Development of Conceptual Understanding and Procedural Knowledge of Mathematics through Inquiry Based Learning Scaffolded by Cognitive Tools

by

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A Thesis Submitted to

The Education University of Hong Kong

in Partial Fulfillment of the Requirement for

the Degree of Doctor of Education

July 2017

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Abstract

This study investigated whether primary school students could develop better conceptual understanding and procedural knowledge on the topic of "area of closed shapes" using the inquiry-based learning approach scaffolded by the cognitive tools developed on the GeoGebra platform. It answered two research questions: (a) Compared with the direct instructional approach, would the inquiry-based learning approach scaffolded by cognitive tools developed on the GeoGebra platform help students develop better conceptual understanding and procedural knowledge? and (b) If the answer to the first research question was affirmative, how did the inquiry-based learning approach scaffolded by the GeoGebra cognitive tools help the students develop better conceptual understanding and procedural knowledge? This study focused on two classes of Grade 5 students with similar mathematics backgrounds. One class was selected randomly to be the experimental group (28 students); the other class became the control group (25 students). The students in the experimental group used GeoGebra cognitive tools to explore how to find the areas of a parallelogram, triangle, and trapezoid. The experimental group students used the inquiry-based instructional model. The control group students learned the same topics through the direct instructional approach without using GeoGebra cognitive tools. The original mathematics teacher of the control group class administered the control group. The researcher of this study administered the experimental group, and the original mathematics teacher of this group sat in as a teacher observer. The students took one pre-test before the study and two post-tests at the middle and end of the study, respectively. The pre-test showed that there were no significant differences between the two groups regarding their conceptual understanding and procedural knowledge of the target topics. The post-tests showed that the experimental group had developed significantly better conceptual understanding than the control group, while there were no

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significant differences between the two groups regarding their development of procedural knowledge. Apparently, because the parallelogram cognitive tool allowed students to interactively transform the parallelogram into a rectangle, they could realize the mathematical formula used for calculating the area of the parallelogram. Similarly, because the triangle and trapezoid cognitive tools allowed students to interactively replicate the shapes to form a parallelogram, they could apprehend the formulas used for calculating the area of these shapes. The GeoGebra cognitive tools also helped the students to visualize the various sets of shape bases and heights. This research revealed that it was difficult for the students to understand that the base of the parallelogram formed by two identical trapezoids was equal to the sum of the upper and lower bases of the trapezoid. Using the pedagogical approach proposed in this study, the teachers could guide the students through this difficulty, step by step. In fact, the GeoGebra cognitive tools, together with all the pedagogical activities carried out in the experimental group, contributed to the students' understanding of the target topics. This study indicated that inquiry-based learning scaffolded by cognitive tools was a promising way to teach mathematics topics like finding the areas of a parallelogram, triangle, and trapezoid.

Keywords: 5E Model, cognitive tools, conceptual understanding, inquiry-based learning, procedural knowledge

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Acknowledgments

I am enormously indebted to my Principal Supervisor, Professor Kong Siu Cheung, for his guidance, encouragement and timely feedback throughout my time in the EdD program. I wish to express my gratitude to the panel members of the thesis proposal presentation panel, Professor Lo Sing Kai and Dr. Man Yiu Kwong, for their support and their valuable feedback on my research study. I also would like to thank the Education University of Hong Kong and every professor who had taught me in these years.

I owe special thanks to the principals, teachers and students of the Jordan Valley St. Joseph's Catholic Primary School, especially Ms. Chan Pui Shan the Vice-principal of the school. This study would not have been possible without their patience and support.

Finally, I would like to thank my family and my fellow schoolmates for their continuous encouragement through words and deeds during the lengthy process of this study.

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List of Abbreviations

BSCS	Biological Sciences Curriculum Study
PCK	Pedagogical Content Knowledge
TPCK	Technological Pedagogical Content Knowledge
5E	Engagement, Exploration, Explanation, Elaboration, and Evaluation

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Chapter 1 – Introduction to the Study

Background of the Study

For the mathematical topic of "area of closed shapes," prevailing education emphasized the procedural knowledge of students while being largely ineffective in developing their conceptual understanding (Baturo & Nason, 1996; Martin & Strutchens, 2000; Kospentaris, Spyrou, & Lappas, 2011). Many researchers had found that students often did not understand the important concepts of *area* (Kospentaris et al., 2011; Hart & Booth, 1984; Kamii & Kysh, 2006; Pitta-Pantazi & Christou, 2009). The conceptual understanding of area was very important because it was the foundation for learning other mathematical topics (Martin & Strutchens, 2000). In view of this ineffectiveness, this study investigated the appropriate pedagogical approach for students to develop both conceptual understanding and procedural knowledge in this mathematics topic.

This study adopted the view that conceptual understanding was about facts and principles, while procedural knowledge was about manipulations and algorithms (de Jong & Ferguson-Hessler, 1996; Baroody, Feil, & Johnson, 2007). It was critical for students to develop both conceptual understanding and procedural knowledge because they developed iteratively—i.e., improving procedural knowledge had a positive effect on conceptual understanding and vice versa (Kilpatrick, Swafford, & Findell, 2001; Baroody et al., 2007; Star, 2007; Rittle-Johnson & Schneider, 2014).

Evidence had shown that inquiry-based learning was an effective way to foster students' conceptual understanding (Haury, 1993; Boaler, 1998a; Boaler, 1998b; Bybee, 2009; Furtak,

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Seidel, Iverson, & Briggs, 2012). Inquiry-based learning was a process in which students were engaged in working out and widely investigating questions, solving problems, and then constructing new understandings and knowledge (Alberta Learning, 2004; Kong & So, 2008). In inquiry-based learning, the students had opportunities to interact with their peers (Wittrock, 1989; King, 1991; Cobb et al., 1991; Webb & Farivar, 1994; Hendry, 1996; Gillies & Ashman, 2003; Bybee et al., 2006; Stein, Engle, Smith, & Hughes, 2008), to explore prior to being explained (Eisenkraft, 2003; Bybee et al., 2006; Stein et al., 2008; Marshall, Horton, & Smart, 2009), and to connect new knowledge to their extant knowledge (Van De Walle, Karp, & Bay-Williams, 2013).

De Jong (2006) suggested that computer-supported cognitive tools might solve the problems encountered by students who were learning under the inquiry-based learning environment. Cognitive tools were mental and computational devices that could support, guide, and facilitate the learner’s cognitive processing (Derry & LaJoie, 1993; Jonassen, 1992). Kong (2011) stated that computer-supported cognitive tools could provide three types of scaffolds, which could support students in constructing their own knowledge without the teacher’s mediation, namely: (a) visual representation, (b) graphical manipulation, and (c) immediate feedback. Iiyoshi, Hannafin, and Wang (2005) suggested that cognitive tools could help students to develop their conceptual understanding.

The teacher’s pedagogical content knowledge, subject matter content knowledge, and knowledge of pedagogy permeated into the cognitive tools so that the tools could support teaching and learning (Ferdig, 2005). According to Shulman (1986), pedagogical content knowledge included “the most powerful analogies, illustrations, examples, explanations, and demonstrations” (p. 9) that represented the subject matter in a way that could be

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comprehended by the students. Pedagogical content knowledge also included “an understanding of what made the learning of specific topics easy or difficult” (Shulman, 1986, p. 9), which meant an understanding of the conceptions and misconceptions the students had on the particular topics. Chan, Kong, and Cheng (2014) identified three common difficulties that students had when they were learning the mathematical topic of “area of closed shapes”; namely, the: (a) lack of the concept of area conservation, (b) failure to identify a base and its corresponding height for area calculation, and (c) misconception that only regular closed shapes had measurable areas and corresponding mathematical formulas for area calculation. These common difficulties could be regarded as the pedagogical content knowledge that the teacher should have realized prior to teaching this topic. Chan et al. (2014) also proposed several cognitive tools developed on the GeoGebra platform to support students to explore the mathematical formulas for calculating the areas of a parallelogram, triangle, and trapezoid so as to address these three common difficulties for students.

Simply memorizing the formulas for calculating the areas of the various shapes was not desirable for students to learn the mathematical concept of area (Baturo & Nason, 1996; Kospentaris et al., 2011; Martin & Strutchens, 2000; Manizade & Mason, 2014). Many scholars had conducted research to investigate students’ construction of their knowledge on this topic by asking the students to explore and develop by themselves the formulas for calculating the areas of closed shapes, especially the area of trapezoids (Manizade & Mason, 2014; Peterson & Saul, 1990; Wanko, 2005). These studies found that middle school, high school, or college students were able to derive different ways or formulas to calculate the area of a trapezoid.

However, there was a paucity of literature examining how integrating cognitive tools with

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inquiry-based learning effected students' conceptual understanding and procedural knowledge. Moreover, there had been little investigation of primary school students regarding their exploration and construction of the formulas for the calculation of the areas of closed shapes. This study aimed to fill these literature gaps.

This study adopted the BSCS 5E Instructional Model, which was an inquiry-based learning instructional model. It consisted of five phases—namely, the engagement, exploration, explanation, elaboration, and evaluation phases (Bybee et al., 2006). The BSCS 5E Instructional Model was chosen because there were extensive empirical findings which indicated it was an effective model. In some cases, it was more effective than alternative teaching approaches in terms of students' learning gains (Bybee et al., 2006; Bybee, 2009).

Purpose of the Study

The purpose of this study was to investigate whether primary school students could develop better conceptual understanding and procedural knowledge on the topic of "area of closed shapes" when they were guided to explore the mathematical formulas for calculating the areas of the shapes using the inquiry-based learning approach scaffolded by cognitive tools developed on the GeoGebra platform.

Research Questions

Specifically, this empirical study was conducted to answer the following two research questions regarding how primary school students learned the areas of a parallelogram, triangle, and trapezoid:

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1. Compared with the direct instructional approach, would the inquiry-based learning approach scaffolded by cognitive tools developed on the GeoGebra platform help students develop better conceptual understanding and procedural knowledge?
2. If the answer to the first research question was affirmative, how did the inquiry-based learning approach scaffolded by the GeoGebra cognitive tools help the students develop better conceptual understanding and procedural knowledge?

Research Methodology

Fifty-three primary school students participated in this study. This study focused on two classes of Grade 5 students aged 9 to 11 with similar mathematics backgrounds at the Jordan Valley St. Joseph's Catholic Primary School in Hong Kong. One class was selected randomly to be the experimental group; the other class became the control group. There were 28 students in the experimental group and 25 students in the control group. The original mathematics teacher of the control group class administered the control group. He had around 10 years of experience teaching mathematics in primary school. The researcher of this study administered the experimental group. He did not have any prior experience teaching mathematics in primary schools. The original mathematics teacher of the experimental group class acted as a teacher observer only. He was present in every class session of the experimental group throughout this study, but he did not administer any of the sessions.

The students in the experimental group used the cognitive tools developed on the GeoGebra platform to explore how to calculate the areas of a parallelogram, triangle, and trapezoid. The experimental group used the BSCS 5E Instructional Model, which was an inquiry-based

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instructional model. First, each student explored individually how to calculate the area of a particular shape using the GeoGebra cognitive tool. Students used a worksheet with topic-specific instructions to guide them in their exploration. After the students completed their individual explorations, the teacher asked them to work in pairs. After that, the teacher then asked the two pairs to join together, forming a group of four, to further discuss and explore. After all the exploration activities, the teacher summoned the students together and started facilitating a whole-class discussion. After the discussion, the teacher summarized the suggestions put forward by the students and explained to the students the correct concepts and formulas.

The teacher taught the control group students the same topics using the direct instructional approach. Control group students used neither the GeoGebra cognitive tools nor the inquiry-based learning approach.

There were one pre-test and two post-tests (namely, Post-test-01 and Post-test-02). All of them were written tests. These three tests consisted of the same set of questions on the areas of a parallelogram, triangle, and trapezoid. The students took the pre-test before the study. After that, both groups of participants attended their respective parallelogram and triangle classes. Then, they took the Post-test-01. Both groups then attended their respective trapezoid classes. Finally, they took the Post-test-02.

Six students from the experimental group and six students from the control group were selected for interviews after Post-test-02. The interviews aimed at providing supplementary information on whether the students actually possessed the conceptual understanding and procedural knowledge that the pre-test and post-tests were intended to assess.

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After the completion of the study, a semi-structured interview with the original mathematics teacher of the experimental group class was conducted. He was present in every class session of the experimental group throughout this study, but he only acted as a teacher observer without facilitating any of the sessions. This interview collected qualitative data regarding the teacher's opinions about the use of the GeoGebra cognitive tools and pedagogical approach for teaching the target topics.

The students in the experimental group were asked to complete a questionnaire after the completion of this study. The questionnaire collected qualitative data regarding the students' opinions about the use of the GeoGebra cognitive tool for their learning experience.

Significance of the Study

The major significance of this study was to provide empirical evidence to support this specific pedagogical approach through which the students could actively construct their own knowledge to explore the mathematical formulas for calculating the area of shapes with the support of prudently designed cognitive tools.

The findings of this study would be valuable to teachers as they could apply this pedagogical approach in their instructional design and to teach using the inquiry-based learning approach with support from the related cognitive tools developed on the GeoGebra platform. The findings of this study would contribute to the repository of pedagogical content knowledge for the topic regarding the areas of the parallelogram, triangle, and trapezoid. The pedagogical content knowledge could either be permeated into the cognitive tools used in this

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study or explicitly applied by the teacher during his interaction with the students during this study. Moreover, the findings of this study would contribute to the repository of knowledge for the advent of the digital classroom wave that Chan (2010) advocated. The digital classroom wave would achieve *individualization*, which would empower every student to attain the required academic level (Chan, 2010).

Organization of this Thesis

Chapter 2 presented the literature review on pedagogical theories, inquiry-based learning, GeoGebra, cognitive tools, technological pedagogical content knowledge, and the digital classroom wave. It detailed the theoretical foundation that supported this study. Chapter 3 detailed the research methodology, including the research design, different pedagogical approaches adopted by the experimental and control groups, description of the participants, assessments and instruments used in this study, data collection methods, data analyses approaches, and ethical considerations. Chapter 4 presented the analyses of the quantitative and qualitative results collected in this study, including the pre-test, post-tests, students' interviews, teacher observer's interview, and students' questionnaires. Chapter 5 presented the implications, conclusion, limitations, and future development.

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Chapter 2 – Review of Literature

Introduction

In this chapter, the factors affecting the students' conceptual understanding and procedural knowledge would be discussed. These factors were related to the pedagogical approaches which were in turn driven by the pedagogical theories. Therefore, this chapter would review the pedagogical theories and the instructional models. The cognitive tools and the technological pedagogical content knowledge would also be reviewed. The research gaps would also be stated at the end of the chapter.

De Jong and Ferguson-Hessler (1996) defined conceptual knowledge as "static knowledge about facts, concepts, and principles that applied within a certain domain" (p. 107) and defined procedural knowledge as knowledge which "contained actions or manipulations that were valid within a domain" (p. 107). Based on de Jong and Ferguson-Hessler's (1996) definitions, Baroody et al. (2007) proposed to define the conceptual knowledge as "knowledge about facts, [generalizations], and principles" (p. 123) and proposed to define the procedural knowledge as "mental actions or manipulations, including rules, strategies, and algorithms, for completing a task" (p. 123). Conceptual knowledge was also referred to as conceptual understanding, and procedural knowledge was also referred to as procedural fluency or procedural skill (Star, 2005). Students with conceptual understanding were able to extend the knowledge to new situations, remember the knowledge more easily, and reconstruct it when forgotten. Students with procedural knowledge were able to analyze the similarities and differences between computational procedures, and perform the procedures flexibly, efficiently, and accurately (Kilpatrick et al., 2001).

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Martin and Strutchens (2000) stated that conceptual understanding of area was very important "because it was one of the most commonly used measurements and it was also the basis for many models used by teachers and textbook authors to explain computational procedures" (p. 223).

Many mathematics textbooks used in the primary schools provided the computational procedure for area computation as well as good explanations on the concepts of area. For example, the textbooks explained that a square unit of any size could be used for comparing the areas of the shapes and that conventionally the size of 1 cm^2 was used for such purpose. They also provided nicely-drawn diagrams to show that two identical triangles or two identical trapezoids could form a parallelogram in order to explain why the areas of the triangles and trapezoids could be calculated using the particular sets of mathematical formulas. Mathematics teachers who taught the students using these textbooks also explained these important concepts accordingly.

With these good textbooks and good experience of mathematics teachers, our students should have developed both conceptual understanding and procedural knowledge with regard to the areas of the closed shapes. However, many researches had found that actually students did not understand the important concepts of area, such as the conservation of area (Kospentaris et al., 2011; Hart & Booth, 1984; Kamii & Kysh, 2006), square as the basic unit of measurement for areas (Kamii & Kysh, 2006), the identification of height and width (Pitta-Pantazi & Christou, 2009), and the area calculation of irregular shapes (Kamii & Kysh, 2006; Hart & Booth, 1984).

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In fact, both procedural knowledge and conceptual understanding were important to students in their learning of mathematics (Star, 2005; Baroody et al., 2007; Star, 2007; Rittle-Johnson & Schneider, 2014). There was extensive evidence indicating that conceptual understanding and procedural knowledge developed iteratively–i.e. improving procedural knowledge had positive effect on conceptual understanding and vice versa (Kilpatrick et al., 2001; Baroody et al., 2007; Star, 2007; Rittle-Johnson & Schneider, 2014). Therefore, it was critical for students to develop both conceptual understanding and procedural knowledge in the learning of mathematics.

There was a variety of factors affecting the development of students' important concepts of area. One of the factors noticed by Baturo and Nason (1996) was that the "students had been passive recipients of their mathematics instruction" (p. 262). Martin and Strutchens (2000) suggested that "the concept of area was often difficult for students to understand, perhaps due to their initial experiences in which it was tied to a formula (such as area = length × width) rather than more conceptual activities such as counting the number of square units it would take to cover a surface" (p. 223). Kospentaris et al. (2011) also had the opinion that "the premature introduction to the quantitative approach to area by use of formulas had been related to the students' difficulties in area measurement" (p. 107). Lochhead (1985) suggested that students' own theory of knowledge (i.e. their epistemology) could also be one of the factors. According to Lochhead, teachers' habit of explaining everything as clearly as they could was one of the reasons causing students to believe that simply memorizing the definitions and rules meant understanding of the subject. Lochhead (1985) was of the opinion that this instructional approach led the students to "think you either knew the answer to a question or you did not" (p. 110) and these "instructions, whether by textbook, lecture, or cookbook laboratory, placed students in the role of copiers" (p. 110). On the other hand,

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Yackel, Cobb, Wood, Wheatley and Merkel (1990) had shown that children were able to develop their own methods for solving mathematical tasks, and there was a variety of solution methods that the children could develop.

Marshall et al. (2009) attributed passive learning to the fact that explanation preceded exploration by saying that “if explanation preceded exploration, which was typical in non-inquiry instruction, students were thrust into passive learning situations that rarely challenged them to confront deficits in prior knowledge or existing alternative conceptions” (p. 509-510).

The factors mentioned above were related to the pedagogical approaches which were in turn driven by the pedagogical theories. The following subsections would review the pedagogical theories, the instructional models, the cognitive tools that could be utilized for the instruction, the technological pedagogical content knowledge, and the digital classroom wave.

Pedagogical Theories

There were two major pedagogical approaches, namely, the objectivism and the constructivism (Wu, Bieber & Hiltz, 2008). Comparing the differences of these two pedagogical theories had been the major theme of the academia (Jonassen, 2001; Cronjé, 2006; Glasser & Bassok, 1989; Leidner & Jarvenpaa, 1995).

On the one hand, objectivism believed that there existed the real world which was “external to humans and independent of human experience” (Jonassen, 2001, p. 57). The objectivist model of learning aimed at understanding this real world. It was teacher-centered and

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advocated knowledge transmission. The teacher or expert, who understood more about the objective truths, transferred the knowledge to the students or learners (Leidner & Jarvenpaa, 1995).

On the other hand, the constructivists shared the view that “reality was a construct that could not be determined independently of the observer” (Jefferies, Carsten-Stahl & McRobb, 2007, p. 113). The constructivist approach was student-centered and advocated knowledge construction. The teacher acted as a coach, and the students were active participants in learning. They collaborated with each other in constructing their knowledge (Knowlton, 2000). In other words, knowledge was constructed through social interaction. As characterized by Savery and Duffy (1996), one of the propositions of constructivism was that “knowledge evolved through social negotiation and through the evaluation of the viability of individual understandings” (p. 136). They further explained that the social negotiation and collaboration helped us to test “our own understanding and examine the understanding of others as a mechanism for enriching, interweaving, and expanding our understanding of particular issues or phenomena” (p. 136).

Jonassen (2001) viewed the objectivism and the constructivism as the polar extremes on a continuum, and he pointed out that “most theorists however took positions that fell somewhere in the middle of the continuum” (p. 57). According to Jonassen (2001), “much of the cognitive psychology and most of the instructional systems technology currently were grounded in objectivism” (p. 62). Kong (2003) noticed that “the constructivists’ view of learning had become widely accepted in recent decades” (p. 24).

Jonassen (2001) suggested that the most realistic instructional approach should fall

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somewhere on the continuum between the objectivist and constructivist positions because, in his opinion, learning comprised both of the objectivistic and constructivistic activities. Cronjé (2006) did not agree with Jonassen's view which treated objectivism and constructivism as the polar extremes on a continuum. He argued that the objectivist and constructivist approaches were complementary rather than conflicting. Nevertheless, he had a similar perspective as Jonassen by saying that "learning events could contain both objectivist and constructivist elements" (Cronjé, 2006, p. 387).

In the report "The BSCS 5E Instructional Model: Origins and Effectiveness" (Bybee et al., 2006), the authors shared the same perspective as Jonassen. They used the term "direct instruction" to denote lecturing and rote memorization, and the term "discovery learning" to denote students discovering all the knowledge themselves without direct instructions from the teachers. They explained that the Biological Sciences Curriculum Study (BSCS) 5E Instructional Model incorporated both of the direct instruction and discovery learning (Bybee et al., 2006).

Inquiry Based Learning

Inquiry-based learning was a process in which students were engaged in working out and widely investigating questions, solving the problems, and then constructing new understandings and knowledge (Alberta Learning, 2004; Kong & So, 2008). The BSCS 5E Instructional Model was an inquiry-based learning instructional model (Marshall et al., 2009; Stamp & O'brien, 2005). There was ample of empirical evidence supporting that the inquiry-based learning approach could enhance students' mastery of subject matter and cultivate their interest in learning the subjects (Bybee et al., 2006). Evidence had also shown

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that inquiry-based learning was efficacious in developing students' conceptual understanding (Haury, 1993; Boaler, 1998a; Boaler, 1998b; Bybee, 2009; Furtak et al., 2012). Boaler (1998a, 1998b) found that students under the traditional direct instructional approach developed the procedural knowledge while students under the inquiry-based learning approach developed both of the procedural and conceptual knowledge.

