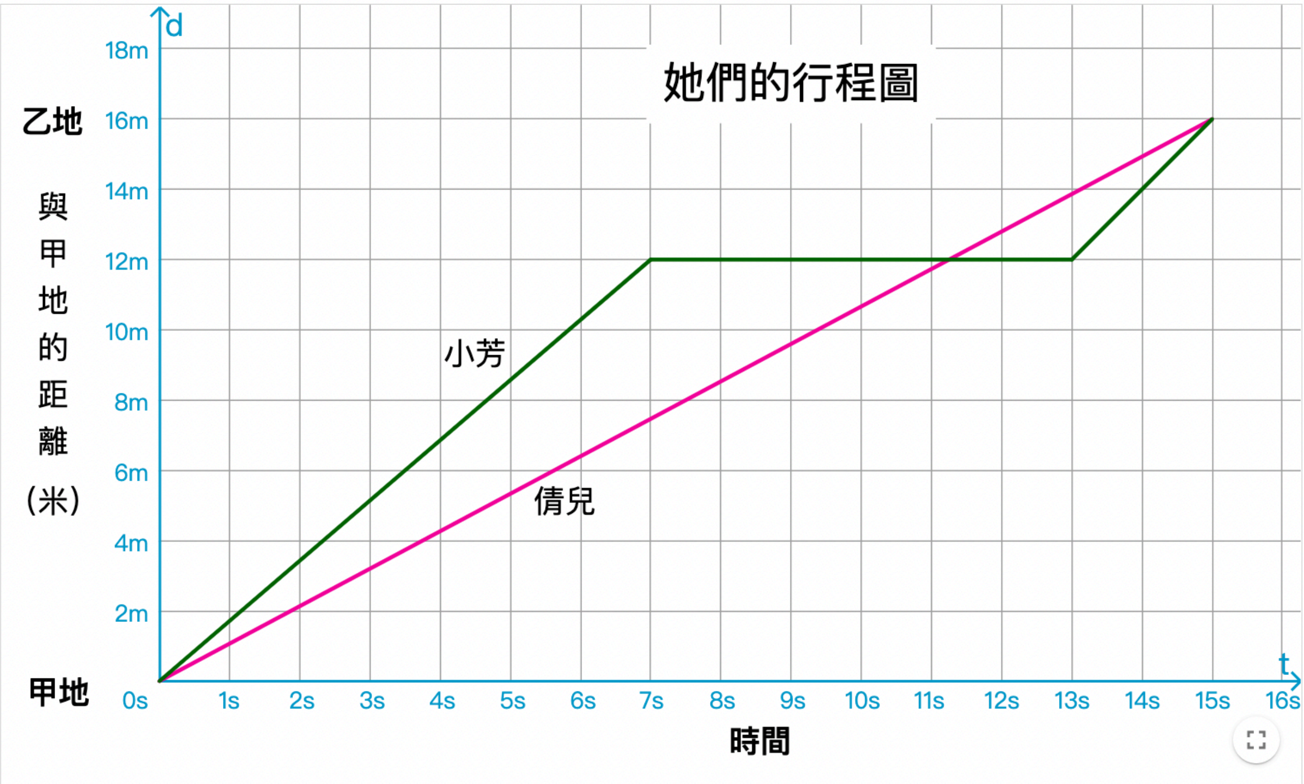
Appendix 2 : Test 1



(a) 他們同一時間開始行程。

勾選所有適當的選項

A ☐ 正確

B ☐ 不正確

https://www.geogebra.org/m/apnvh5zz

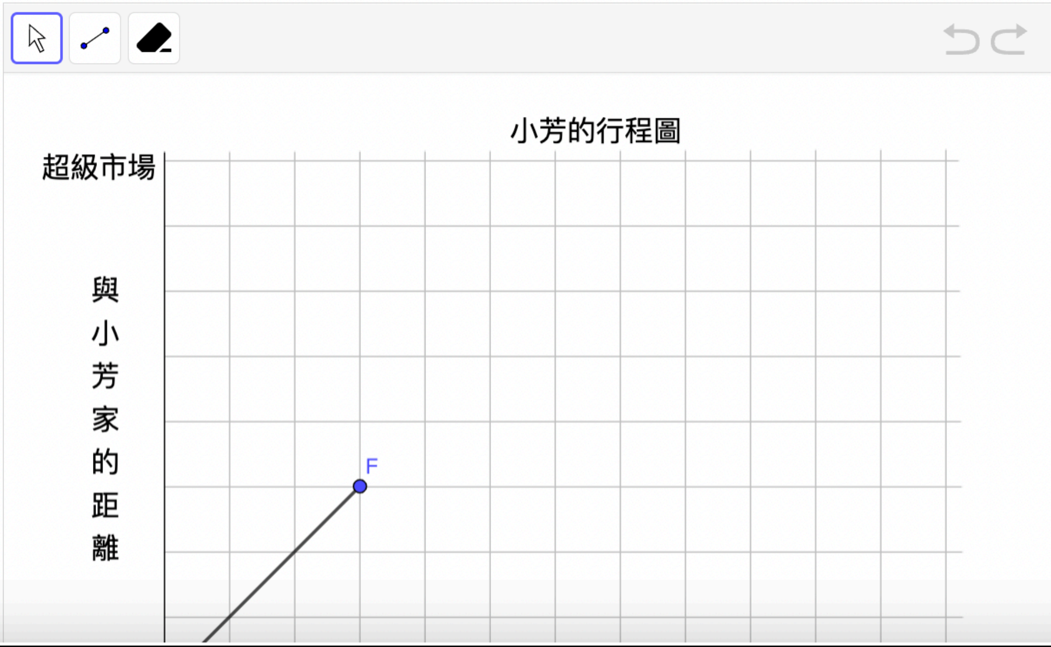
Appendix 3 : Test 2

1. 小莉家到圖書館的路程長 500 米。小莉從家步行到圖書館，需要 400 秒，

   她步行的平均速率是______米每秒。（列式作答）

Aa π 請填入你的答案 ...

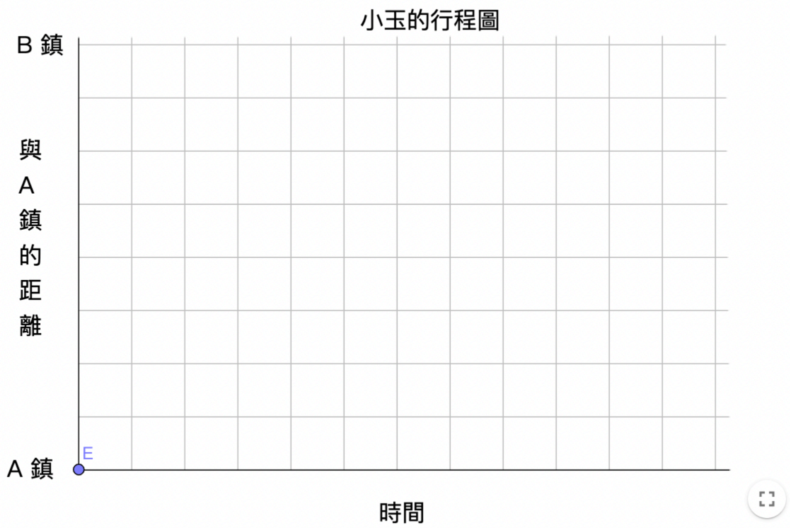
2. 小芳到超級市場買東西，途中小芳發現沒有足夠金錢，她便以較快的速率折返家中。回到家中休息一會後，便以原來（首段路）的速率行走到超級市場。完成下圖。



https://www.geogebra.org/m/wjthfnuw

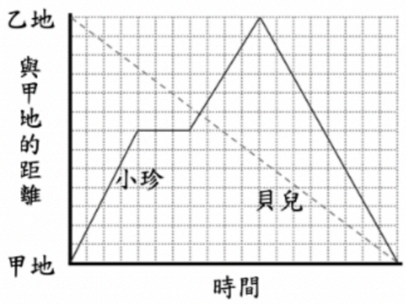
Appendix 4 : Task in interview

1. 小玉駕車由 B 鎮駛到 A 鎮，途中遇上交通擠塞，車子便停下來一會。往後以比首段路較慢的速率行駛，然後到達 A 鎮。待在 A 鎮一會後，以首段路相同速率回到 B 鎮。完成下圖。



2.

圖(3)：小珍和貝兒的行程圖



以下那些句子能準確形容圖(3)？

I. 小珍和貝兒所用時間相同，因此他們的平均速率相同。
II. 小珍的平均速率比貝兒快。
III. 小珍比貝兒多走一倍路程。
IV. 小珍和貝兒全程都是同向而行。

(A) (I)和(IV)。
(B) (II)和(III)。
(C) (II)和(IV)。
(D) (II)，(III)和(IV)。

以上那些句子能準確形容圖 (3) ？

勾選所有適當的選項

- A ☐ (I) 和 (II)
- B ☐ (II) 和 (III)
- C ☐ (II) 和 (IV)
- D ☐ (II) 、 (III) 和 (IV)

https://www.geogebra.org/m/mn9uwent