There were pedagogical approaches which were similar to the inquiry-based learning approach and they had been called, for example, reform-oriented teaching, inquiry mathematics or problem-based learning in the research literature (Hahkioniemi & Leppaaho, 2012). This type of instructional approaches often proceeded in three phases: (a) launch phase, (b) explore phase, and (c) discuss and summarize phase (Stein et al., 2008).

Different inquiry-based instructional models consisted of different phases. The Atkin-Karplus Learning Cycle consisted of three phases, namely, the exploration, invention, and discovery phases (Bybee et al., 2006). The BSCS 5E Instructional Model consisted of five phases, namely, the engagement, exploration, explanation, elaboration, and evaluation phases (Bybee et al., 2006). The 7E Model consisted of seven phases, namely, the elicit, engage, explore, explain, elaborate, extend, and evaluate phases (Eisenkraft, 2003). The 4E x 2 Instructional Model was based on three major constructs (i.e. metacognitive reflection, inquiry instructional models, and formative assessment) in which the inquiry instructional models consisted of four phases, namely, the engage, explore, explain, and extend phases (Marshall et al., 2009).

One of the emphases of the inquiry-based learning approach was to let the students to have opportunities to interact with the peers, discuss, explain, and justify their solutions and

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interpretations (Wittrock, 1989; King, 1991; Cobb et al., 1991; Webb & Farivar, 1994; Hendry, 1996; Gillies & Ashman, 2003; Bybee et al., 2006; Stein et al., 2008). The inquiry-based learning approach facilitated students to construct their knowledge (King, 1991); aided students to understand the subject matter (Brown & Campione, 1986); promoted a positive effect on students' learning and performance (Gillies & Ashman, 2003); helped peers to learn by means of similar language and vocabulary (Webb & Farivar, 1994); and promoted students' self-reflection on their thinking (Yackel et al., 1990).

Another emphasis of the inquiry-based learning approach was that the exploration phase should precede the explanation phase (Eisenkraft, 2003; Bybee et al., 2006; Stein et al., 2008; Marshall et al., 2009) so that the students would have the opportunity to connect new knowledge to their extant knowledge (Van De Walle et al., 2013). There was empirical evidence indicating that offering opportunities for exploration before instruction supported the development of both conceptual understanding and procedural knowledge (Rittle-Johnson & Schneider, 2014).

In view of the empirical findings of the research literature mentioned above, an inquiry-based learning model was adopted in the current study. Specifically, this study used the BSCS 5E Instructional Model because of the extensive empirical findings which indicated the BSCS 5E Instructional Model was an effective model and, in some cases, it was more effective than alternative approaches (Bybee et al., 2006; Bybee, 2009). As mentioned above, the BSCS 5E Instructional Model consisted of five phases, namely, the engagement, exploration, explanation, elaboration, and evaluation phases (Bybee et al., 2006). The engagement phase exposed the students' prior knowledge and engaged the students in the upcoming learning activities. The exploration phase provided activities for students to explore the target topic,

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establish relationships, observe patterns, and formulate concepts, processes, and skills. The explanation phase provided opportunities for students to explain their understandings to others and listen to others' explanations. The explanation phase also provided opportunities for the teacher to explain to the students the correct concepts, processes, and skills. The elaboration phase allowed the students to apply their newly acquired knowledge to closely related but new situations. The evaluation phase assessed the students' understanding on the target topic. These five phases could be run through in one or multiple lessons. They could be run in different order. In this study, the students were asked to work with their fellow students to explore, discuss, explain, and justify their solutions and interpretations. After that, the teacher facilitated the whole-class discussion and summarized the approaches suggested by the students. For more details of the research design, please refer to Chapter 3 (Research Methodology) of this report.

GeoGebra – A Cognitive Tool

De Jong (2006) suggested that computer-supported cognitive tools might solve the problems encountered by the students when they were using the inquiry-based approach to learn. Cognitive tools were mental and computational devices that could support, guide, and facilitate the learner's cognitive processing (Derry & LaJoie, 1993; Jonassen, 1992). Jonassen and Reeves (1996) referred cognitive tools as "technologies, tangible or intangible, that enhanced the cognitive powers of human beings during thinking, problem solving, and learning" (p. 693).

According to Jonassen (1992), cognitive tools were learner-controlled knowledge construction tools which facilitated learners to extend their mind and construct their

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knowledge. Jonassen and Reeves (1996) viewed computer-based cognitive tools as intellectual partners which allowed leaners to off-load the unproductive tasks to the computers so that the learners could focus on the cognitive processing that they did best. “Cognitive tools provided an environment and vehicle that often required learners to think harder about the subject matter domain being studied while generating thoughts that would be difficult without the tool” (Jonassen, 1992, p. 5). One of the best way to learn was to teach others about the subject matter (Jonassen & Reeves, 1996; Chan, 2010). Computer-based cognitive tools empowered the learners to be the instructional designers by providing the tools for the learners to analyze the world, interpret the events, organize their personal knowledge, and represent what they knew to others. (Jonassen & Reeves, 1996; Ertmer & Ottenbreit-Leftwich, 2013).

It had long been recognized that learning was mediated by tools, signs, or manipulatives (Van Hiele, 1999; Duffy & Cunningham, 1996; Cramer, Post & delMas, 2002; Van De Walle et al., 2013; Hwang & Hu, 2013). For the learning of mathematics, “the use of manipulatives had proven helpful for assisting children in further developing of their concepts, procedures, and other aspects of mathematics” (Hwang, & Hu, 2013, p. 309). Cramer et al. (2002) had conducted a study and found that students using the curriculum which placed particular emphasis on manipulatives outperformed the control group in solving the order and estimation tasks involving fractions.

Tools and manipulatives were also advocated in inquiry-based learning. In order to let the students to interact directly with the material world, the BSCS 5E Instructional Model also supported the use of “physical manipulation of substances, organisms, and systems; interactions with simulations; interactions with actual (not artificially created) data; analysis

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of large databases; and remote access to instruments and observations, for example, via World Wide Web links" (Bybee et al., 2006, p. 16). Iiyoshi et al. (2005) suggested that "cognitive tools could facilitate conceptual understanding by supporting efforts to test presumed relationships between newly organized knowledge and existing knowledge" (p. 289). For the learning of mathematics, Van De Walle et al. (2013) were of the opinion that technology "permitted students to focus on mathematical ideas, to reason, and to solve problems in ways that were often impossible without these tools. …… enhanced the learning of mathematics by allowing for increased exploration, enhanced representation, and communication of ideas" (p. 3). Kong (2011) stated that computer-supported cognitive tools could provide three types of scaffolds which could support the students in constructing their own knowledge without teacher's mediation: (a) visual representation, (b) graphical manipulation, and (c) immediate feedback. In an empirical study, Kong (2011) found that the cognitive tool could effectively support students in gaining better conceptual understanding and procedural knowledge in learning fractions. Moreover, "students demonstrated higher levels of motivation for learning mathematics when they were allowed to interact with their peers while using computer-supported cognitive tools" (Kong, 2011, p. 1852).

For the learning of geometry, some schools had already adopted the dynamic geometry software as manipulatives for their geometry curricula (Hohenwarter, Jarvis & Lavicza, 2009; Lavicza & Papp-Varga, 2010; Hwang, & Hu, 2013). The dynamic geometry software could be used to "enhance the teaching and learning of geometry. ……. enabled the teacher or individual students to generate and manipulate geometrical diagrams quickly and explore relationships using a range of examples" (Jones, 2002, p. 133). Battista (2002) provided several episodes to show how the dynamic geometry software could be used to develop students' genuine understanding and reasoning about shapes instead of passively memorizing

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the rules.

Kong and Li (2007) found that cognitive tools could facilitate learner-centered exploration and help students to gain significantly in the learning of perimeter of closed shapes. Karadag and McDougall (2011) provided several examples of using GeoGebra (a dynamic geometry software) as cognitive tool to explore, explain, and model mathematical concepts as well as the connections between the mathematical concepts. Chan et al. (2014) also provided examples of using GeoGebra to support students' exploratory learning of the concepts of the areas of the closed shapes. They proposed mathematics teachers to use GeoGebra for students' exploratory learning in order to address three common difficulties in learning the concepts of area, namely: (a) the lack of the concept of area conservation; (b) the failure to identify a base and its corresponding height for area calculation; and (c) the misconception that only regular closed shapes had measurable area and corresponding mathematical formulas for area calculation.

GeoGebra had been chosen for the current study. It was a platform on which a wide range of tools and resources related to mathematics and science could be developed. For this study, three cognitive tools had been developed on the GeoGebra platform, namely, the parallelogram cognitive tool, the triangle cognitive too, and the trapezoid cognitive tool. GeoGebra was chosen because it was free-of-charge, easy-to-use, multi-language (including Chinese language which was widely used in the schools in Hong Kong), assessable simply using the web browser, having a large user base and developer community, and promoting ongoing teacher professional learning and support (Hohenwarter et al., 2009).

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Technological Pedagogical Content Knowledge

According to Shulman (1986), the pedagogical content knowledge (PCK) included “the most powerful analogies, illustrations, examples, explanations, and demonstrations” (p. 9) that could represent the subject matter in a way which could be comprehended by the students as well as “an understanding of what made the learning of specific topics easy or difficult” (p. 9). Shulman (1986, 1987) argued that the proficient teachers possessed PCK in addition to the subject matter content knowledge and the general teaching strategies. PCK “represented the blending of content and pedagogy into an understanding of how particular topics, problems, or issues were organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (Shulman, 1987, p. 8). Figure 1 illustrated the PCK proposed by Shulman (1986).

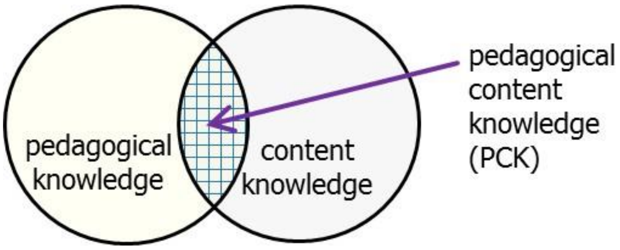

Figure 1: Pedagogical Content Knowledge (PCK).

Building on the Pedagogical Content Knowledge Framework, Koehler and Mishra (2009) explained that in developing good teaching, the technological knowledge, the pedagogical knowledge, and the content knowledge should not be treated as three separate and independent components. Instead, all these knowledge should be considered together. Mishra and Koehler (2006) called this blended knowledge the technological pedagogical content knowledge (TPCK). They further elaborated that “TPCK was the basis of good teaching with

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technology and required an understanding of the representation of concepts using technologies; pedagogical techniques that used technologies in constructive ways to teach content; knowledge of what made concepts difficult or easy to learn and how technology could help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies could be used to build on existing knowledge and to develop new epistemologies or strengthen old ones" (Mishra & Koehler, 2006, p. 1029). Figure 2 illustrated the TPCK and PCK under the framework proposed by Mishra and Koehler (2006).

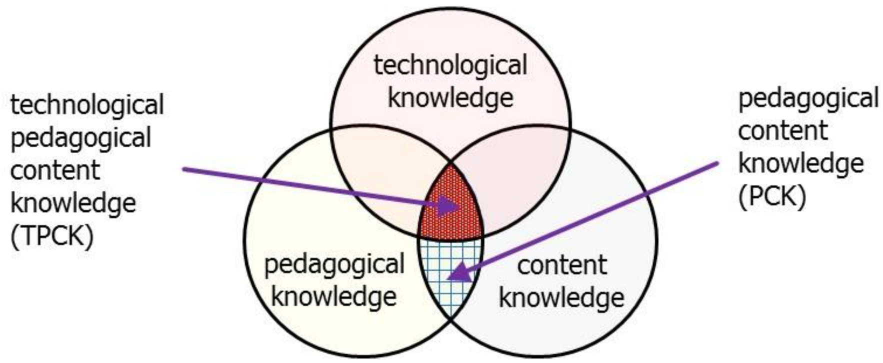

Figure 2: Technological Pedagogical Content Knowledge (TPCK) and Pedagogical Content Knowledge (PCK).

For the topic on "area of closed shapes", it was important for the students to see the relationship between parallelogram and rectangle as well as the relationship between parallelogram and triangle (or trapezoid). For example, two identical triangles could always form a parallelogram. Teachers might use two identical paper triangles to demonstrate this to the students. This was the pedagogical content knowledge (PCK) of the topic.

In the current study, technology (i.e., cognitive tools developed on the GeoGebra platform) was used so that the students could easily modify the various shapes, cut and recombine a parallelogram into rectangle, replicate an identical triangle (or trapezoid), and rotate the

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replicated triangle (trapezoid) to form a parallelogram. This was the technological pedagogical content knowledge (TPCK) of this topic which encompassed the knowledge of using technology to make concepts easy to learn. Moreover, worksheet with topic-specific instructions (which were regarded as PCK by Shulman (1987)) was provided to the students to guide them in their exploration. For example, for exploring how to calculate the area of the parallelogram, the students were provided with a worksheet with the parallelogram-specific instructions. Please refer to Appendix A for the worksheet used in this study. The teachers also exercised their pedagogical content knowledge when they facilitated the students' learning in the classes.

This study explored an effective pedagogical approach to help students to construct their conceptual understanding and procedural knowledge on the target topics by integrating the technology with the PCK of the topics.

Digital Classroom Wave

Chan (2010) believed that the digital classroom wave was imminent. This wave would be made possible by the availability of e-books and e-boards. Chan (2010) referred e-books as any computing devices that the students used in a classroom, intended to replace the current printed textbooks. According to Chan (2010), in the digital classroom, the students would learn directly from their e-books or they would learn together in groups mediated by the e-books. By that time, the teachers would no longer need to spend most of their time in instructional activities as the e-book had shared their teaching workload. The role of the teacher would change to becoming the personal mentor of each of the students. Under the digital classroom wave, *individualization* would be achieved which would empower every

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student to fulfill the level of academic achievement required by the standard curriculum (Chan, 2010). The findings of this study would contribute to the repository of knowledge for the advent of the digital classroom wave advocated by Chan (2010).

Research Gaps

However, there was a paucity of literature examining the effect of integrating cognitive tools with inquiry-based learning on students' conceptual understanding and procedural knowledge. Moreover, there had been little investigation on the primary school students regarding their exploration and construction of the formulas for the calculation of the areas of closed shapes. This study aimed to fill these gaps in the literature.

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Chapter 3 – Research Methodology

Introduction

As mentioned in the previous chapters, there was a paucity of literature examining the effect of integrating cognitive tools with inquiry-based learning on students' conceptual understanding and procedural knowledge. Moreover, there had been little investigation on the primary school students regarding their exploration and construction of the formulas for the calculation of the areas of closed shapes. This study aimed to fill these gaps in the literature.

In this research, empirical study was conducted to answer the following two research questions regarding the primary school students' learning of the areas of parallelogram, triangle and trapezoid:

1. Compared with the direct instructional approach, would the inquiry-based learning approach scaffolded by cognitive tools developed on the GeoGebra platform help students develop better conceptual understanding and procedural knowledge?
2. If the answer to the first research question was affirmative, how did the inquiry-based learning approach scaffolded by the GeoGebra cognitive tools help the students develop better conceptual understanding and procedural knowledge?

In this research, the students were asked to take the pre-test and post-tests. The results collected from these tests provided empirical data which could be analyzed quantitatively using statistical procedures and provided objective outcomes. In addition to the quantitative data, qualitative data were also collected through teacher observer's interview, students'

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interviews, and students' questionnaire. The qualitative data could provide further information for the understanding of the outcomes of the quantitative analyses.

Background of the Study Site

This was a quasi-experimental study with experimental and control groups. However, the two groups of participants were not randomly assigned to groups for different treatment. This study took place in October and November of 2015. The study site, Jordan Valley St. Joseph's Catholic Primary School, had two classes of Grade 5 students who had similar mathematics background. One class was selected randomly to be the experimental group and the other class was the control group.

The students in the experimental group used the cognitive tools developed on the GeoGebra platform to explore how to calculate the areas of the parallelogram, triangle, and trapezoid. Every Grade 5 student in this school possessed a Samsung Galaxy Tab tablet and the GeoGebra cognitive tools were installed in the students' tablets. By means of GeoGebra, the students in the experimental group could perform the explorations interactively by cutting the shapes into smaller pieces, manipulating them, and re-combining the pieces to form different shapes onscreen. The BSCS 5E Instructional Model was used in the experimental group. The students in the control group was taught how to calculate the areas of the parallelogram, triangle, and trapezoid through the direct instructional approach. No GeoGebra cognitive tools were used in the control group and the inquiry-based learning approach was not adopted. The control group would provide data for comparing the effectiveness of the proposed pedagogical approach with that of the traditional instructional method.

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This study was conducted during regular school hours. Each class session lasted for 35 minutes. For both of the experimental and control groups, the same number of class sessions were spent on each of the topics, namely parallelogram, triangle, and trapezoid. Therefore, the learning time of both groups were the same. Both groups attended 8 class sessions. Table 1 summarized the number of class sessions spent on each topic.

Table 1: Number of class sessions spent on each topic.

Topic	Class sessions spent on the topic
parallelogram	4 class sessions (i.e. 140 minutes)
triangle	2 class sessions (i.e. 70 minutes)
trapezoid	2 class sessions (i.e. 70 minutes)

Research Design

Experimental Group

The BSCS 5E Instructional Model was adopted for the experimental group. The model consisted of five phases, namely, the engagement, exploration, explanation, elaboration, and evaluation phases (Bybee et al., 2006).

For the experimental group, every two class sessions formed a “teaching cycle”. There were four teaching cycles in total and they were: (a) parallelogram - part A; (b) parallelogram - part B; (c) triangle; and (d) trapezoid. At the start of each teaching cycle, the teacher briefly explained to the students the purpose of that particular teaching cycle and engaged them on the instructional tasks (i.e. the **engagement phase**). The engagement phase was followed by the exploration phase. The **exploration phase** started with individual exploration in which

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each student explored individually using the GeoGebra cognitive tools. The exploration was a guided exploration as there was evidence indicating that guided exploration was more effective than unguided ones (Angeli & Valanides, 2009). Worksheet was provided to the students to provide guidance for them to conduct their exploration. For example, for exploring how to calculate the area of the parallelogram, a worksheet with the following guidance was provided to the students (this guidance was modified from Pitta-Pantazi & Christou, 2009, p. 13):

1. Can you measure the area of the parallelogram? (Hint: It may be helpful if you can rearrange it into a rectangle.)
2. Measure the area of the rectangle you have created by counting the number of squares in the rectangle. What do you observe?
3. What is the relationship between the base and height of the rectangle and the area of the rectangle?
4. What is the relationship between area of the rectangle and area of the original parallelogram?
5. What is the relationship between the base of the rectangle and the base of the original parallelogram?
6. What is the relationship between the height of the rectangle and the height of the original parallelogram?
7. Can you describe a way in which we can find the area of a parallelogram?
8. Complete the following: Area of Parallelogram = _______________
9. How many pairs of base and height of the parallelogram can be found?
10. How many ways can be used to find the area of parallelogram?

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Please refer to Appendix A for the worksheet used in each of the teaching cycles. Following the guidance on the worksheet, the students would manipulate the corresponding GeoGebra cognitive tools to perform the individual exploration. Please refer to the next subsection for details of the cognitive tools used in this study.

After the completion of the individual exploration, the teacher asked the students to work in pairs. Each pair of students discussed with each other to solve the unresolved issues that they had encountered during the individual exploration. They also explained to each other their findings and justifications (i.e. the continuation of the **exploration phase** plus the **explanation phase**). After that, the teacher asked two pairs to join together, forming a group of four. The two pairs exchanged their findings and justifications (i.e. the continuation of the **exploration phase** plus the **explanation phase**). After all the exploration activities, the teacher summoned the students together and started facilitating a whole-class discussion. After the discussion, the teacher summarized the suggestions put forward by the students and explained to the students the correct concepts and formulas (i.e. also the **explanation phase**). In order for the students to work effectively during the small group discussions, the following ground rules were introduced to the students (Webb and Farivar (1994) regarded them as helping behavior):

- everyone should have equal participation and equal opportunity to explain his or her ideas
- attentive listening
- indicated agreement or disagreement
- elaborated to the peer students instead of giving the answer only
- asked peer students to explain their ideas

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– no insulting

– no yelling

Table 2: Activities that the experimental group performed.

Class session	Activities	Phases of BSCS 5E Instructional Model	Actual date the activities occurred	Duration (minutes)
1	Pre-test		6 Oct 2015	30
2 - 3	Teaching Cycle 1 – Introduction of the ground rules – Discover the formula for calculating the area of parallelogram (i.e. Parallelogram - Part A)	engagement, exploration, explanation	15 Oct 2015	70
4 - 5	Teaching Cycle 2 – Further discover the characteristics of the area of parallelogram (i.e. Parallelogram - Part B)	engagement, exploration, explanation	19 Oct 2015 & 20 Oct 2015	70
6 - 7	Teaching Cycle 3 – Discover how to find the area of triangle	engagement, exploration, explanation	22 Oct 2015	70
8	Post-test-01	elaboration, evaluation	26 Oct 2015	30
9 - 10	Teaching Cycle 4 – Discover how to find the area of trapezoid	engagement, exploration, explanation	27 Oct 2015 & 28 Oct 2015	70
11	Post-test-02	elaboration, evaluation	29 Oct 2015	30

Table 2 summarized the activities that the experimental group performed in this study as well as the actual dates on which the activities occurred. Each class session lasted for 35 minutes. As shown in Table 2, there were one pre-test and two post-tests (namely Post-test-01 and Post-test-02). The students were given 30 minutes to complete each of these tests. These three tests consisted of the same set of questions on the areas of parallelogram, triangle, and trapezoid. For details of these tests, please refer to the section “Assessment and Instrument” in this chapter.

The two post-tests were conducted to evaluate the conceptual understanding and procedural

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knowledge of the students with regard to the target topics (i.e. the **elaboration** and **evaluation phases**). Both groups of participants attended their respective parallelogram and triangle classes first. Then, they took the Post-test-01. After that, both groups continued the study and attended their respective trapezoid classes. Finally, they took the Post-test-02.

GeoGebra Cognitive Tools

Figure 3 to Figure 8 illustrated the parallelogram cognitive tool used by the students in the experimental group. They manipulated it by following the guidance on the parallelogram worksheet (please refer to Appendix A1 and A2 for the parallelogram worksheet).

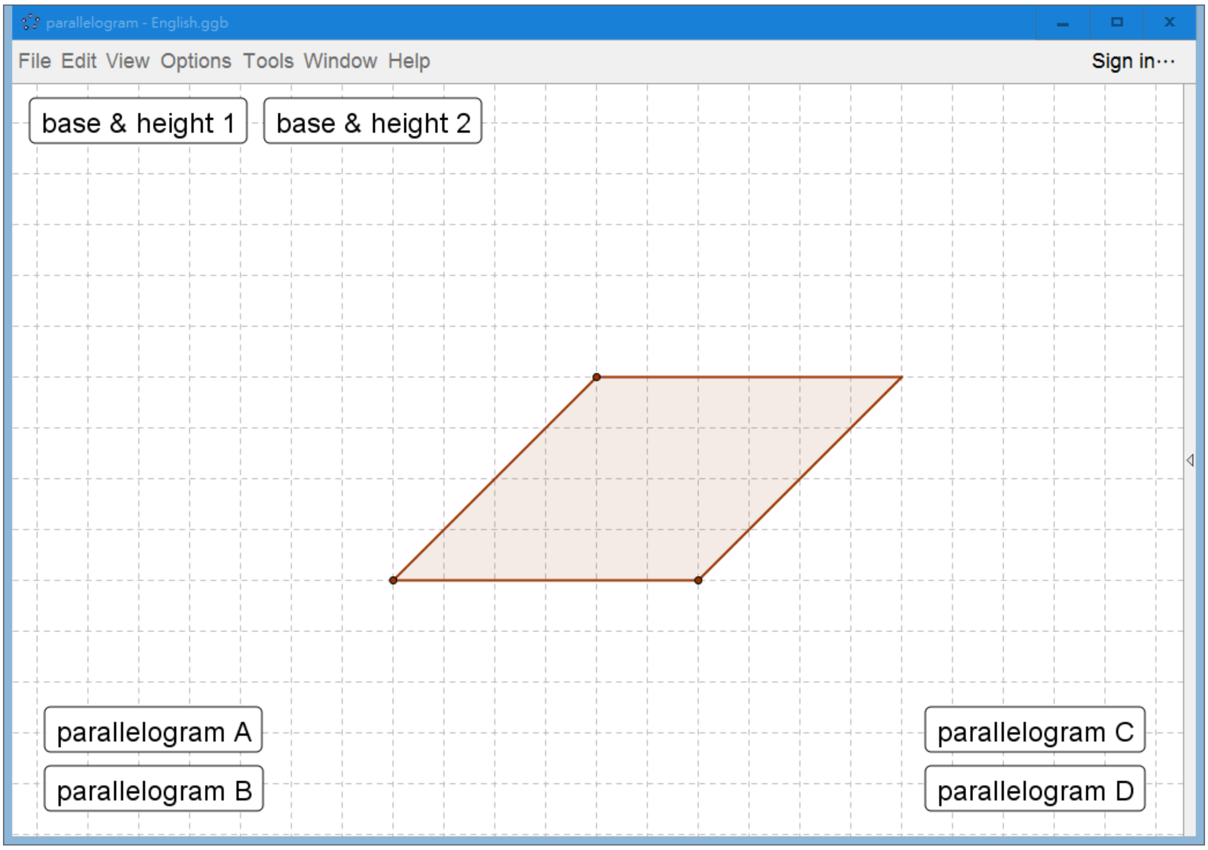

Figure 3: The initial screen of the parallelogram cognitive tool.

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Figure 3 showed the initial screen of the parallelogram cognitive tool. Figure 4 and Figure 5 illustrated that the parallelogram cognitive tool displayed one of the two sets of heights and bases when the students clicked the corresponding “base and height” button. The cognitive tool allowed the students to cut the parallelogram into two pieces and slide one of the pieces to turn the parallelogram into a rectangle (as illustrated in Figure 6). By turning the parallelogram into a rectangle dynamically using the cognitive tool, the students could explore the mathematical formula used for calculating the area of the parallelogram.

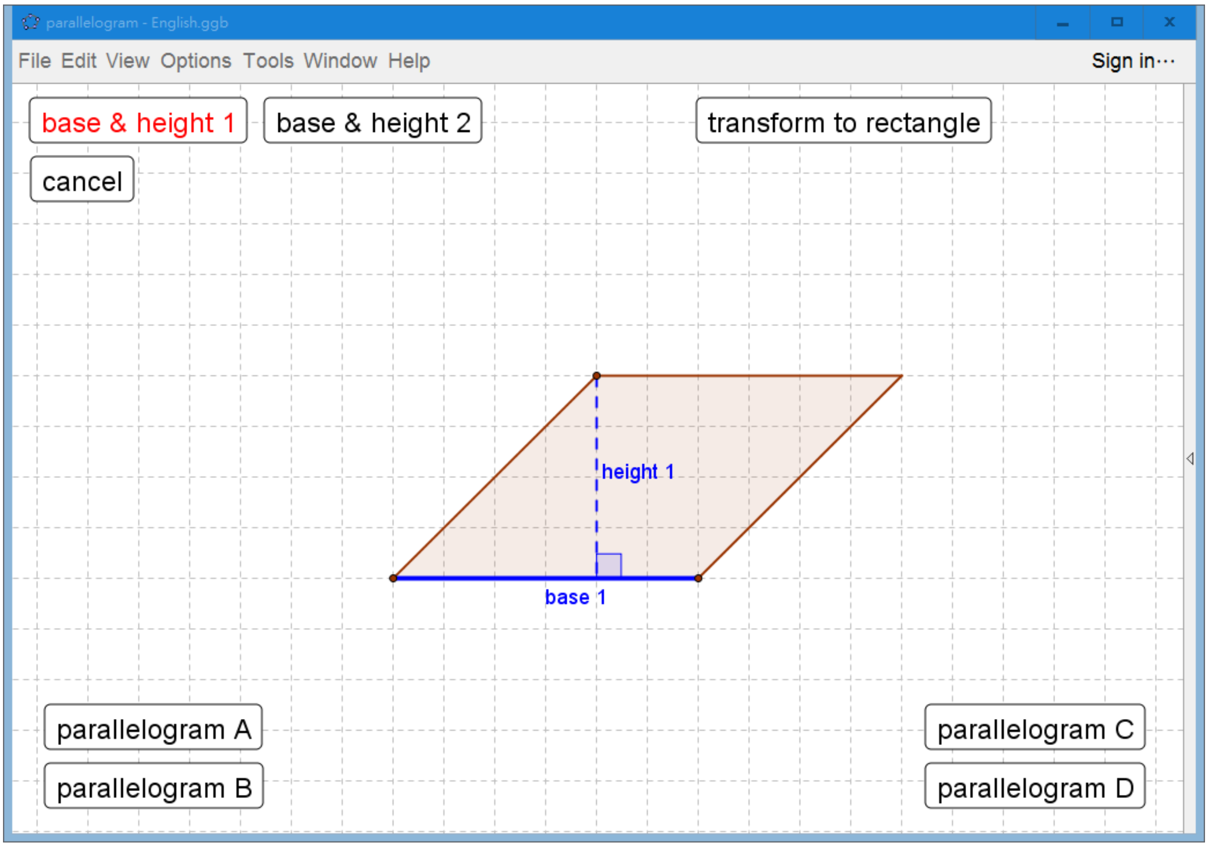

Figure 4: The parallelogram cognitive tool displayed the first set of height and base if the student clicked the “base & height 1” button.

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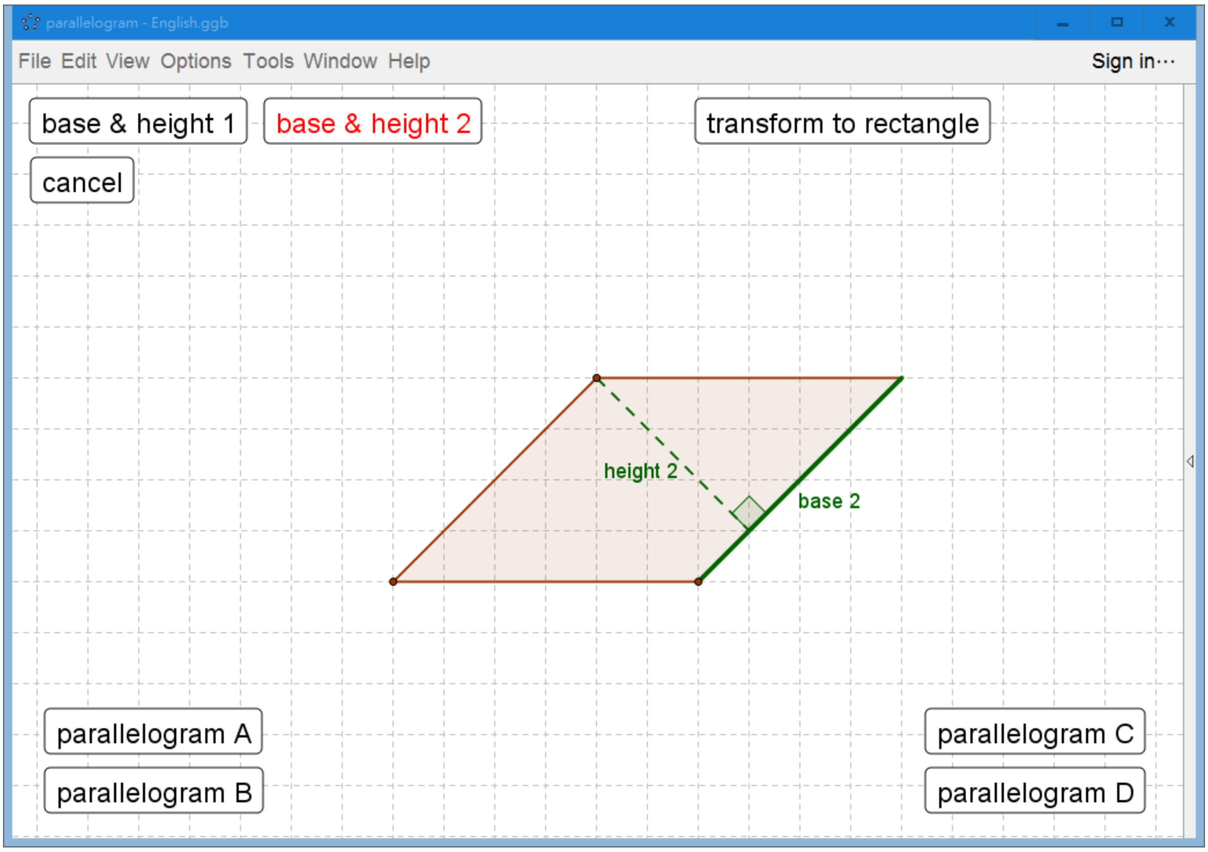

Figure 5: The parallelogram cognitive tool displayed the second set of height and base if the student clicked the "base & height 2" button.

The students could drag the vertices of the parallelogram to change its shape (Figure 7). By doing so, the students could observe how the corresponding height and base were changed dynamically so as to explore their relationship, especially the fact that the height and base were perpendicular to each other. Figure 8 showed the four different shapes of parallelograms which were predefined in the cognitive tool. By comparing these predefined shapes, the students could see that as long as the heights and bases of the two parallelograms were identical, their areas would be the same even though their areas might look different.

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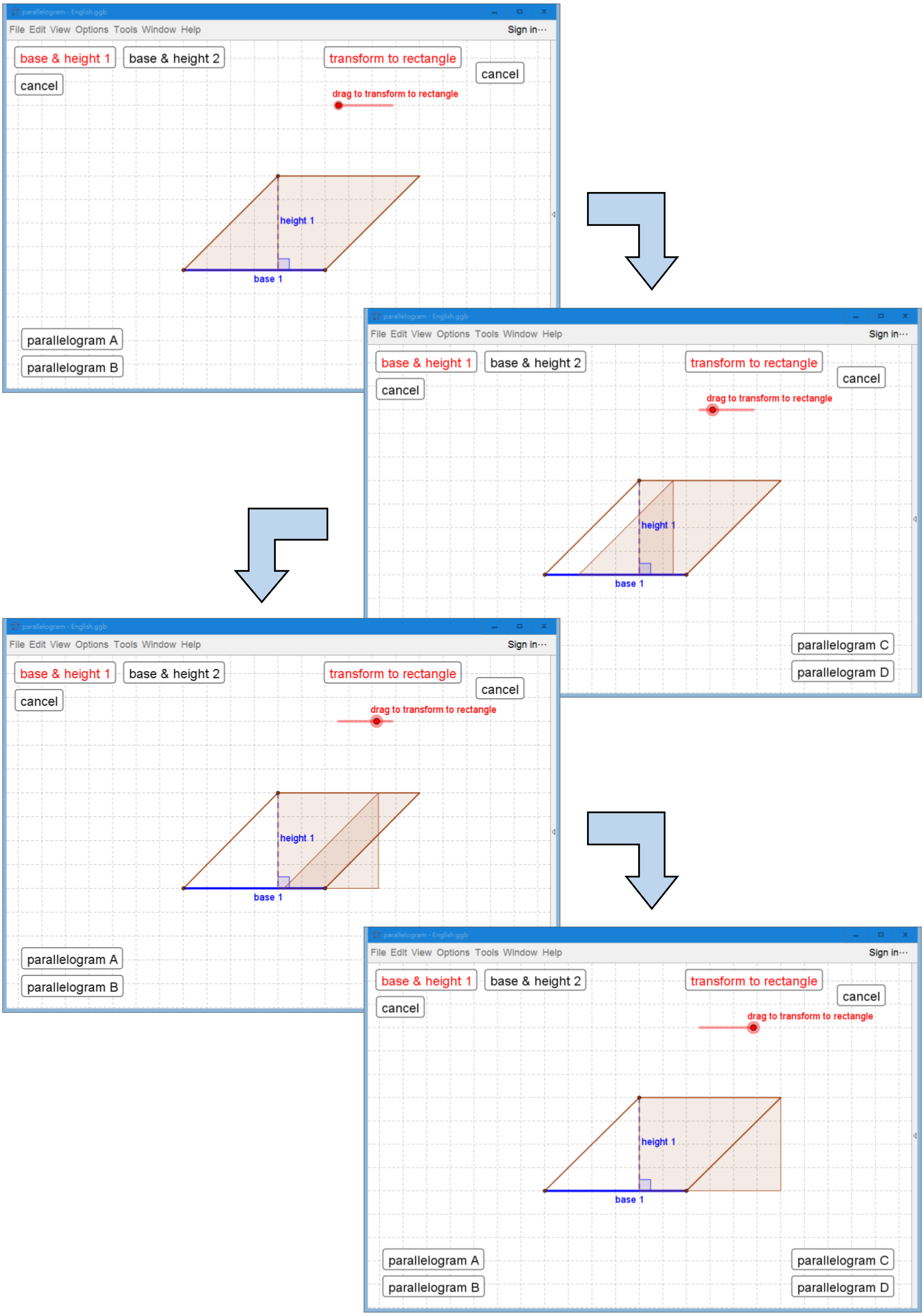

Figure 6: The parallelogram cognitive tool allowed students to cut the parallelogram into two pieces and slide one of the pieces to turn the parallelogram into a rectangle.

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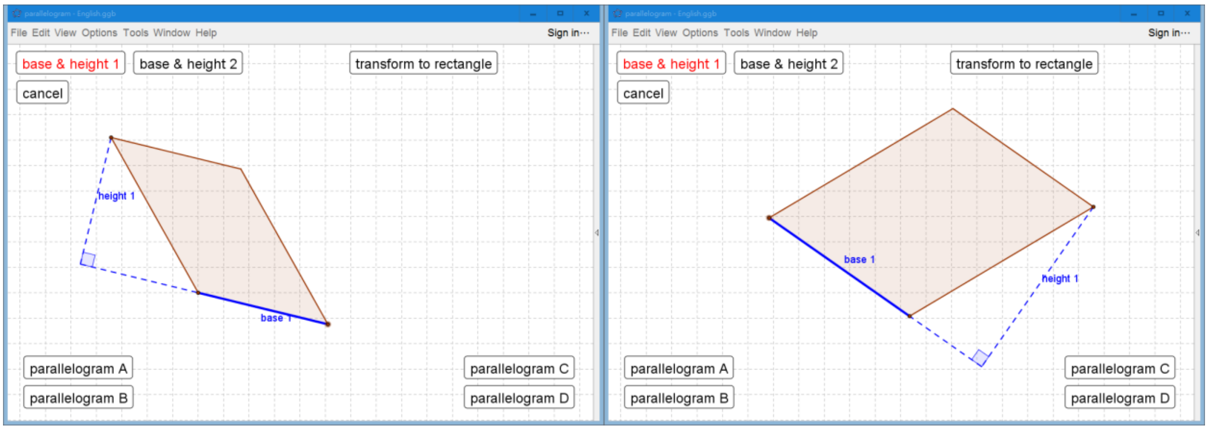

Figure 7: The parallelogram cognitive tool allowed students to change the shape of the parallelogram.

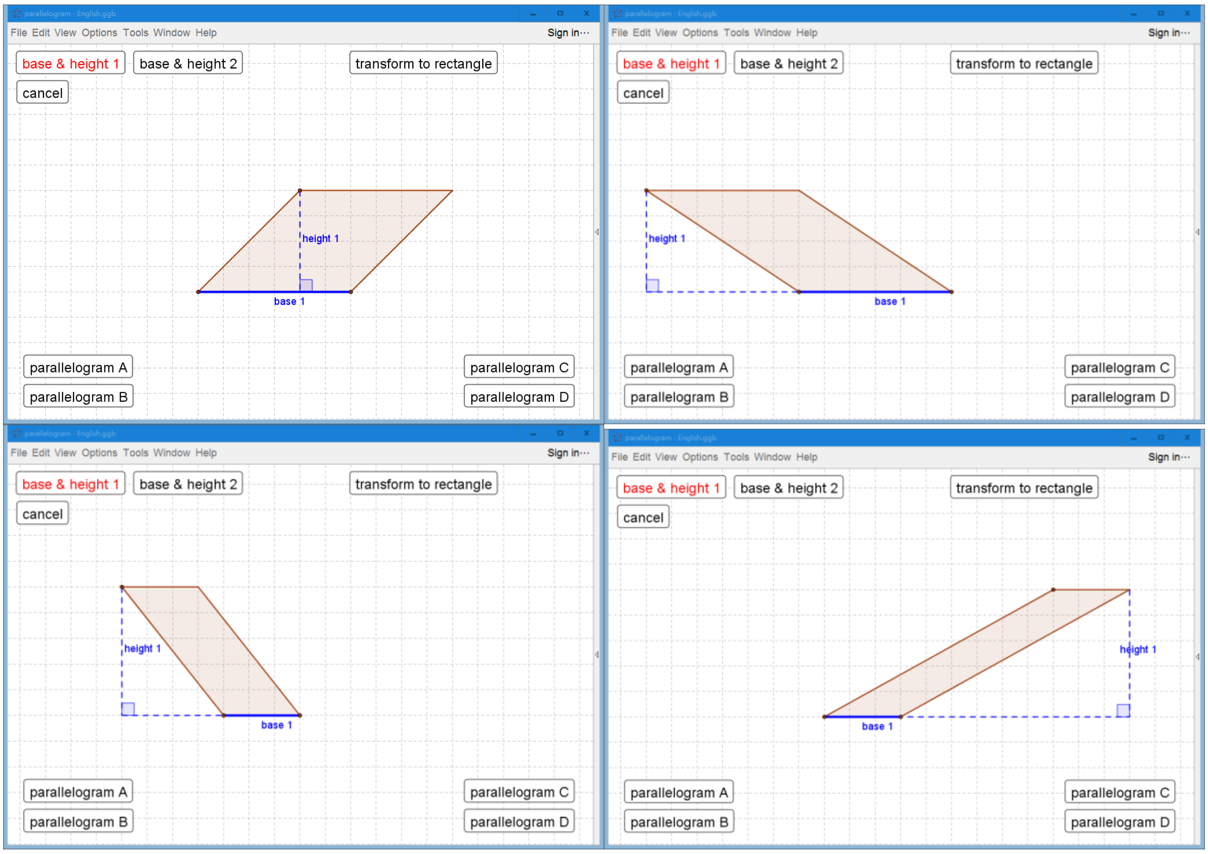

Figure 8: Four different shapes of parallelograms predefined in the cognitive tool.

Figure 9 to Figure 13 illustrated the triangle cognitive tool used by the students in the experimental group. They manipulated it by following the guidance on the triangle worksheet (please refer to Appendix A3 for the triangle worksheet).

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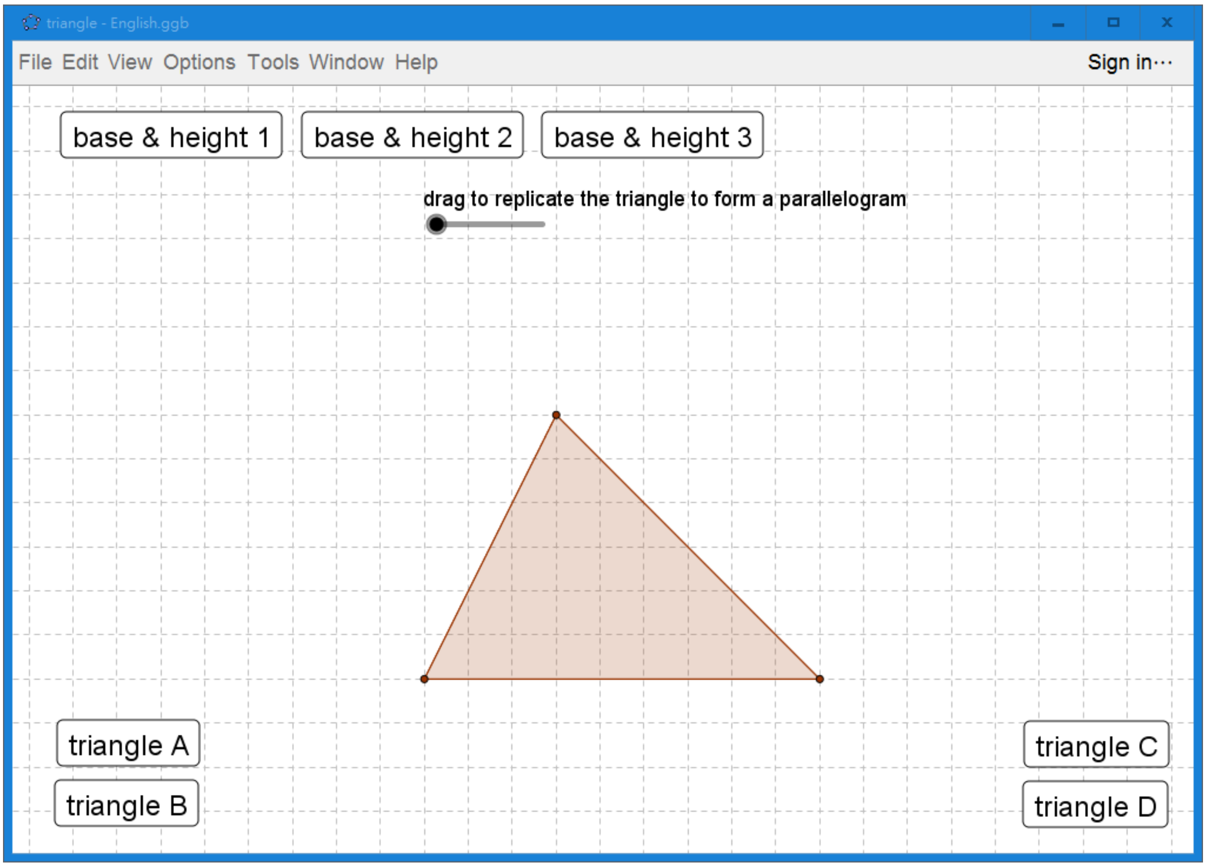

Figure 9: The initial screen of the triangle cognitive tool.

Figure 9 showed the initial screen of the triangle cognitive tool. Figure 10 illustrated that the triangle cognitive tool displayed one of the three sets of heights and bases when the students clicked the corresponding “base and height” button. The cognitive tool allowed the students to replicate an identical triangle and rotate the replicated triangle in order to form a parallelogram with these two identical triangles (as illustrated in Figure 11). By forming a parallelogram dynamically using the cognitive tool, the students could explore the mathematical formula used for calculating the area of the triangle.

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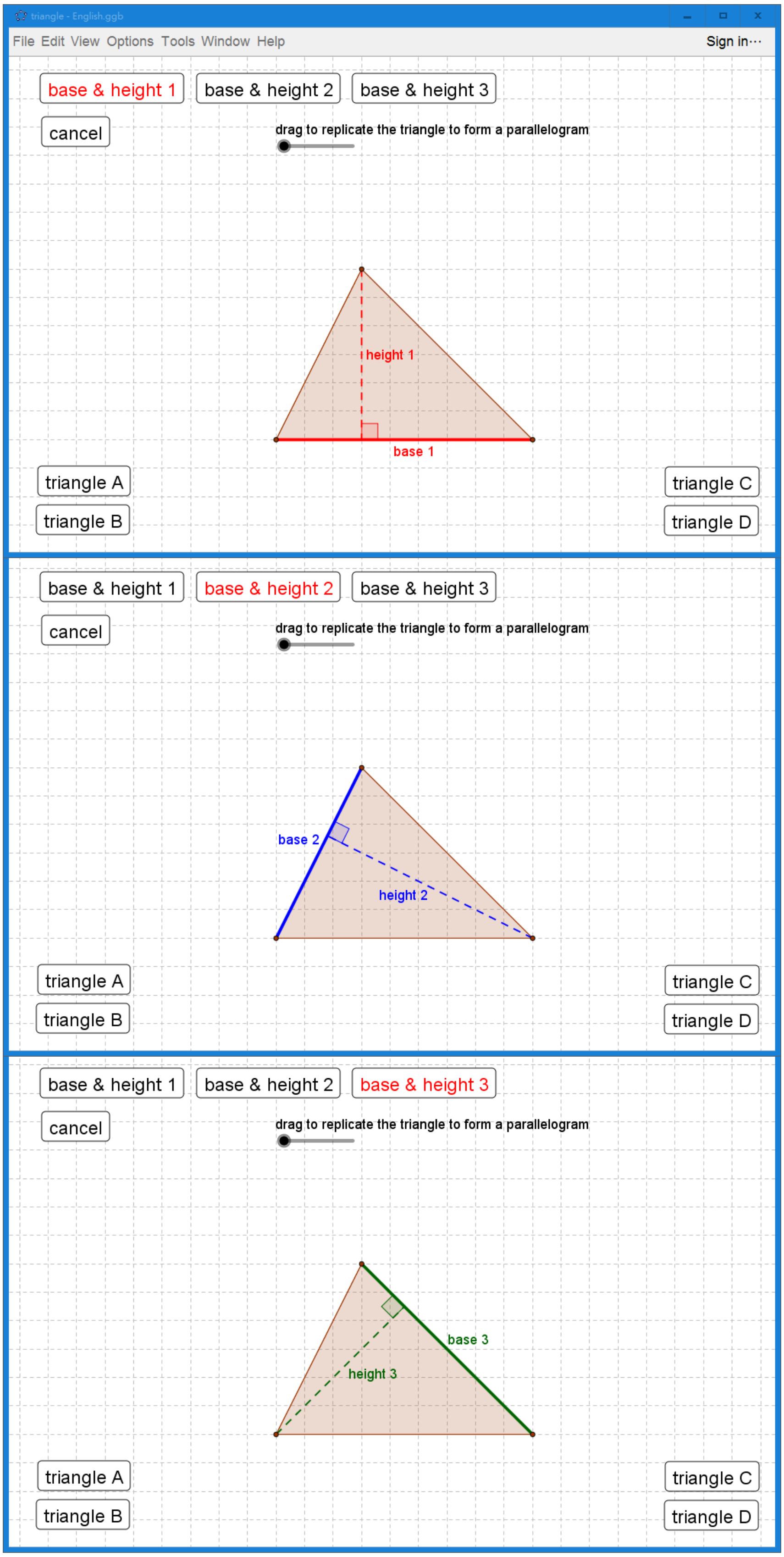

Figure 10: Triangle cognitive tool displayed one of the three sets of heights and bases when students clicked the corresponding "base & height" button.

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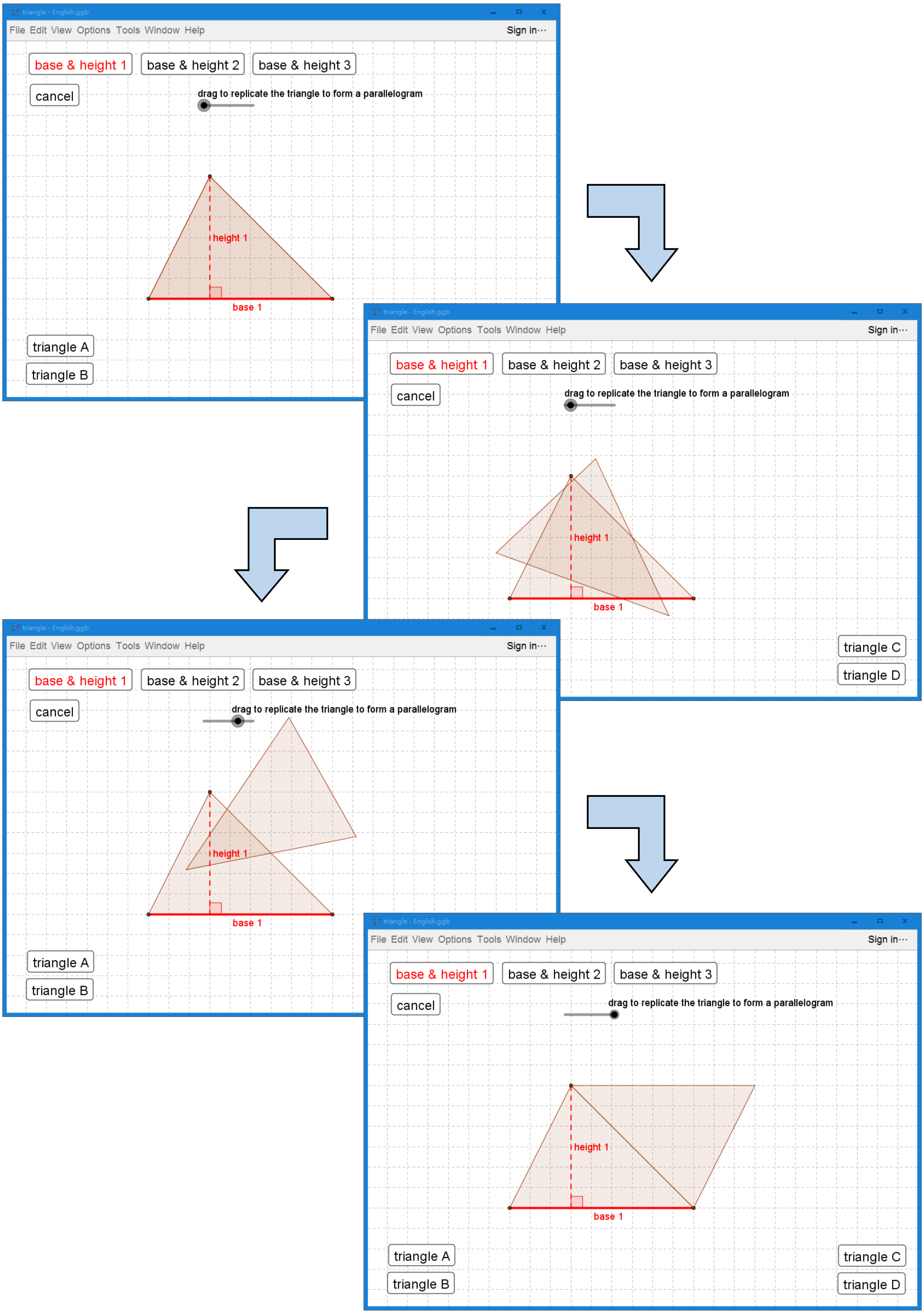

Figure 11: The cognitive tool allowed the students to replicate an identical triangle and rotate the replicated triangle to form a parallelogram with these two identical triangles.

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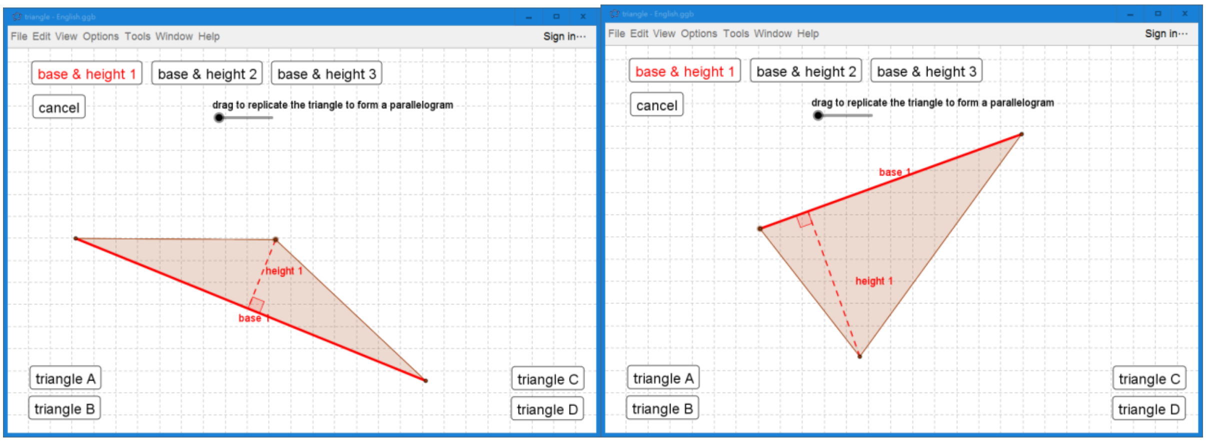

Figure 12: The triangle cognitive tool allowed students to change the shape of the triangle in order to explore the relationship between height and base.

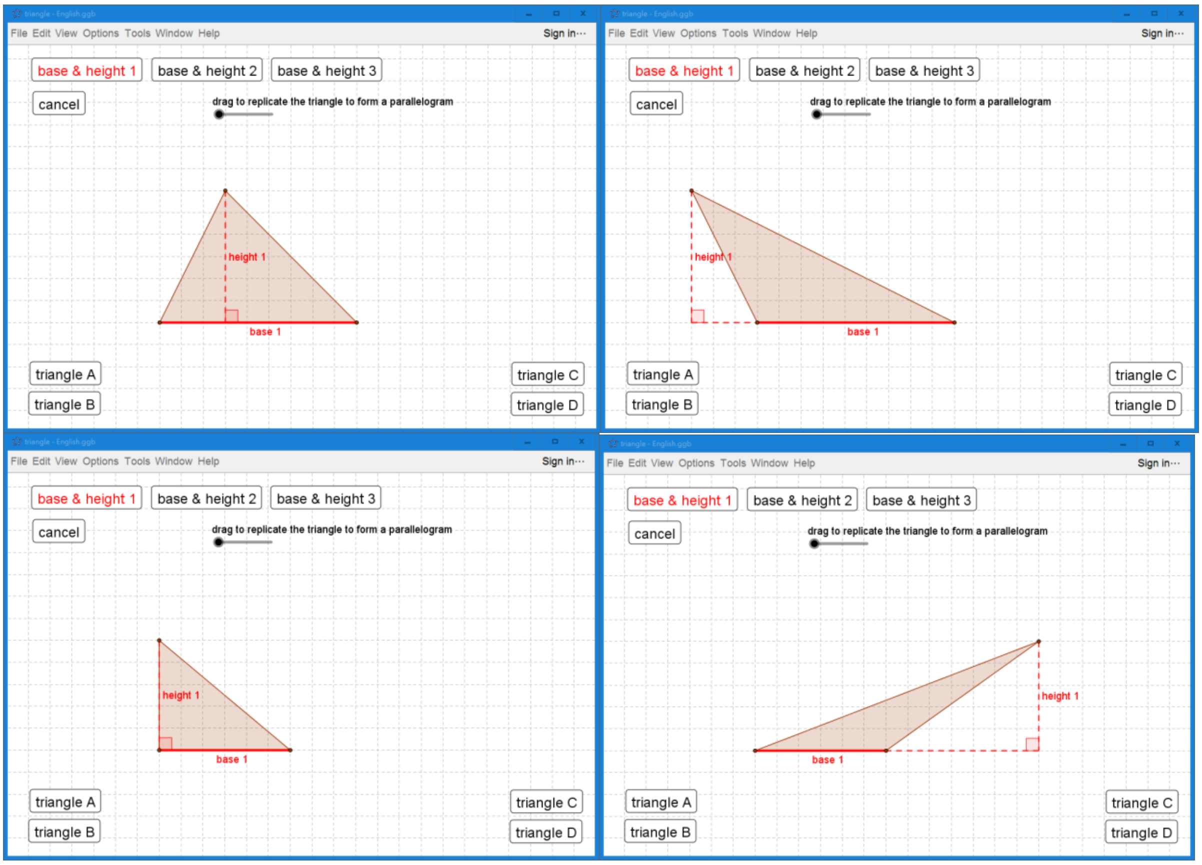

Figure 13: Four different shapes of triangles predefined in the cognitive tool.

The students could drag the vertices of the triangle to change its shape (Figure 12). By doing

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so, the students could observe how the corresponding height and base were changed dynamically so as to explore their relationship, especially the fact that the height and base were perpendicular to each other. Figure 13 showed the four different shapes of triangles which were predefined in the cognitive tool. By comparing these predefined shapes, the students could see that as long as the heights and bases of the two triangles were identical, their areas would be the same even though their areas looked different.

Figure 14 to Figure 18 illustrated the trapezoid cognitive tool used by the students in the experimental group. They manipulated it by following the guidance on the trapezoid worksheet (please refer to Appendix A4 for the trapezoid worksheet). Figure 14 showed the initial screen of the trapezoid cognitive tool.

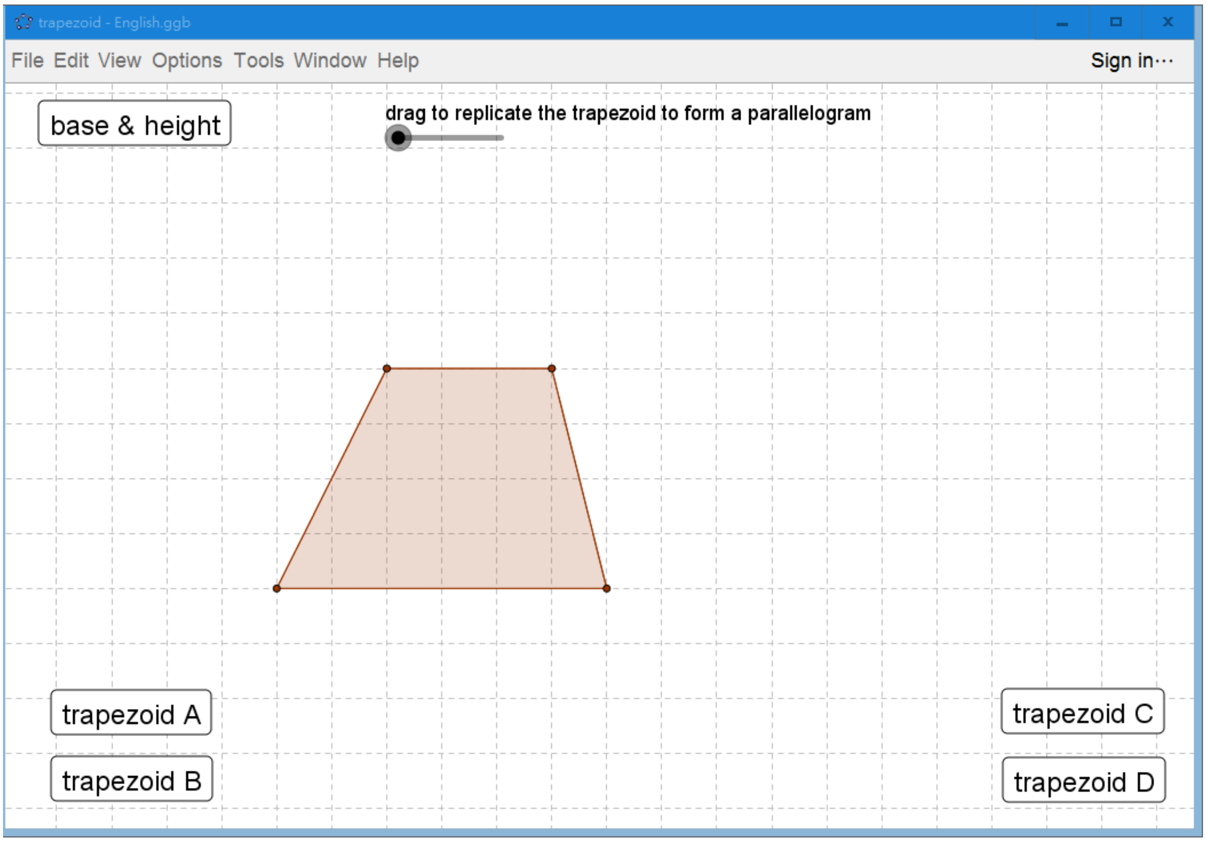

Figure 14: The initial screen of the trapezoid cognitive tool.

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Figure 15 illustrated that the trapezoid cognitive tool displayed the height, upper base and lower base when the students clicked the "height and bases" button.

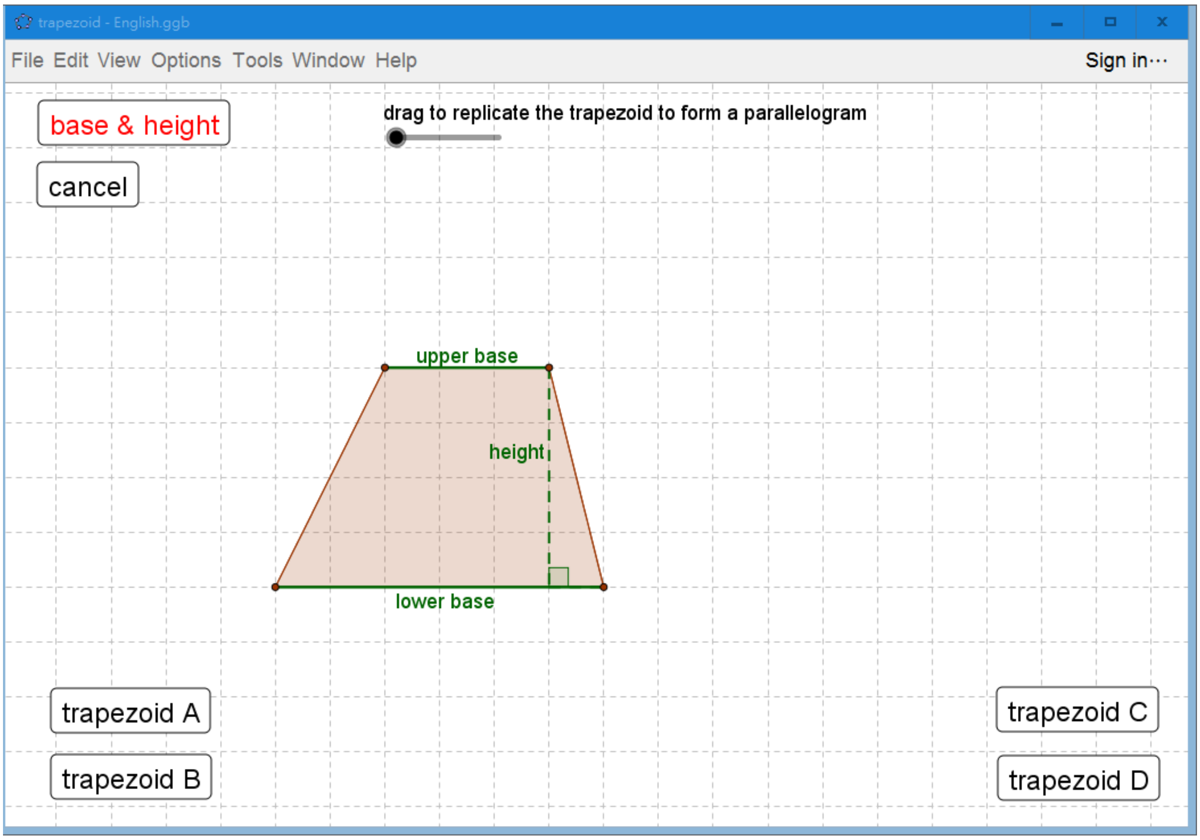

Figure 15: The trapezoid cognitive tool displayed the height, upper base and lower base when the students clicked the "base & height" button.

The cognitive tool allowed the students to replicate an identical trapezoid and rotate the replicated trapezoid in order to form a parallelogram with these two identical trapezoids (as illustrated in Figure 16). By forming a parallelogram dynamically using the cognitive tool, the students could explore the mathematical formula used for calculating the area of the trapezoid.

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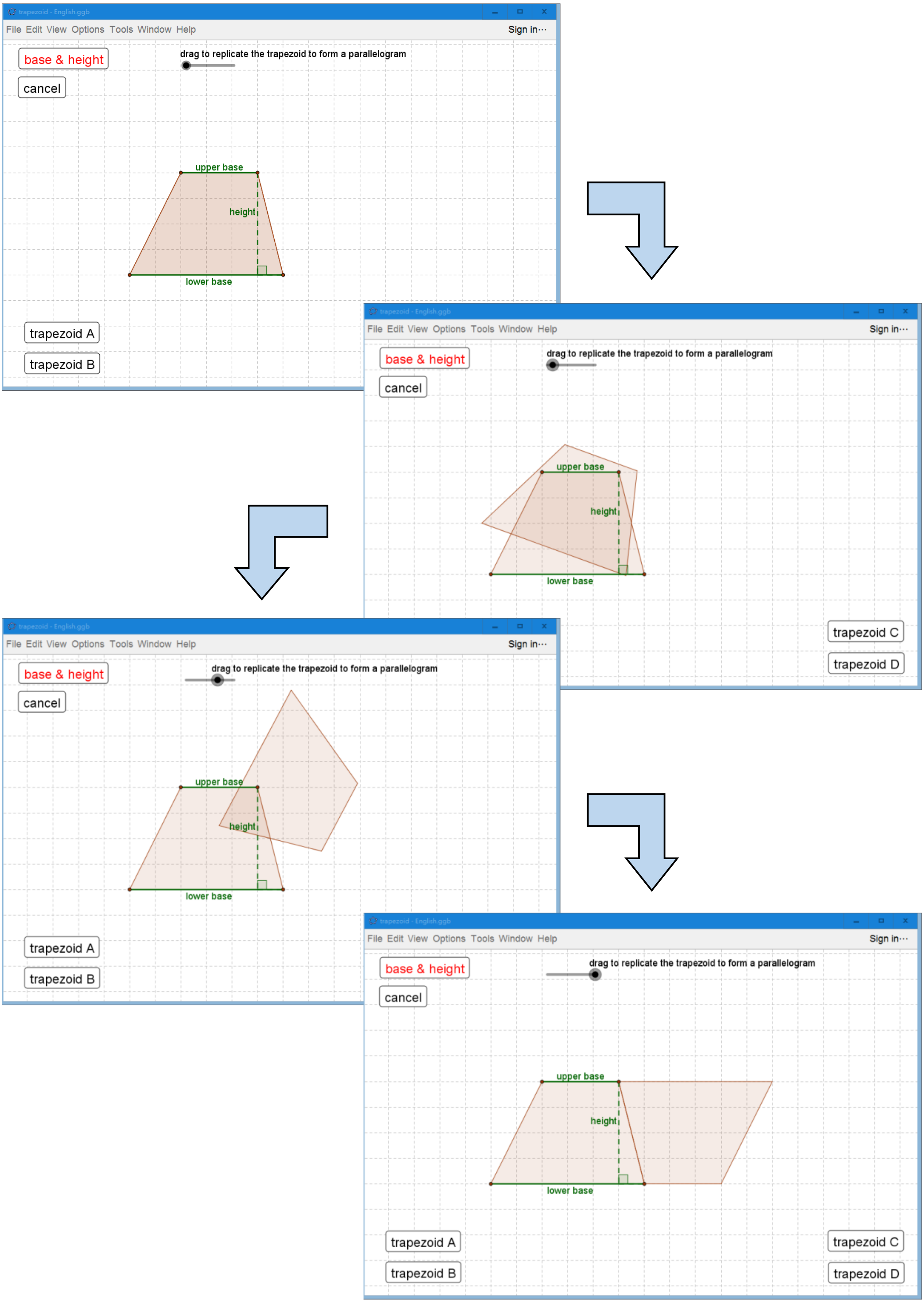

Figure 16: The cognitive tool allowed the students to replicate an identical trapezoid and rotate the replicated trapezoid to form a parallelogram with these two identical trapezoids.

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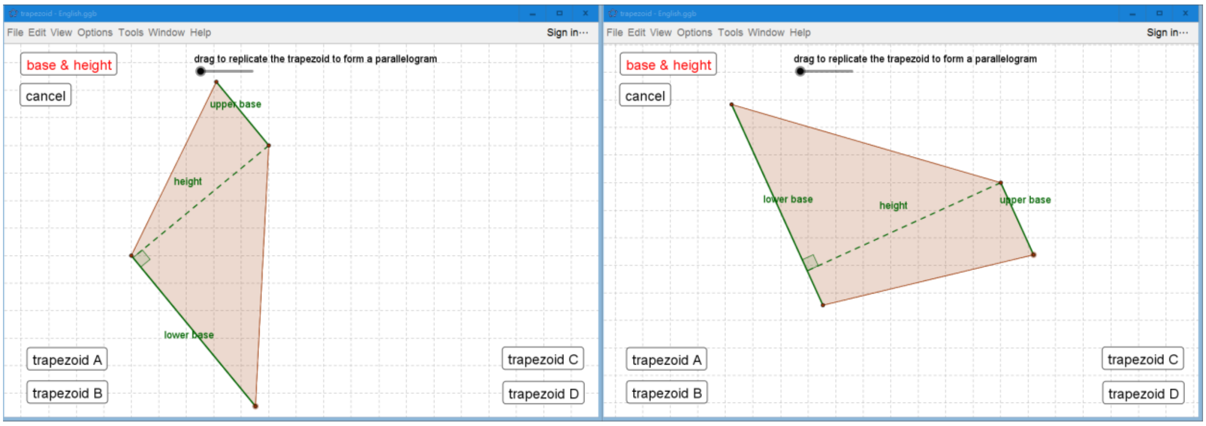

Figure 17: The trapezoid cognitive tool allowed students to change the shape of the trapezoid in order to explore the relationship between height and bases.

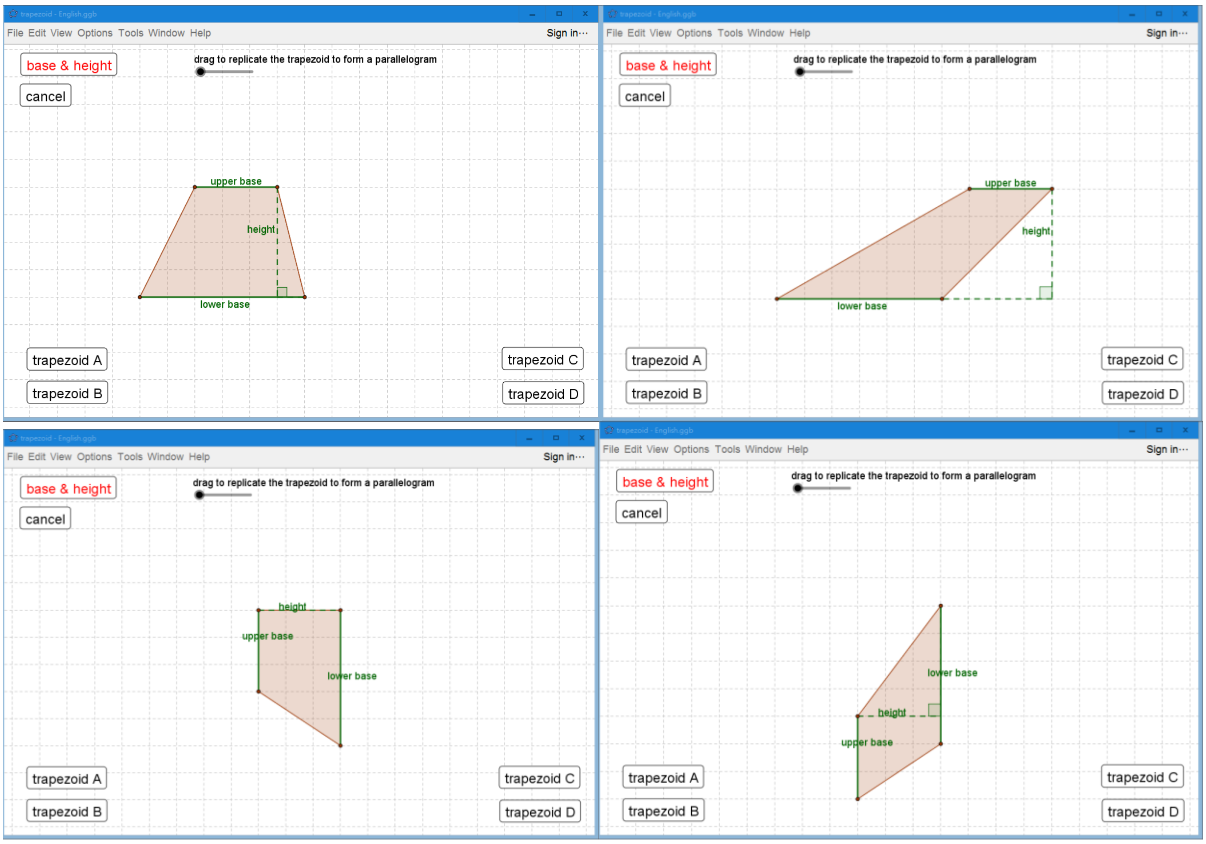

Figure 18: Four different shapes of trapezoids predefined in the cognitive tool.

The students could drag the vertices of the trapezoid to turn it into different shapes of

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trapezoid (Figure 17). By doing so, the students could observe how the corresponding height and bases were changed dynamically so as to explore their relationship, especially the fact that the height was perpendicular to both of the upper and lower bases. Figure 18 showed the four different shapes of trapezoid which were predefined in the cognitive tool. By comparing these predefined shapes, the students could see that as long as the heights and bases of the two trapezoids were identical, their areas would be the same even though their areas looked different.

Control Group

For the control group, the classes resembled the traditional school classes in which the teacher provided direct instructions regarding the concepts, the procedure and the formulas for the calculation of the areas of parallelogram, triangle and trapezoid.

Table 3: Activities of the control group.

Class Session	Activities	Actual date the activities occurred	Duration (minutes)
1	Pre-test	12 Oct 2015	30
2 - 5	Direct instruction on the concepts and computational procedure of the area of parallelogram	19 Oct 2015, 20 Oct 2015, 22 Oct 2015 & 23 Oct 2015	140
6 - 7	Direct instruction on the concepts and computational procedure of the area of triangle	26 Oct 2015 & 27 Oct 2015	70
8	Post-test-01	27 Oct 2015	30
9 - 10	Direct instruction on the concepts and computational procedure of the area of trapezoid	28 Oct 2015 & 29 Oct 2015	70
11	Post-test-02	29 Oct 2015	30

First of all, the teacher explained to the students the basic concepts of the topic. He then

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presented to the students the formula used for calculating the area of the particular shape. After that, he chose a few questions from the textbook and demonstrated to the student how the area could be calculated using the formula. No cognitive tools were used in the control group and the classes were not conducted using the inquiry-based learning approach. Table 3 summarized the activities of the control group. Each class session lasted for 35 minutes. Same as those students in the experimental group, the students in the control group were also asked to complete the same pre-test, Post-test-01, Post-test-02. The students were given 30 minutes to complete each of these tests. The test results of the control group would provide data for comparing the effectiveness of the proposed pedagogical approach with that of the traditional instructional method.

In-class Exercise and Homework

For both of the experimental and control groups, the students were asked to do in-class exercises and homework as deemed fit by their class teachers. The exercises contained questions on the topics of parallelogram, triangle, and trapezoid. Please refer to Appendix B for the sample in-class exercises and homework questions that the students were asked to do.

The experimental group students completed 54 questions in total, and the control group students finished 69 questions in total. Therefore, the control group students completed 28% more questions than the experimental group students. The differences in the exercises were mainly due to the school teachers' preference and their considerations on the progress of the classes. Table 4 and Figure 19 summarized the actual number of questions on each topic that each of the groups completed during different period of this study. Before taking the Post-test-01, the students in the experimental group completed 26 questions on the topic of

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parallelogram while they did not do any questions on the topic of triangle. On the other hand, before taking the Post-test-01, the control group completed 26 questions on parallelograms and 10 questions on triangles. Before taking the Post-test-02, the experimental group did not answer any questions on the topic of trapezoid while the control group answered 16 questions on trapezoid.

Table 4: The actual number of questions on each topic that each group had done during different period.

Period	Group	Topics			Total
		Parallelogram	Triangle	Trapezoid	
Before Post-test-01	Experimental	26	-	-	26
	Control	26	10	-	36
After Post-test -01 & before Post-test-02	Experimental	5	23	-	28
	Control	4	13	16	33

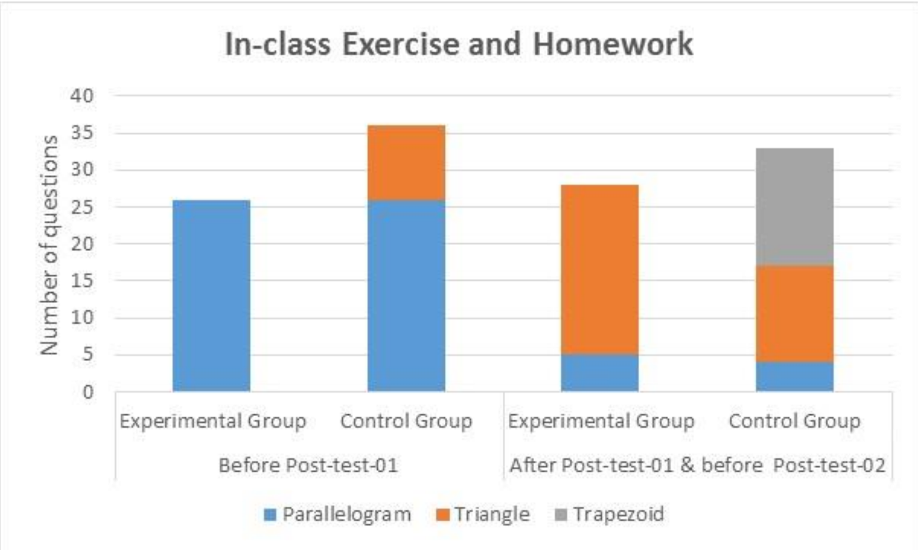

Figure 19: The actual number of questions on each topic that each group had done during different period.

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Participants

Students

There were 28 students (15 girls and 13 boys) in the experimental group and 25 students (14 girls and 11 boys) in the control group. The average age of the students in the experimental and control groups were 10.07 and 10.00 respectively. Table 5 summarized the profile of the two groups of students. All the students who participated in this study were full-time primary students studying in Grade 5. Their age ranged from 9 to 11. The students, their parents (or guardians), and the participating school had given informed consent regarding the students' participation in the research.

Table 5: Profile of the experimental and control groups.

Profile	Experimental group	Control group
Number of students	28	25
Ratio of girls to boys	15:13	14:11
Average age	10.07	10.00

Teachers

In this study, the original mathematics teacher of the experimental group class only acted as an observer. He had around 10 years of experience in teaching mathematics in primary and secondary schools. Although he was present in every class session of the experimental group throughout this study, he did not facilitate any of the teaching cycles. Instead, the researcher of this study was responsible for administering the experimental group according to the research design mentioned in the preceding sections. He did not have any experience in

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teaching mathematics in primary schools. For the control group, the original mathematics teacher of the control group class was responsible for teaching the control group throughout this study. He had around 10 years of experience in teaching mathematics in primary school.

Assessment and Instrument

Pre-test and Post-tests

The pre-test, Post-test-01 and Post-test-02 were written tests. These three tests consisted of the same set of questions on the areas of parallelogram, triangle, and trapezoid. There were pros and cons for using the same set of questions or different questions in the pre-test and post-tests. In this research, the same set of questions was used to facilitate the comparison of students' conceptual understanding and procedural knowledge at different stage of the research. Both of the experimental and control groups were administered the same tests and they were given the same duration (i.e. 30 minutes) to complete each of these tests. The test paper was collected immediately after each test. The teachers were not allowed to discuss with the students about the questions or their test scores during the whole research period. Moreover, the students were requested not to discuss with anyone about the questions in the tests.

These three tests consisted of three types of questions which aimed at testing the three types of important understanding on area (testing of these three important aspects of understanding were proposed by Pitta-Pantazi and Christou (2007) and Pitta-Pantazi and Christou (2009)). The first type was the "Recognition" type which asked the students to identify the shapes with the same area or to identify the base and height of the shape. For example, a triangle was

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presented and the students was required to indicate the possible pairs of base and height. The second type was the “Construction” type which asked the students to construct the shapes with the same areas by means of drawing on a graph paper. For example, the student was asked to draw two different parallelograms with the same area but one of them should have a base twice the length of the other one. The third type was the “Computation” type which asked the students to calculate the area of the various shapes. For example, the student was required to state the formula used for calculating the area of a trapezoid and then calculate the area using the formula.

Please refer to Appendix C for the pre-test and post-tests used in this study. In each of the tests, there were 16 questions. Table 6 summarized the different types of questions in these tests. There were 4 questions on parallelogram. Two of them asked the students to identify the parallelograms with the same area or to identify the base and height of the parallelogram. One question asked the students to construct two parallelograms with the same area. One questions asked the students to calculate the area of the parallelogram. There were 5 questions on triangle, 5 questions on trapezoid, and 2 questions on square or rectangle.

Table 6: Question types in the pre-test and post-tests.

Types of questions	Square / Rectangle	Parallelogram	Triangle	Trapezoid
Identify the shapes with the same area / identify the base and height of the shape	-	2	3	2
Construct the shapes with the same area	-	1	1	1
Calculate the area of the shape	2	1	1	2

Each question in the pre-test and post-tests aimed at assessing either students’ conceptual understanding or procedural knowledge. This study adopted the view that conceptual understanding was about the facts and principles while the procedural knowledge was about

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the manipulations and algorithms (de Jong & Ferguson-Hessler, 1996; Baroody et al., 2007). These views were followed in determining whether the question in the pre-test and post-tests aimed at assessing students' conceptual understanding or procedural knowledge. Table 7 summarized which questions in the pre-test and post-tests aimed at assessing students' conceptual understanding and which ones aimed at assessing students' procedural knowledge. There were 10 questions which aimed at assessing students' conceptual understanding and there were 6 questions which aimed at assessing students' procedural knowledge.

Table 7: Pre-test / post-tests questions aiming at assessing students' conceptual understanding or procedural knowledge.

Question no.	Brief description of the question	Aimed at assessing
1	Calculate the area of a square	Procedural knowledge
2	Calculate the area of a rectangle	Procedural knowledge
3	Identify which parallelograms have the same area	Conceptual understanding
4	Identify the correct base and height of the parallelogram	Conceptual understanding
5	Calculate the area of a parallelogram	Procedural knowledge
6	Identify which triangle have the same area	Conceptual understanding
7	Identify the correct base and height of the triangle	Conceptual understanding
8	Identify the correct base and height of the triangle	Conceptual understanding
9	Calculate the area of a triangle	Procedural knowledge
10	Calculate the area of a trapezoid	Procedural knowledge
11	Identify the correct base and height of the trapezoid	Conceptual understanding
12	Identify which trapezoid have the same area	Conceptual understanding
13	Calculate the area of a trapezoid	Procedural knowledge
14	Construct two triangles with the same area	Conceptual understanding
15	Construct two trapezoids with the same area	Conceptual understanding
16	Construct two parallelograms with the same area but one of them should have a base twice the length of the other one	Conceptual understanding

After both of the control and experimental groups had attended all of their respective classes of this research, the students were expected to attain a certain level of procedural knowledge and conceptual understanding on the target topics. In terms of procedural knowledge, the students should know the mathematical formulas for calculating the areas of the various

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shapes and be able to compute the correct areas. In terms of conceptual understanding, the students should be able to:

- see that the parallelogram could be cut and re-combined to form a rectangle;
- understand that the area of the parallelogram was equal to the area of the rectangle formed;
- understand that the base and height of the parallelogram was equal to the base and height of the rectangle formed respectively;
- identify the two sets of base and height of a parallelogram;
- see that two identical triangles could always form a parallelogram;
- understand that the area of the triangle was half of the area of the parallelogram formed;
- understand that the base and height of the triangle was equal to the base and height of the parallelogram formed respectively;
- identify the three sets of base and height of a triangle;
- see that two identical trapezoid could always form a parallelogram;
- understand that the area of the trapezoid was half of the area of the parallelogram formed;
- understand that the sum of the upper and lower bases of the trapezoid was equal to the base of the parallelogram formed;
- understand that the height of the trapezoid was equal to the height of the parallelogram formed; and
- identify the bases and height of a trapezoid.

Each question in the pre-test and post-tests aimed at assessing different procedural knowledge and conceptual understanding. Table 8 listed out the procedural knowledge and conceptual understanding that each question aimed at assessing.

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Table 8: Pre-test / post-tests questions aiming at assessing different procedural knowledge and conceptual understanding.

Question no.	Procedural Knowledge	Conceptual Understanding
1	Students should know that the area of a square could be found by multiplying the length of one side by itself, and be able to compute the correct area of the square.	Students should understand that multiplying the length of one side of the square by itself would find the number of “unit squares” that covered the square.
2	Students should know that the area of a rectangle could be found by multiplying the length by the width, and be able to compute the correct area of the rectangle.	Students should understand that multiplying the length by the width of the rectangle would find the number of “unit squares” that covered the rectangle.
3	Not application (because this was a question which aimed at assessing students’ conceptual knowledge only)	Students should see that as long as the lengths of the bases were the same and the lengths of the heights were also the same, the parallelograms would have the same areas. Alternatively, students should be able to make use of the formula (base $\times$ height) to ensure that the areas of the parallelograms were the same.
4	Not application	Students should know that the height of a parallelogram was the distance between the base and the opposite side parallel to the base.
5	Students should know that the area of a parallelogram could be found by multiplying the base by the height, and be able to compute the correct area of the parallelogram.	Students should see that the parallelogram could be cut and re-combined to form a rectangle, and understand that the area of the parallelogram was equal to the area of the rectangle formed. They should also understand that the base and height of the parallelogram was equal to the base and height of the rectangle formed respectively.
6	Not application	Students should see that as long as the lengths of the bases were the same and the lengths of the heights were also the same, the triangles would have the same areas. Alternatively, students should be able to make use of the formula (base $\times$ height $\div$ 2) to ensure that the areas of the triangles were the same.
7	Not application	Students should be able to identify the heights and the corresponding bases of the triangle.
8	Not application	Students should be able to identify the corresponding bases of the heights of the triangles.
9	Students should know that the area of a triangle could be found by the formula (base $\times$ height $\div$ 2), and be able to compute the correct area of the triangle.	Students should see that two identical triangles could form a parallelogram, and understand that the area of the triangle was half of the area of the parallelogram formed. They should also understand that the base and height of the triangle was equal to the base and height of the parallelogram formed respectively.

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10	Students should know that the area of a trapezoid could be found by the formula ((upper base + lower base) × height ÷ 2)), and be able to compute the correct area of the trapezoid.	Students should see that two identical trapezoid could form a parallelogram, and understand that the area of the trapezoid was half of the area of the parallelogram formed. They should also understand that the sum of the upper and lower bases of the trapezoid was equal to the base of the parallelogram formed, and understand that the height of the trapezoid was equal to the height of the parallelogram formed.
11	Not application	Students should know that the height of a trapezoid was the distance between the upper and lower bases.
12	Not application	Students should see that as long as the lengths of the upper bases, lower bases and heights were the same, the trapezoids would have the same areas. Alternatively, students should be able to make use of the formula ((upper base + lower base) × height ÷ 2)) to ensure that the areas of the trapezoids were the same.
13	Students should know that the area of a trapezoid could be found by the formula ((upper base + lower base) × height ÷ 2)), and be able to compute the correct area of the trapezoid.	Students should see that two identical trapezoid could form a parallelogram, and understand that the area of the trapezoid was half of the area of the parallelogram formed. They should also understand that the sum of the upper and lower bases of the trapezoid was equal to the base of the parallelogram formed, and understand that the height of the trapezoid was equal to the height of the parallelogram formed.
14	Not application	Students should be able to make use of the formula (base × height ÷ 2) to ensure that the areas of the two triangles were the same.
15	Not application	Students should be able to make use of the formula ((upper base + lower base) × height ÷ 2)) to ensure that the areas of the two trapezoids were the same.
16	Not application	Students should be able to make use of the formula (base × height) to ensure that the areas of the two parallelograms were the same.

<u>Student Interview</u>

After the completion of this study, six students from the experimental group and six students from the control group were selected for interviews. The interviews aimed at providing supplementary information on whether the students possessed the conceptual understanding and procedural knowledge that the pre-test and post-tests intended to assess. Interviews could be used for exploring the students’ ways of thinking and provided rich evidence on whether

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there were any misunderstandings (Van De Walle et al., 2013). Kospentaris et al. (2011) found that even though the students could get the correct answers in the written test, but they showed that they did not fully understand the underlying concepts when they were interviewed.

For the students in the experimental group, all the students were ranked according to the marks they got in Post-test-02. The six students with the rank number of 5n + 1 (where n starts from the number zero) were selected for the interview. As the selection was based on the students' performance in Post-test-02, this selection method ensured that representatives of high achiever, moderate achiever and low achiever were selected for interviews. Thus, the interviews could provide a rich set of data on conceptual understanding and procedural knowledge of students with different level of performance. Six students from the control group were also selected for interviews using the same mechanism.

An appointment for the interview was made in advance. The interview was conducted within one week after the completion of this study. Each of the selected students were interviewed separately. Each interview lasted around 15 minutes. No computer was used during the interview for the students to refer to the cognitive tools used in this study. Instead, the test paper that the students completed in Post-test-02 was presented to them and they were asked how they came up with the answers for each of the questions in Post-test-02. The following questions were asked one by one during the interview:

1) For question 1, why did you use this method to find the area of this square? Why could it be found by multiplying the length of one side of the square by itself?
2) For question 2, why did you use this method to find the area of this rectangle? Why

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could it be found by multiplying the length by the width of the rectangle?

3) For question 3, how did you figure out that these parallelograms had the same area?

4) For question 4, how did you figure out that these parallelograms had the correct bases and heights indicated in the diagram?

5) For question 5, why did you use this method to find the area of this parallelogram? Why could it be found by multiplying the base by the height of the parallelogram?

6) For question 6, how did you figure out that these triangles had the same area?

7) For question 7, how did you figure out that these two were the heights of this triangle? What were their corresponding bases?

8) For question 8, how did you figure out that these were the corresponding bases of the heights?

9) For question 9, why did you use this method to find the area of this triangle? Why could it be found by the formula (base × height ÷ 2)?

10) For question 10, why did you use this method to find the area of this trapezoid? Why could it be found by the formula ((upper base + lower base) × height ÷ 2)?

11) For question 11, how did you figure out that these trapezoids had the correct bases and heights indicated in the diagram?

12) For question 12, how did you figure out that these trapezoids had the same area?

13) For question 13, why did you use this method to find the area of this trapezoid? Why could it be found by the formula ((upper base + lower base) × height ÷ 2)?

14) For question 14, how did you know that these two triangles had the same area?

15) For question 15, how did you know that these two trapezoids had the same area?

16) For question 16, how did you know that these two parallelograms had the same area?

The main purpose of the interviews was to determine whether the students had attained the

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required procedural knowledge and conceptual understanding that the particular question aimed at assessing. Please refer to Table 8 for the procedural knowledge and conceptual understanding that each question aimed at assessing.

The whole interview was tape-recorded. Please refer to the section “Ethical Considerations” in this chapter for the procedures of handling the tape-recordings of the interviews.

Interview of Teacher Observer

A semi-structured interview with the original mathematics teacher of the experimental group class was conducted after the completion of this study. He was present in every class session of the experimental group throughout this study, but he only acted as a teacher observer without facilitating any of the sessions. This interview collected qualitative data regarding this teacher observer’s opinions about the use of these cognitive tools for teaching the computation of the areas of a parallelogram, triangle and trapezoid.

An appointment for the interview was made in advance. The interview was conducted within one week after the completion of this study. It was a one-to-one semi-structured interview which lasted around 30 minutes. The interview questions were not given to the teacher observer before the interview. No computer was used during the interview for the teacher observer to refer to the cognitive tools used in this study. The following questions were asked one by one during the interview:

1) Were the cognitive tools easy for students to use?
2) Did the cognitive tools foster teacher-student interactions?

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3) Could the students use the cognitive tools to discuss with their classmates?

4) Could the cognitive tools help the students to:

   a) understand the concept of area conservation?

   b) understand that irregular shapes (e.g. trapezoid) also had measurable areas which could be calculated by mathematical formulas?

   c) understand why the areas of the various shapes could be calculated using particular mathematical formulas?

   d) identify the height and base of the various shapes?

5) What were the main purposes of teaching area? Had the cognitive tools met these purposes?

6) Were there any aspects that needed to be improved regarding the cognitive tools?

7) How would you evaluate this pedagogical approach?

8) Presuming that you would use these cognitive tools next year, how would you make use of them?

9) Was this pedagogical approach worth promoting? If yes, how would you promote it?

10) What would be your overall comments regarding this pedagogical approach?

The whole interview was tape-recorded. Please refer to the section “Ethical Considerations” in this chapter for the procedures of handling the tape-recordings of the interviews.

Questionnaire

The students in the experimental group were asked to complete a questionnaire after the completion of this study. The questionnaire collected qualitative data regarding the students’ opinions about the use of this cognitive tool for their learning.

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Please refer to Appendix D for the full set of evaluation items in the questionnaire. These evaluation items were adopted and modified from Kong and Li (2007) and Kong (2011).

Data Collection

In addition to the result of the written tests, interviews and questionnaires mentioned in the preceding sections, the following data were also collected:

1. students' profile, including age and gender
2. video tapes of all the whole-class discussions
3. video tapes of the selected groups of students (in each teaching cycle, a group of four students were randomly selected for video recording)
4. tape-recordings of all the interviews

Raters

There were two raters in this study. Rater-01 was the researcher of this study who did not have any prior experience in teaching mathematics in primary schools. Rater-02 was an experienced teacher who had around 15 years of experience in teaching mathematics in primary schools. These two raters were responsible for marking the pre-test and post-tests, and rating the students' interviews.

First of all, the two raters met together to agree on the followings:

1. the marking scheme of the pre-test and post-tests (please refer to Appendix E for the

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marking scheme agreed by the two raters), and

2. the level of procedural knowledge and conceptual understanding that each of the questions in the pre-test and post-tests aimed at assessing (please refer to Table 8 for the required level of procedural knowledge and conceptual understanding agreed by the two raters); and
3. the coding for rating the students' interviews (please refer to Table 9 for the coding agreed by the two raters).

Table 9: The coding for rating the students' interviews agreed by the two raters.

Code	Description
P	the student had procedural knowledge only
C	the student had conceptual understanding only
B	the student had both of the procedural knowledge and conceptual understanding
N	the student neither had procedural knowledge nor conceptual understanding
L	there was lack of information to draw any conclusions regarding students' knowledge

After that, the two raters marked the pre-test and post-tests and rated the students' interviews independently. For the pre-test and post-tests, the raters went through each of the tests taken by the students and marked them according to the marking scheme as shown in Appendix E. For the students' interviews, the raters listened to the tape-recordings of each of the students' interviews. Based on the tape-recordings, the raters determined whether the students' verbal explanation to each of the questions proved the students had attained the required procedural knowledge and conceptual understanding that the particular question aimed at assessing (please refer to Table 8 for the procedural knowledge and conceptual understanding that each question aimed at assessing). For the particular question, if the student had attained the required procedural knowledge only, the rater would give a "P" to that particular question of that particular student. If the student had attained the required conceptual understanding only,

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the rater would give a “C”. If the student had attained both of the procedural knowledge and conceptual understanding, the rater would give a “B”. If the student had attained neither the procedural knowledge nor the conceptual understanding, the rater would give an “N”. If the rater could not draw any conclusions after listening to the tape-recording, the rater would give an “L”. Please refer to Table 9 for the coding.

After the raters had independently completed marking the tests and rating the students’ interviews, they met again to discuss about the results to see if there were any mistakes in their individual marking and rating. In case there were any mistakes, the raters would correct their individual marking and rating accordingly.

After the raters completed the rating, interrater reliability was calculated to estimate the degree of consensus between the two raters regarding their marking of the tests and their rating of the students’ interviews. The Cohen's Kappa statistics were computed using IBM SPSS Statistics version 21 to show the interrater reliability.

Data Analyses

Quantitative Data Analyses

The results collected from the pre-test and post-tests provided empirical data for quantitative analyses. These quantitative data were analyzed using IBM SPSS Statistics version 21.

Independent t-tests on the pre-test scores were performed to see whether there were any significant differences between the two groups before the research. Independent t-tests on the

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post-tests scores were performed to analyze whether there were any significant differences between the two groups after the two groups had attended their respective classes. In order to attain a statistical power of 80%, each group should have at least 30 participants (VanVoorhis & Morgan, 2007). According to VanVoorhis and Morgan (2007), 80% was the minimum statistical power for an ordinary study. According to Nachar (2008), when the two independent groups were not large normally distributed samples, the Mann-Whitney U test was less at risk than the independent t-test to give a wrongfully significant result. The Mann-Whitney U test was a nonparametric statistical test which did not require the samples to be normally distributed (Nachar, 2008). Zimmerman (1987) found that when the samples were small in size, either the Mann-Whitney U test or the independent t-test could be more statistically power depending on different situation. As a result, in this research, the Mann-Whitney U tests were conducted to supplement the results of the independent t-tests.

Paired t-tests on the scores obtained by the experimental group (and the control group) in the pre-test and post-tests were performed to analyze whether there were any significant improvement in the experimental group (and the control group). The Wilcoxon signed rank tests were conducted to supplement the results of the paired t-tests. The Wilcoxon signed rank test was a nonparametric statistical test equivalent to the paired t-test while it did not require the samples to be normally distributed.

Qualitative Data Analyses

The qualitative data collected from the students' interviews were analyzed to further examine whether the student had gained conceptual understanding and procedural knowledge on the target topics. The qualitative data collected from the teacher observer's interview were

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analyzed to evaluate the teaching effect and the potential of using this cognitive tool to support the teaching process of these topics. The qualitative data collected from the students' questionnaire were analyzed to investigate the learning effect and the potential of using these cognitive tools to support the learning process of the target topics.

These qualitative data provided further information for the understanding of the outcomes of the quantitative analyses.

Ethical Considerations

Informed consent were obtained from the student, the parents (or the guardians), and the participating school regarding the students' participation in this study. They had been provided with the information sheet which fully explained the details of this study. Please refer to Appendix G and H for the school's consent form and the parents' consent form respectively.

Student names and HKID numbers were not collected. The students were only identified by a code assigned for this particular study. As a result, the personal identities of the participants would not be disclosed easily.

Permission were obtained in advance from participants to videotape the class sessions as well as tape-recording the interviews. Permission would be obtained in advance from participants before the videos or tape-recordings were used for public dissemination.

All the data collected (including videos) were kept in strictly confidential and would only be

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used for the purpose of this study. Entered data would be stored on a password-protected computer, while original hard copies of the questionnaires and tests as well as videos and tape-recordings were stored in a locked office until 5 years past publication.

At the end of each teaching cycle, the teacher summarized the discussions in the experimental group and explained to the students the correct concepts and formulas so that all the experimental group students would have the opportunities to learn the target topics. As the cognitive tools had beneficial effects to the students, the control group were equally treated and given the opportunity to explore the target topics using the cognitive tools after the completion of the research. After the Post-test-02, a class session was arranged for the control group to work on the GeoGebra cognitive tools used in this study. They were given a combined and simplified worksheet which guided them through the exploration with the cognitive tools. Please refer to Appendix A5 for the combined and simplified worksheet.

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Chapter 4 – Results and Discussions

Introduction

The results collected from the pre-test and post-tests provided empirical data for quantitative analyses. These quantitative data were analyzed to address the following research questions:

1. Compared with the direct instructional approach, would the inquiry-based learning approach scaffolded by cognitive tools developed on the GeoGebra platform help students develop better conceptual understanding and procedural knowledge?
2. If the answer to the first research question was affirmative, how did the inquiry-based learning approach scaffolded by the GeoGebra cognitive tools help the students develop better conceptual understanding and procedural knowledge?

The qualitative data collected from the students' interviews were analyzed to further examine whether the student had gained conceptual understanding and procedural knowledge on the target topics. The qualitative data collected from the teacher observer's interview were analyzed to evaluate the teaching effect and the potential of using this cognitive tool to support the teaching process of these topics. The qualitative data collected from the students' questionnaire were analyzed to investigate the learning effect and the potential of using these cognitive tools to support the learning process of the target topics.

Table 10 summarized the results of the quantitative and qualitative data analyses in this study. They were further elaborated in the subsequent sections.

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Table 10: Summary of the results of the quantitative and qualitative data analyses in this study.

	Experimental Group		Control Group	
	M	SD	M	SD
Quantitative Analyses				
Overall Test Performance				
Pre-test	4.82	3.32	4.44	3.45
Post-test-02 *	10.61	4.02	8.20	3.29
Conceptual questions				
Pre-test	2.29	2.23	2.12	2.17
Post-test-02 **	5.46	3.23	3.40	2.29
Procedural questions				
Pre-test	2.54	1.62	2.32	1.65
Post-test-02	5.14	1.15	4.80	1.58
Trapezoid conceptual questions				
Pre-test	0.68	0.72	0.52	0.65
Post-test-01	0.96	0.88	0.68	0.69
Post-test-02 **	1.57	1.10	0.68	0.75
Base and height questions				
Pre-test	0.93	1.05	0.84	0.80
Post-test-01	1.93	1.21	1.80	0.82
Post-test-02 **	2.43	1.35	1.44	1.12
Qualitative Analyses				
"Construction" type questions				
Creative drawings in the pre-test and post-tests showed that the students were able to identify the correct bases and heights of the various shapes. (For details, please refer to the subsection "Overcame Difficulty 'Failure to Identify a Base and the Corresponding Height'" in Chapter 4.)	Many students in the experimental group produced these kinds of creative drawings in the post-tests		Only one student in the control group produced these kinds of creative drawings in the post-tests	
Interview of Teacher Observer				
The interview provided the teacher observer's opinions about the use of these cognitive tools and pedagogical approach for teaching the target topics. (For details, please refer to the section "Interview of Teacher Observer" in Chapter 4.)	The teacher observer strongly agreed that the cognitive tools were able to help students clearly understand the concept of area and this pedagogical approach was the right approach for teaching area.		N/A	
Student Questionnaire				
The questionnaire provided the students' opinions about the use of this cognitive tool for their learning. (For details, please refer to the section "Student Questionnaire (for Experimental Group Only)" in Chapter 4.)	The students' overall perceptions were that the cognitive tools could help them to learn the key concepts of area, and learn the computation of the areas of the various shapes.		N/A	
Student Interview				
The student's interviews provided supplementary information on whether the students possessed the conceptual understanding and procedural knowledge. (For details, please refer to the section "Student Interview" in Chapter 4.)	The students' interviews showed that, in general, the marks that the students obtained in Post-test-02 reflected their conceptual understanding and procedural knowledge.		The students' interviews showed that, in general, the marks that the students obtained in Post-test-02 reflected their conceptual understanding and procedural knowledge.	

*p < 0.05.
**p < 0.01.

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Interrater Reliability

Two raters, namely Rater-01 and Rater-02, were responsible for marking the pre-test and post-tests, and rating the students' interviews. After the raters completed the rating, the Cohen's Kappa statistics were computed for the interrater reliability. For details of the actions taken by the raters, please refer to the section "Raters" in Chapter 3.

After the raters had independently completed marking the pre-test and post-tests, they met together to discuss about the results to see if there were any mistakes in their individual marking. Some mistakes were found in each of the raters' marking. After correcting these mistakes, it turned out that the marks given by the two raters were identical for each of the questions in all the three tests. It was because these mathematics questions had clear and objective answers. As there was a perfect agreement between the two raters on the marking of the pre-test and post-tests, there was no need to compute the Cohen's Kappa statistic for the marking of these tests. The Cohen's Kappa value would definitely be a "1".

On the other hand, there were differences between the two raters in rating the students' interviews. Please refer to Appendix I for the ratings given by each of the raters after they had listened to the tape-recordings of the students' interviews. The Cohen's Kappa statistic computed based on these ratings was 0.907. According to Crano, Brewer and Lac (2015), a Cohen's Kappa value greater than 0.75 indicated that an *excellent* level of interrater reliability had attained. Therefore, there was a high degree of consensus between the raters in terms of their rating of the students' interviews.

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Effectiveness of Adopting Inquiry-Based Learning Approach Scaffolded by Cogntive Tool on Students' Learning

There were 16 questions in each of the pre-test and post-tests. Each question carried 1 mark. Therefore, the maximum marks that the students could obtained were 16. Figure 20 showed the average marks that each group obtained in the pre-test and Post-test-02.

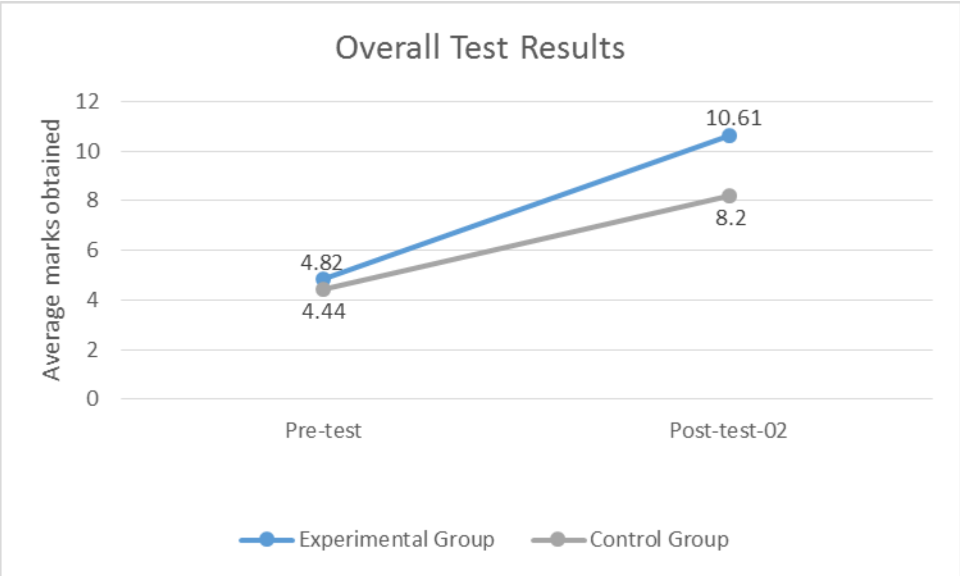

Figure 20: The average marks that each group obtained in the pre-test and Post-test-02.

Table 11: Comparison of the overall test results between the control group and the experimental group.

Test	Experimental Group			Control Group			95% CI for Mean Difference	t	df
	M	SD	n	M	SD	n			
Pre-test	4.82	3.32	28	4.44	3.45	25	-2.25, 1.49	-0.41	51
Post-test-02	10.61	4.02	28	8.20	3.29	25	-4.45, -0.37	-2.37*	51

*p < 0.05.

In order to answer the ***first research question***, independent t-tests were performed to compare the overall achievement of the control group and the experimental group in the pre-test and Post-test-02 respectively (Table 11). Results of the independent t-test for the pre-test showed

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that the mean score did not differ significantly between the experimental group (M = 4.82, SD = 3.32, n = 28) and the control group (M = 4.44, SD = 3.45, n = 25) at the 0.05 level of significance (t = -0.41, df = 51, p = 0.684 (two-tailed)). On the other hand, results of the independent t-test for Post-test-02 showed that the mean score differed significantly between the experimental group (M = 10.61, SD = 4.02, n = 28) and the control group (M = 8.20, SD = 3.29, n = 25) at the 0.05 level of significance (t = -2.37, df = 51, p = 0.022 (two-tailed)). These independent t-test results showed that the students in the experimental group performed ***significantly better*** than those in the control group after both groups had attended all of their respective classes.

Mann-Whitney U tests were also conducted to compare the overall test results between the control group and the experimental group in the pre-test and Post-test-02 respectively. Same as the independent t-tests shown in Table 11, the results of the Mann-Whitney U tests also showed that there were no significant differences between the control and experimental groups in the pre-test (U = 317, p = 0.547 (two-tailed)) while there were significant differences between the two groups in Post-test-02 (U = 230, p = 0.032 (two-tailed)). In the pre-test, the means of the ranks for the control group and experimental group were 25.66 and 28.20 respectively. In the Post-test-02, the means of the ranks for the control group and experimental group were 22.20 and 31.29 respectively.

Further analyses showed that, in fact, both of the experimental and control groups had significantly developed their conceptual understanding and procedural knowledge. This was shown in Table 12 to Table 14.

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Table 12: Paired t-test of students' overall test results between pre-test and Post-test-02.

Group	Pre-test		Post-test-02			95% CI for Mean Difference		
	M	SD	M	SD	n		t	df
Experimental	4.82	3.32	10.61	4.02	28	-7.13, -4.45	-8.86***	27
Control	4.44	3.45	8.20	3.29	25	-4.84, -2.68	-7.18***	24

***p < 0.001.

Table 12 showed the paired t-tests which compared each group of students' overall achievement in the pre-test and Post-test-02. Results of the paired t-test for the experimental group showed that the mean score differs significantly between pre-test (M = 4.82, SD = 3.32) and Post-test-02 (M = 10.61, SD = 4.02) at the 0.05 level of significance (t = -8.86, df = 27, n = 28, p < 0.001 (two-tailed)). Results of the paired t-test for the control group also showed that the mean score differs significantly between pre-test (M = 4.44, SD = 3.45) and Post-test-02 (M = 8.20, SD = 3.29) at the 0.05 level of significance (t = -7.18, df = 24, n = 25, p < 0.001 (two-tailed)). These paired t-test results showed that both of the experimental and control groups had performed ***significantly better*** after they had attended their respective classes.

Wilcoxon signed rank tests were also conducted to compare each group of students' overall achievement in the pre-test and Post-test-02. Same as the paired t-tests shown in Table 12, the results of the Wilcoxon signed rank tests also showed that there were significant differences between pre-test and Post-test-02 for the experimental group (Z = -4.467, p < 0.001 (two-tailed)) as well as for the control group (Z = -4.130, p < 0.001 (two-tailed)). For the experimental group, the mean of the negative ranks was 0.00 and the mean of the positive ranks was 13.50. For the control group, the mean of the negative ranks was 2.50 and the mean of the positive ranks was 12.43.

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Table 13: Paired t-test of students' results on conceptual questions between pre-test and Post-test-02.

Group	Pre-test		Post-test-02			95% CI for Mean Difference		
	M	SD	M	SD	n		T	df
Experimental	2.29	2.23	5.46	3.23	28	-4.34, -2.01	-5.59***	27
Control	2.12	2.17	3.40	2.29	25	-1.99, -0.57	-3.72**	24

**p < 0.01.
***p < 0.001.

Table 13 showed the paired t-tests which compared each group of students' achievement in the pre-test and Post-test-02 on questions aimed at assessing their conceptual understanding. Note that there were 10 questions which aimed at assessing students' conceptual understanding and each question carried 1 mark (please refer to Table 7 for details of those questions). Therefore, the maximum marks that the students could obtained for conceptual questions were 10 only. Results of the paired t-test for the experimental group showed that the mean score differs significantly between pre-test (M = 2.29, SD = 2.23) and Post-test-02 (M = 5.46, SD = 3.23) at the 0.05 level of significance ($t = -5.59$, $df = 27$, $n = 28$, $p < 0.001$ (two-tailed)). Results of the paired t-test for the control group also showed that the mean score differs significantly between pre-test (M = 2.12, SD = 2.17) and Post-test-02 (M = 3.40, SD = 2.29) at the 0.05 level of significance ($t = -3.72$, $df = 24$, $n = 25$, $p = 0.001$ (two-tailed)). These paired t-test results showed that both of the experimental and control groups had ***significantly developed*** their conceptual understanding after they had attended their respective classes.

Wilcoxon signed rank tests were also conducted to compare each group of students' achievement in the pre-test and Post-test-02 on questions aimed at assessing their conceptual understanding. Same as the paired t-tests shown in Table 13, the results of the Wilcoxon signed rank tests also showed that there were significant differences between pre-test and

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Post-test-02 for the experimental group (Z = -4.119, $p < 0.001$ (two-tailed)) as well as for the control group (Z = -3.094, p = 0.002 (two-tailed)). For the experimental group, the mean of the negative ranks was 0.00 and the mean of the positive ranks was 11.50. For the control group, the mean of the negative ranks was 5.75 and the mean of the positive ranks was 11.69.

Table 14: Paired t-test of students' results on procedural questions between pre-test and Post-test-02.

Group	Pre-test		Post-test-02			95% CI for Mean Difference		
	M	SD	M	SD	n		t	df
Experimental	2.54	1.62	5.14	1.15	28	-3.20, -2.02	-9.06***	27
Control	2.32	1.65	4.80	1.58	25	-3.11, -1.85	-8.10***	24

***$p < 0.001$.

Table 14 showed the paired t-tests which compared each group of students' achievement in the pre-test and Post-test-02 on questions aimed at assessing their procedural knowledge. Note that there were 6 questions which aimed at assessing students' procedural knowledge and each question carried 1 mark (please refer to Table 7 for details of those questions). Therefore, the maximum marks that the students could obtained for procedural questions were 6 only. Results of the paired t-test for the experimental group showed that the mean score differs significantly between pre-test (M = 2.54, SD = 1.62) and Post-test-02 (M = 5.14, SD = 1.15) at the 0.05 level of significance (t = -9.06, df = 27, n = 28, $p < 0.001$ (two-tailed)). Results of the paired t-test for the control group also showed that the mean score differs significantly between pre-test (M = 2.32, SD = 1.65) and Post-test-02 (M = 4.80, SD = 1.58) at the 0.05 level of significance (t = -8.10, df = 24, n = 25, $p < 0.001$ (two-tailed)). These paired t-test results showed that both of the experimental and control groups had ***significantly developed*** their procedural knowledge after they had attended their respective classes.

Wilcoxon signed rank tests were also conducted to compare each group of students'

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achievement in the pre-test and Post-test-02 on questions aimed at assessing their procedural knowledge. Same as the paired t-tests shown in Table 14, the results of the Wilcoxon signed rank tests also showed that there were significant differences between pre-test and Post-test-02 for the experimental group ($Z = -4.325$, $p < 0.001$ (two-tailed)) as well as for the control group ($Z = -4.047$, $p < 0.001$ (two-tailed)). For the experimental group, the mean of the negative ranks was 0.00 and the mean of the positive ranks was 12.50. For the control group, the mean of the negative ranks was 0.00 and the mean of the positive ranks was 11.00.

However, when we compared which group had developed better conceptual understanding on the target topics, it was found that the students in the experimental group had developed ***significantly better*** conceptual understanding than those in the control group after both groups had attended all of their respective classes. Figure 21 showed the average marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' conceptual understanding. Note that there were 10 questions which aimed at assessing students' conceptual understanding and each question carried 1 mark (please refer to Table 7 for details of those questions). Therefore, the maximum marks that the students could obtained for conceptual questions were 10 only.

Table 15 showed the independent t-tests which compared the marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' conceptual understanding. Results of the independent t-test for the pre-test showed that the mean score did not differ significantly between the experimental group ($M = 2.29$, $SD = 2.23$, $n = 28$) and the control group ($M = 2.12$, $SD = 2.17$, $n = 25$) at the 0.05 level of significance ($t = -0.27$, $df = 51$, $p = 0.785$ (two-tailed)). On the other hand, the results of the independent t-test for the Post-test-02 showed that the mean score differed significantly between the

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experimental group (M = 5.46, SD = 3.23, n = 28) and the control group (M = 3.40, SD = 2.29, n = 25) at the 0.05 level of significance (t = -2.71, df = 48.66, p = 0.009 (two-tailed)). These independent t-test results showed that the students in the experimental group had developed ***significantly better*** conceptual understanding than those in the control group after both groups had attended their respective classes.

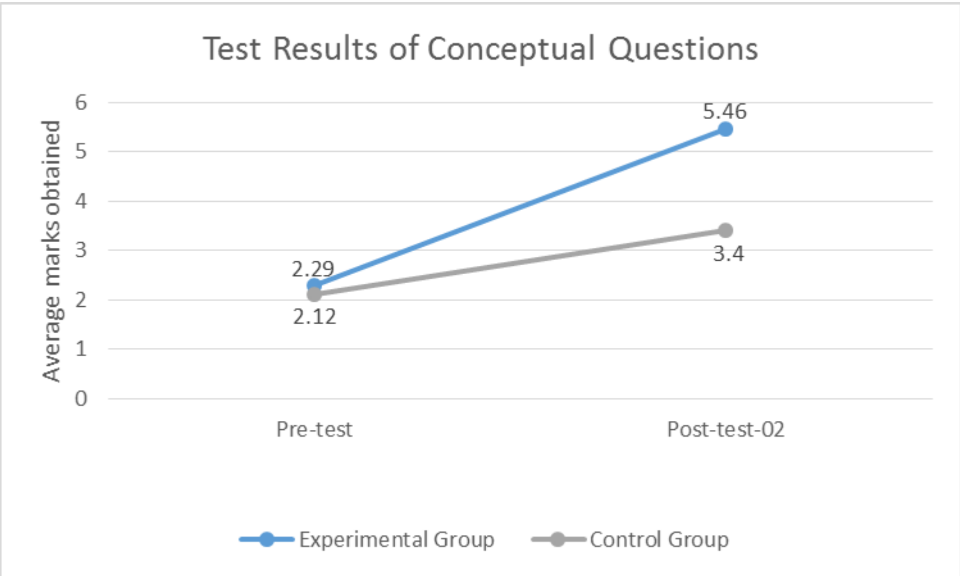

Figure 21: The average marks that each group obtained in the pre-test and Post-test-02 on conceptual questions.

Table 15: Comparison of the results on conceptual questions between the control group and the experimental group.

Test	Experimental Group			Control Group			95% CI for Mean Difference	t	df
	M	SD	n	M	SD	n			
Pre-test	2.29	2.23	28	2.12	2.17	25	-1.38, 1.05	-0.27	51
Post-test-02	5.46	3.23	28	3.40	2.29	25	-3.60, -0.53	-2.71**	48.66

**p < 0.01.

Mann-Whitney U tests were also conducted to compare the marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' conceptual understanding. Same as the independent t-tests shown in Table 15, the results of the Mann-Whitney U tests also showed that there were no significant differences between the

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control and experimental groups in the pre-test (U = 326, p = 0.654 (two-tailed)) while there were significant differences between the two groups in Post-test-02 (U = 238, p = 0.043 (two-tailed)). In the pre-test, the means of the ranks for the control group and experimental group were 26.02 and 27.88 respectively. In the Post-test-02, the means of the ranks for the control group and experimental group were 22.50 and 31.02 respectively.

On the other hand, in terms of the development of procedural knowledge, there were ***no significant differences*** between the experimental group and the control group after both groups had attended all of their respective classes. Figure 22 showed the average marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' procedural knowledge. Note that there were 6 questions which aimed at assessing students' procedural knowledge and each question carried 1 mark (please refer to Table 7 for details of those questions). Therefore, the maximum marks that the students could obtained for procedural questions were 6 only.

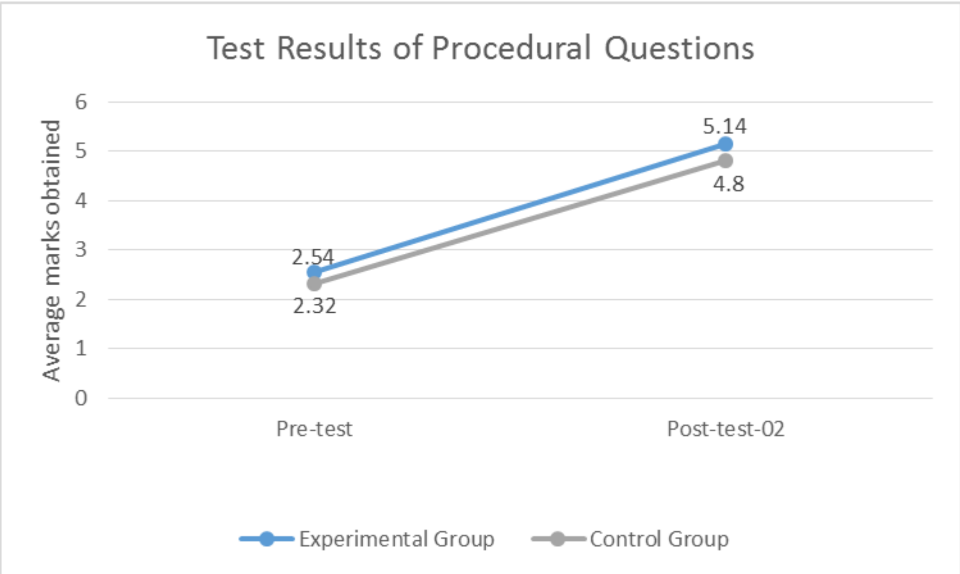

Figure 22: The average marks that each group obtained in the pre-test and Post-test-02 on procedural questions.

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Table 16: Comparison of the results on procedural questions between the control group and the experimental group.

Test	Experimental Group			Control Group			95% CI for Mean Difference	t	df
	M	SD	n	M	SD	n			
Pre-test	2.54	1.62	28	2.32	1.65	25	-1.12, 0.69	-0.48	51
Post-test-02	5.14	1.15	28	4.80	1.58	25	-1.10, 0.41	-0.91	51

Table 16 showed the independent t-tests which compared the marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' procedural knowledge. Results of the independent t-test for the pre-test showed that the mean score did not differ significantly between the experimental group (M = 2.54, SD = 1.62, n = 28) and the control group (M = 2.32, SD = 1.65, n = 25) at the 0.05 level of significance (t = -0.48, df = 51, p = 0.634 (two-tailed)). Results of the independent t-test for the Post-test-02 also showed that the mean score did not differ significantly between the experimental group (M = 5.14, SD = 1.15, n = 28) and the control group (M = 4.80, SD = 1.58, n = 25) at the 0.05 level of significance (t = -0.91, df = 51, p = 0.367 (two-tailed)). These independent t-test results showed that, in terms of the development of procedural knowledge, there were ***no significant*** differences between the experimental group and the control group after both groups had attended their respective classes. This provided evidence that the pedagogical approach adopted by the experimental group was ***as effective as*** the direct instructional approach in developing students' procedural knowledge even though the experimental group did less practices in terms of in-class exercises and homework than the control group.

In terms of procedural knowledge, the students in the control group were well-prepared for the post-tests. Firstly, the teacher of the control group emphasized on the students' procedural knowledge. At the beginning of the class, the teacher explained to the students the basic concepts of the topic. He then presented to the students the formula used for calculating the area of the particular shape. After that, he chose a few questions from the textbook and

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demonstrated to the student how the area could be calculated using the formula. He then asked the students to do a few questions in class. At the end of the class, he asked the students to do a few questions on area calculation at home. Secondly, the control group had done 28% more in-class exercises and homework than the experimental group.

On the other hand, the students in the experimental group had less practices in terms of procedural knowledge. Nevertheless, as the pedagogical approach adopted by the experimental group helped the students to developed significantly better conceptual understanding of the target topics, their conceptual understanding had exerted positive effect on their procedural knowledge. This was because the conceptual understanding and procedural knowledge developed iteratively (i.e. improving conceptual understanding had positive effect on procedural knowledge and vice versa). As a result, the students in the experimental group performed as well as those in the control group in terms of the development of their procedural knowledge.

Mann-Whitney U tests were also conducted to compare the marks that each group obtained in the pre-test and Post-test-02 on those questions which aimed at assessing students' procedural knowledge. Same as the independent t-tests shown in Table 16, the results of the Mann-Whitney U tests also showed that there were no significant differences between the control and experimental groups in the pre-test ($U = 315$, $p = p = 0.505$ (two-tailed)) and there were no significant differences between the two groups in Post-test-02 ($U = 314$, $p = 0.485$ (two-tailed)). In the pre-test, the means of the ranks for the control group and experimental group were 25.58 and 28.27 respectively. In the Post-test-02, the means of the ranks for the control group and experimental group were 25.56 and 28.29 respectively.

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The feedback from the students' questionnaire also showed that 26 out of the 28 students in the experimental group agreed or strongly agreed that the cognitive tools could help them to learn the key concepts on the topics of area. Moreover, 25 out of the 28 students in the experimental group agreed or strongly agreed that the cognitive tools could help them to learn the computation of the areas of the various shapes. For details of the students' questionnaire, please refer to the section "Student Questionnaire (for Experimental Group Only)" in this chapter.

In the teacher observer's interview, the teacher observer also said that when he marked the subsequent homework submitted by the students, he noticed that the students in the experimental group had developed very good conceptual understanding. He quoted an example which he said he had never seen any students performed the calculation in such a way before. For details of this example, please refer to the section "Interview of Teacher Observer" in this chapter.

How Did the Proposed Pedagogical Approach Support Students' Learning?

Further analyses were conducted in order to find out how the inquiry-based learning approach scaffolded by the GeoGebra cognitive tools could help the students develop better conceptual understanding (i.e. to answer the ***second research question***). It was found that the pedagogical approach adopted in the experimental group was especially more effective than the direct instructional approach in helping the students to construct their conceptual understanding on the area of ***trapezoid***. This was difficult for students to comprehend through traditional direct instructional approach. The following subsections further explained how the pedagogical approach adopted in the experimental group made the difference.

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Moreover, the qualitative data collected from the students' questionnaire, teacher observer's interview and students' drawings in the pre-test and post-tests revealed that the pedagogical approach adopted in the experimental group was able to assist students to overcome the following two common difficulties in learning the concepts of area (the analyses and findings were elaborated in the following subsections):

– the failure to identify a base and its corresponding height for area calculation; and
– the misconception that only regular closed shapes had measurable area and corresponding mathematical formulas for area calculation.

As mentioned earlier in Chapter 2, there was extensive evidence indicating that conceptual understanding and procedural knowledge developed iteratively. Improving conceptual understanding had positive effect on procedural knowledge and vice versa (Kilpatrick et al., 2001; Baroody et al., 2007; Star, 2007; Rittle-Johnson & Schneider, 2014). As the students in the experimental group had significantly developed their conceptual understanding of the target topics, their conceptual understanding had exerted positive effect on their procedural knowledge even though they did less practices in terms of in-class exercises and homework.

The Proposed Pedagogical Approach was Effective in Developing Students' Conceptual Understanding on ***Trapezoid***

This research revealed that the pedagogical approach adopted in the experimental group was especially more effective than the direct instructional approach in helping the students to construct their conceptual understanding on the area of trapezoid. We divided the pre-test or post-tests questions into four categories: (a) trapezoid conceptual questions; (b) trapezoid

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procedural questions; (c) non-trapezoid conceptual questions; and (d) non-trapezoid procedural questions. We then performed analyses on each of these categories. “Trapezoid conceptual questions” were those questions which aimed at assessing students’ conceptual understanding on the area of trapezoid. “Trapezoid procedural questions” were those questions which aimed at assessing students’ procedural knowledge on the area of trapezoid. “Non-Trapezoid conceptual questions” were those questions which aimed at assessing students’ conceptual understanding on the topics other than trapezoid. “Non-Trapezoid procedural questions” were those questions which aimed at assessing students’ procedural knowledge on the topics other than trapezoid. Non-trapezoid questions consisted of questions of square, rectangle, parallelogram and triangle (i.e. there were no trapezoid questions).

Table 17 showed these four categories and it summarized the number of questions in each of the categories. In each of the pre-test or post-tests, there were 3 questions which tested the students’ conceptual understanding on trapezoid; 2 questions which tested the students’ procedural knowledge on trapezoid; 7 questions which tested the students’ conceptual understanding on square, rectangle, parallelogram or triangle; and 4 questions which tested the students’ procedural knowledge on square, rectangle, parallelogram or triangle.

Table 17: Number of questions in the four categories, namely, trapezoid conceptual questions, trapezoid procedural questions, non-trapezoid conceptual questions, and non-trapezoid procedural questions.

	Conceptual	Procedural
Trapezoid	3	2
Non-trapezoid	7	4

Analyses were conducted on each of these categories. It was found that, for the trapezoid conceptual questions, there were significant differences between the control and experimental

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groups after both groups had attended the trapezoid classes (Table 18 to Table 20). However, there were no significant differences between the two groups for the trapezoid procedural questions after both groups had attended the trapezoid classes. There were also no significant differences between the two groups for the non-trapezoid procedural and non-trapezoid conceptual questions after both groups had attended the parallelogram and triangle classes. It was also found that after attended the trapezoid classes, the experimental group had developed better conceptual understanding on parallelogram and triangle.

Figure 23 showed the average marks that each group obtained on trapezoid conceptual questions in the pre-test. Post-test-01, and Post-test-02 respectively. Note that there were 3 trapezoid conceptual questions and each question carried 1 mark. Therefore, the maximum marks that the students could obtained for "trapezoid conceptual questions" were 3 only.

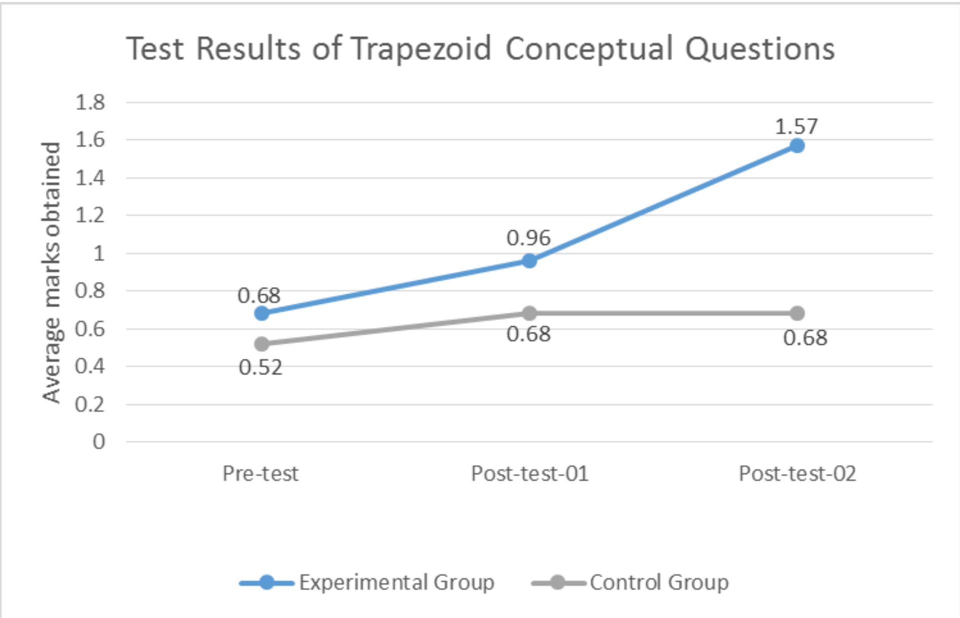

Figure 23: The average marks that each group obtained on trapezoid conceptual questions.

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Table 18: Comparison of the results on trapezoid conceptual questions between the control group and the experimental group.

Test	Experimental Group			Control Group			95% CI for Mean Difference	t	df
	M	SD	n	M	SD	n			
Pre-test	0.68	0.72	28	0.52	0.65	25	-0.54, 0.22	-0.83	51
Post-test-01	0.96	0.88	28	0.68	0.69	25	-0.72, 0.16	-1.30	51
Post-test-02	1.57	1.10	28	0.68	0.75	25	-1.41, -0.38	-3.47**	47.74

**p < 0.01.

Table 18 showed the independent t-tests which were conducted to compare the students' achievement on the trapezoid conceptual questions between the control group and the experimental group. Results of the independent t-test for the pre-test showed that the mean score did not differ significantly between the experimental group (M = 0.68, SD = 0.72, n = 28) and the control group (M = 0.52, SD = 0.65, n = 25) at the 0.05 level of significance (t = -0.83, df = 51, p = 0.408 (two-tailed)). Results of the independent t-test for Post-test-01 showed that the mean score did not differ significantly between the experimental group (M = 0.96, SD = 0.88, n = 28) and the control group (M = 0.68, SD = 0.69, n = 25) at the 0.05 level of significance (t = -1.30, df = 51, p = 0.201 (two-tailed)). On the other hand, the results of the independent t-test for Post-test-02 showed that the mean score differed significantly between the experimental group (M = 1.57, SD = 1.10, n = 28) and the control group (M = 0.68, SD = 0.75, n = 25) at the 0.05 level of significance (t = -3.47, df = 47.74, p = 0.001 (two-tailed)). These independent t-test results showed that, for the topic of "area of trapezoid", the students in the experimental group had developed ***significantly better*** conceptual understanding than those in the control group after both groups had attended their respective trapezoid classes.

Mann-Whitney U tests were also conducted to compare the students' achievement on the "trapezoid conceptual questions" between the control group and the experimental group.

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Same as the independent t-tests shown in Table 18, the results of the Mann-Whitney U tests also showed that there were no significant differences between the control and experimental groups in the pre-test (U = 308, p = 0.403 (two-tailed)) and Post-test-01 (U = 290, p = 0.255 (two-tailed)) while there were significant differences between the two groups in Post-test-02 (U = 190, p = 0.003 (two-tailed)). In the pre-test, the means of the ranks for the control group and experimental group were 25.32 and 28.50 respectively. In the Post-test-01, the means of the ranks for the control group and experimental group were 24.62 and 29.13 respectively. In the Post-test-02, the means of the ranks for the control group and experimental group were 20.60 and 32.71 respectively.

Paired t-tests were performed to compare the students' achievement on the trapezoid conceptual questions in the pre-test, Post-test-01, Post-test-02 (Table 19 and Table 20).

Table 19: Paired t-test of students' results on trapezoid conceptual questions between pre-test and Post-test-01.

Group	Pre-test		Post-test-01			95% CI for Mean Difference		
	M	SD	M	SD	n		t	df
Experimental	0.68	0.72	0.96	0.88	28	-0.63, 0.06	-1.69	27
Control	0.52	0.65	0.68	0.69	25	-0.44, 0.12	-1.16	24

Table 19 showed the paired t-tests which compared each group of students' achievement in the pre-test and Post-test-01 on trapezoid conceptual questions. Results of the paired t-test for the experimental group showed that the mean score did not differ significantly between pre-test (M = 0.68, SD = 0.72) and Post-test-01 (M = 0.96, SD = 0.88) at the 0.05 level of significance (t = -1.69, df = 27, n = 28, p = 0.103 (two-tailed)). Results of the paired t-test for the control group also showed that the mean score did not differ significantly between pre-test (M = 0.52, SD = 0.65) and Post-test-01 (M = 0.68, SD = 0.69) at the 0.05 level of significance (t = -1.16, df = 24, n = 25, p = 0.256 (two-tailed)). There were no significant

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differences between pre-test and Post-test-01 for both groups because the trapezoid classes were held after the Post-test-01. These paired t-test results showed that both of the experimental and control groups had not developed their conceptual knowledge on trapezoid before they attended the trapezoid classes.

Wilcoxon signed rank tests were also conducted to compare each group of students' achievement in the pre-test and Post-test-01 on trapezoid conceptual questions. Same as the paired t-tests shown in Table 19, the results of the Wilcoxon signed rank tests also showed that there were no significant differences between pre-test and Post-test-01 for the experimental group ($Z = -1.710$, $p = 0.087$ (two-tailed)) as well as for the control group ($Z = -1.155$, $p = 0.248$ (two-tailed)). For the experimental group, the mean of the negative ranks was 4.50 and the mean of the positive ranks was 7.50. For the control group, the mean of the negative ranks was 6.50 and the mean of the positive ranks was 6.50.

On the other hand, as shown in Table 20, there were ***significant*** differences between Post-test-01 and Post-test-02 for the experimental group while there was no significant difference between Post-test-01 and Post-test-02 for the control group.

Table 20: Paired t-test of students' results on trapezoid conceptual questions between Post-test-01 and Post-test-02.

Group	Post-test-01		Post-test-02			95% CI for Mean Difference		
	M	SD	M	SD	n		t	df
Experimental	0.96	0.88	1.57	1.10	28	-1.03, -0.18	-2.92**	27
Control	0.68	0.69	0.68	0.75	25	-0.32, 0.32	0.00	24

**$p < 0.01$.

Table 20 showed the paired t-tests which compared each group of students' achievement in the Post-test-01 and Post-test-02 on trapezoid conceptual questions. Results of the paired

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t-test for the experimental group showed that the mean score differed significantly between Post-test-01 (M = 0.96, SD = 0.88) and Post-test-02 (M = 1.57, SD = 1.10) at the 0.05 level of significance (t = -2.92, df = 27, n = 28, p = 0.007 (two-tailed)). On the other hand, the results of the paired t-test showed for the control group that the mean score did not differ significantly between Post-test-01 (M = 0.68, SD = 0.69) and Post-test-02 (M = 0.68, SD = 0.75) at the 0.05 level of significance (t = 0.00, df = 24, n = 25, p = 1.000 (two-tailed)). These paired t-test results showed that the experimental had ***significantly developed*** their conceptual knowledge on trapezoid after they had attended the trapezoid classes. On the other hand, the control groups had not significantly developed their conceptual knowledge on trapezoid even after they had attended the trapezoid classes.

Wilcoxon signed rank tests were also conducted to compare each group of students' achievement in the Post-test-01 and Post-test-02 on trapezoid conceptual questions. Same as the paired t-tests shown in Table 20, the results of the Wilcoxon signed rank tests also showed that there were significant differences between Post-test-01 and Post-test-02 for the experimental group (Z = -2.539, p = 0.011 (two-tailed)) while there were no significant differences for the control group (Z = 0.000, p = 1.000 (two-tailed)). For the experimental group, the mean of the negative ranks was 8.25 and the mean of the positive ranks was 7.96. For the control group, the mean of the negative ranks was 5.50 and the mean of the positive ranks was 6.50.

The abovementioned independent t-tests and paired t-tests showed that the pedagogical approach adopted by the experimental group was ***more effective than*** the direct instructional approach in developing students' conceptual understanding on the area of trapezoid.

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Pedagogical Content Knowledge Regarding the Topic on Area of ***Trapezoid***

Several phenomena were noticed in the class sessions of the control group and experimental group which helped us to understand why the pedagogical approach adopted in this study was more effective in facilitating students to construct their conceptual understanding in trapezoid. These phenomena enriched our pedagogical content knowledge regarding the mathematical topics on the area of trapezoid.

In the trapezoid discovery sessions, only some of the students in the experimental group were able figure out that the *sum of the upper and lower bases* of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids during individual exploration or group exploration activities without teacher's assistance. Many students needed to seek help from the teacher during group exploration (i.e. even after they had discussed with fellow students in groups of two or four). Initially, both the teacher observer and the researcher of this study presumed that it would be easy for the students to understand that the *sum of the upper and lower bases* of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. That was because in the previous triangle sessions of the experimental group, (a) the students had learned that two identical triangles could form a parallelogram, and (b) almost all of them had been able to figure out, without assistance, that the base of the triangle was equal to the base of the parallelogram formed by two identical triangles. Thus, when they noticed that the students were not able to easily transfer this understanding from the triangle to the trapezoid sessions, they were surprised. The teachers' presumption was incorrect because, for triangle, the base of the triangle was exactly the same as the base of the parallelogram formed by the two identical triangles. On the other hand, one of the bases of the trapezoid was not exactly the same as the base of the parallelogram formed

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by the two identical trapezoids. It was the *sum of the upper and lower bases*, which was equal to the base of the parallelogram. It was the *sum of the upper and lower bases*, which confused the students. Therefore, at the beginning, many students were not able to relate it to what they had learned in the triangle classes. As a result, only some of the students were able to understand it without teacher's mediations.

In the trapezoid discovery sessions of the experimental group, several groups of students approached the teacher and asked how to work out the mathematical formula for calculating the area of the trapezoid. Instead of directly telling the students the formula, the teacher used the cognitive tool to guide the students to construct the formula by themselves. First of all, the teacher asked the students to suggest the formula that could be used for calculating the trapezoid area. Some students would suggest multiplying the *upper base of the trapezoid* by the *height of the trapezoid* divided by 2. Some would suggest multiplying the *lower base of the trapezoid* by the *height of the trapezoid* divided by 2. These suggested formulas revealed the fact that it was difficult for the students to see the base of the parallelogram formed by two identical trapezoids was actually equal to the *sum of the upper and lower bases* of the trapezoid. In order to guide the students to get through this difficulty, the teacher asked the students to ignore the trapezoids and focus on the parallelogram instead (i.e. the parallelogram formed by the two identical trapezoids). The teacher then asked the students to identify the base of that parallelogram. In each group of 4 students, at least 3 students were able to identify the correct base of the parallelogram displayed on the screen of the cognitive tool. The teacher went on to ask the students to suggest how to calculate the base of that parallelogram. In each group of 4 students, at least 3 students either suggested *upper base* or *lower base* only. They were still not able to figure out that the base of the parallelogram was equal to the *sum of the upper and lower bases* of the trapezoid until the teacher further asked

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them by pointing to the upper base and then pointing to the lower base displayed on the screen of the cognitive tool. By then, almost all the 4 students in the group realized that those two lines which formed the base of the parallelogram were the *upper base* and *lower base* of the two identical trapezoids. The teacher continued by asking them to identify the height of the parallelogram; to calculate the area of the parallelogram; and to suggest the formula for calculating the area of the trapezoid. Almost all the students were then able to answer these subsequent questions without any difficulties.

In the experimental group, the teacher acted as a mentor. This arrangement allowed students to proactively seek help from the teacher so that most of the students could effectively construct their conceptual understanding regarding the area of trapezoid. In the students' questionnaire, they also indicated that the combination of the (a) parallelogram and triangle teaching sessions, (b) trapezoid cognitive tool, (c) in-class discussions with fellow students, (e) teacher's whole-class explanation, and (f) teacher's individual coaching in the class helped them further understand the formula for calculating the area of trapezoid. That meant all of these pedagogical activities together with the cognitive tools contributed to their understanding of this formula. For details of the students' questionnaire, please refer to the section "Student Questionnaire (for Experimental Group Only)" in this chapter.

In the control group, the teacher using the traditional direct instructional approach patiently explained to the students that, similar to what they had learned in the triangle classes, the sum of the upper and lower bases of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. He then presented to the students the formula used for calculating the area of a trapezoid. After that, he chose a few questions from the textbook and demonstrated to the student how the trapezoid area could be calculated using the formula. On

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28 October 2015 (please refer to Table 3 for the dates of the classes), he spent 35 minutes to explain the concepts and the formula, demonstrated the calculation, and asked the students to do a few questions in class. Then, he asked the students to do a few questions on trapezoid area calculation at home. In the class session held on 29 October 2015 (i.e. the other 35 minutes), the teacher explained to the students the answers of the trapezoid questions they had done and asked the students to do a few more trapezoid questions in class. It seemed that the teacher of the control group also presumed that it would be easy for students to understand the concepts and formula for the area of trapezoid after they had learned the area of triangle. However, this presumption was incorrect. This incorrect presumption was one of the factors contributing to the ineffectiveness of the direct instruction approach in teaching the concepts of "area of trapezoid". Other factors included the lack of exploration activities and the lack of student-to-student and student-to-teacher interactive discussions for students in the control group to clarify their doubts. All these factors putting together contributed to the relative ineffectiveness of the direct instruction approach adopted in the control group comparing to the pedagogical approach adopted in the experimental group.

Overcame Difficulty "Failure to Identify a Base and the Corresponding Height"

One of the common difficulties that the students had when they were learning the mathematical topic on "area of the closed shapes" was the failure to identify a base and its corresponding height for area calculation (Chan et al., 2014). The cognitive tools adopted in this study were able to help the students to address this difficulty. The visual display in the cognitive tools helped the students to visualize the various sets of base and height of the shapes. For example, the parallelogram cognitive tool displayed one of the two sets of heights and bases when the students clicked the corresponding "base and height" button so that the

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students could visualize that each parallelogram had two sets of base and height. In the cognitive tool, the students could also drag the vertices of the parallelogram to turn it into different shapes of parallelograms. By doing so, the students could observe how the corresponding height and base were changed dynamically so as to explore and understand their relationship, especially the fact that the height and base were perpendicular to each other. For triangles, the triangle cognitive tool displayed one of the three sets of heights and bases when the students clicked the corresponding "base and height" button so that the students could visualize that each triangle had three sets of base and height. The students could also drag the vertices of the triangle to turn it into different shapes of triangles and observe how the corresponding height and base were changed dynamically so as to explore and understand their relationship. Please refer to the subsection "GeoGebra Cognitive Tools" in Chapter 3 for details of these cognitive tools. In addition to the manipulation of the cognitive tools, all the pedagogical activities carried out in the experimental group also contributed to helping students to address this difficulty. Pedagogical activities included exploration prior to explanation, student-to-student interaction, teacher-to-student interaction, and whole-class discussions. Exploration prior to explanation provided opportunities for students to challenge the shortcomings in their prior knowledge or existing conceptions as well as providing opportunities for them to connect new knowledge to their extant knowledge. Student-to-student interaction provided opportunities for students to interact with the peers, discuss, explain, and justify their solutions and interpretations so as to promote students' self-reflection on their thinking and facilitate students to construct their knowledge. Teacher-to-student interaction and whole-class discussions provided opportunities for the teacher to guide the students towards the correct conceptions. The empirical evidence described in the subsequent paragraphs showed that the proposed pedagogical approach was able to help students to address this difficulty.

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It was found that, after both groups had attended all of their respective classes, the students in the experimental group had developed ***significantly better*** capability in identifying the correct bases and heights of the various shapes. Figure 24 showed the average marks that each group obtained in the pre-test, Post-test-01, and Post-test-02 on questions that aimed at assessing students' ability in identifying the correct bases and heights. There were 4 questions which aimed at assessing this ability and each question carried 1 mark. Therefore, the maximum marks that the students could obtain were 4 only.

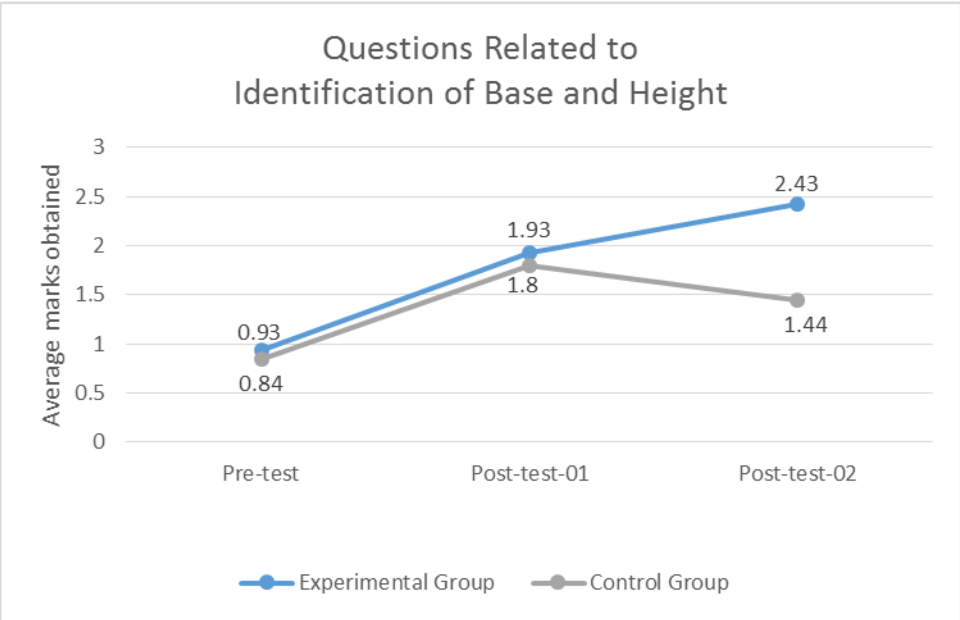

Figure 24: The average marks that each group obtained on questions related to the identification of base and height.

Table 21 showed the independent t-tests which compared the students' achievement on questions that aimed at assessing students' ability in identifying the correct bases and heights. Results of the independent t-test for the pre-test showed that the mean score did not differ significantly between the experimental group ($M = 0.93$, $SD = 1.05$, $n = 28$) and the control group ($M = 0.84$, $SD = 0.80$, $n = 25$) at the 0.05 level of significance ($t = -0.34$, $df = 51$, $p =$

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0.734 (two-tailed)). Results of the independent t-test for Post-test-01 showed that the mean score did not differ significantly between the experimental group (M = 1.93, SD = 1.21, n = 28) and the control group (M = 1.80, SD = 0.82, n = 25) at the 0.05 level of significance (t = -0.46, df = 47.54, p = 0.650 (two-tailed)). On the other hand, the results of the independent t-test for Post-test-02 showed that the mean score differed significantly between the experimental group (M = 2.43, SD = 1.35, n = 28) and the control group (M = 1.44, SD = 1.12, n = 25) at the 0.05 level of significance (t = -2.89, df = 51, p = 0.006 (two-tailed)). These independent t-test results showed that the students in the experimental group had developed ***significantly better*** capability in identifying the correct bases and heights of the various shapes after both groups had attended all of their respective classes.

Table 21: Comparison of students' achievement on questions related to the identification of base and height.

Test	Experimental Group			Control Group			95% CI for Mean Difference	t	df
	M	SD	n	M	SD	n			
Pre-test	0.93	1.05	28	0.84	0.80	25	-0.61, 0.43	-0.34	51
Post-test-01	1.93	1.21	28	1.80	0.82	25	-0.71, 0.45	-0.46	47.54
Post-test-02	2.43	1.35	28	1.44	1.12	25	-1.68, -0.30	-2.89**	51

**p < 0.01.

Mann-Whitney U tests were also conducted to compare the students' achievement on questions that aimed at assessing students' ability in identifying the correct bases and heights. Same as the independent t-tests shown in Table 21, the results of the Mann-Whitney U tests also showed that there were no significant differences between the control and experimental groups in the pre-test (U = 349, p = 0.985 (two-tailed)) and Post-test-01 (U = 324, p = 0.624 (two-tailed)) while there were significant differences between the two groups in Post-test-02 (U = 209, p = 0.010 (two-tailed)). In the pre-test, the means of the ranks for the control group and experimental group were 26.96 and 27.04 respectively. In the Post-test-01, the means of

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the ranks for the control group and experimental group were 25.94 and 27.95 respectively. In the Post-test-02, the means of the ranks for the control group and experimental group were 21.36 and 32.04 respectively.

Note that the Post-test-01 was conducted prior to the trapezoid discovery sessions while the Post-test-02 was conducted after the trapezoid sessions. As mentioned in the previous subsection, it was difficult for the students to understand that the base of the parallelogram formed by two identical trapezoids was equal to the *sum of the upper and lower bases* of the trapezoid. During their trapezoid discovery sessions, most of the students in the experimental group were able to understand this with the guidance and challenges from the teacher. In this process, the students were challenged to recall their prior knowledge on parallelogram and triangle regarding the bases and the corresponding heights. This process further improved the students' ability in the identification of the correct bases and heights of the parallelogram and triangle as well.

In the students' questionnaire, the students in the experimental group also agreed that the interactive visual representation of the cognitive tools were able to assist them in identifying the bases and the corresponding heights of the various shapes. For details of the students' questionnaire, please refer to the section "Student Questionnaire (for Experimental Group Only)" in this chapter.

In addition, when the raters marked the students' pre-test and post-tests, the raters had noticed that many students in the experimental group drew the various shapes in creative ways in the post-tests. These creative drawings showed that the students were able to identify the correct bases and heights of the various shapes. On the other hand, only one student in the control

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group produced these kinds of creative drawings after he was taught the topics through the direct instructional approach.

Questions 14 and 15 of the pre-test or post-tests asked the students to draw the triangles and trapezoids respectively. Question 14 asked the students to draw two triangles with the same area but with different shapes. Question 15 asked the students to draw two trapezoids with the same area but with different shapes. In answering Question 14 and 15, many students in the experimental group drew creative shapes of triangles and trapezoids after they had attended the discovery sessions of this study. In this subsection, the drawings of four students from the experimental group (namely Student A, Student B, Student C and Student D) were selected for discussion. The drawings of the only student in the control group was also illustrated at the end of this subsection.

Figure 25 showed the answers that Student A provided for Question 14 in the pre-test and Post-test-01. In the pre-test, Student A drew two small right-angle triangles. It was easy for him to manually count the unit squares (or areas) occupied by these two small right-angle triangles in order to ensure that they had the same area. After attending the discovery sessions of this study, Student A drew the triangles in a more creative way in Post-test-01. For the triangles he drew in Post-test-01, it would be hard for him to manually count the unit squares (or areas) occupied by the triangles. His drawings in Post-test-01 showed that he understood that as long as the triangles had the same heights and bases, their areas would always be the same. His drawings in Post-test-01 also showed that he could correctly identify the heights and bases of these two triangles even though these triangles were not right-angle triangles. It would be reasonable to believe that the discovery sessions of this study could assist Student A to develop his capability in identifying the correct base and height of triangle.

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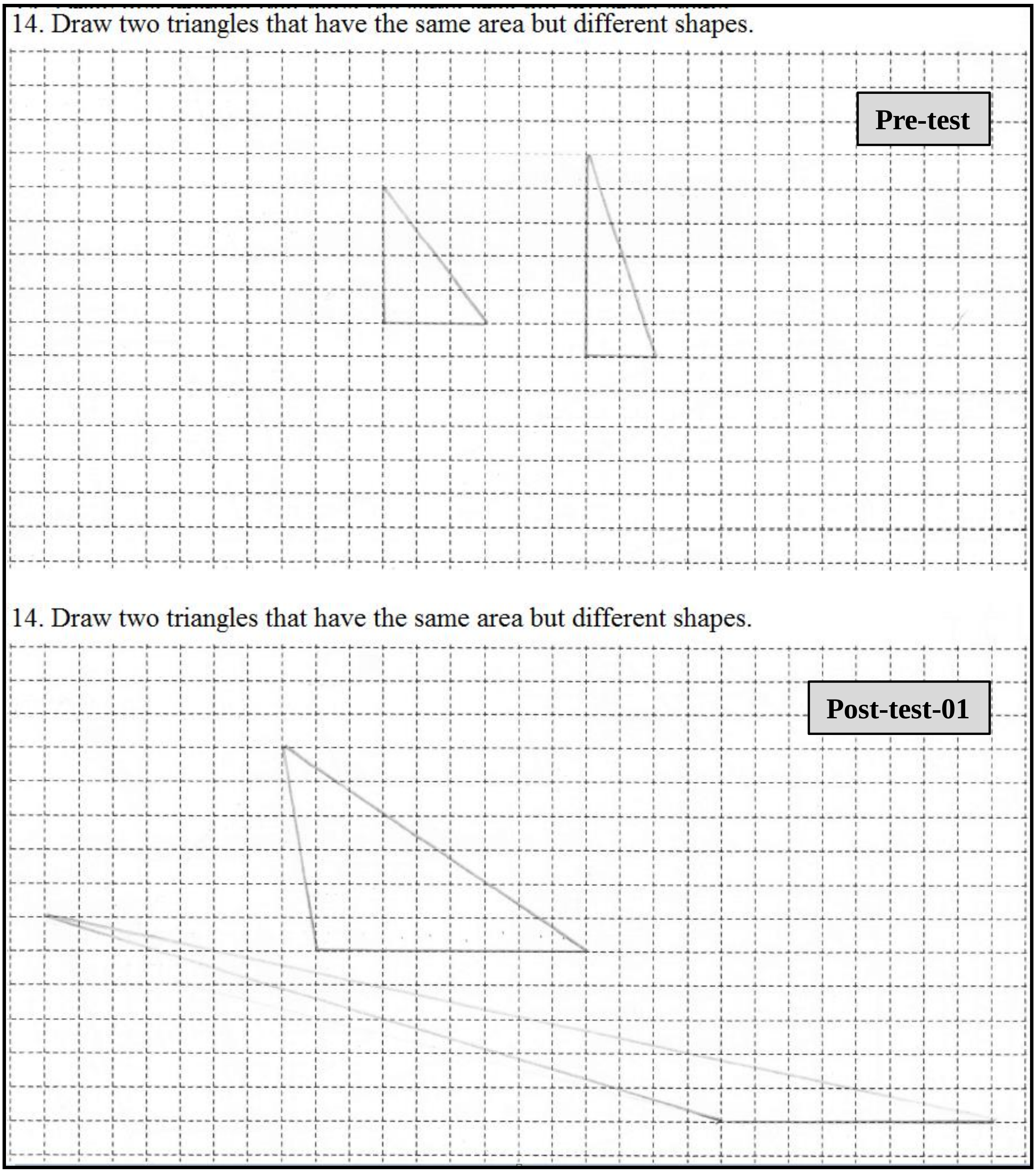

Figure 25: Answers that Student A provided for Question 14 in pre-test and Post-test-01.

Figure 26 and Figure 27 showed the answers that Student B provided for Question 14 and Question 15 respectively. As shown in Figures 26 and 27, Student B drew the triangles and trapezoids in more creative ways after attending the discovery sessions of this study.

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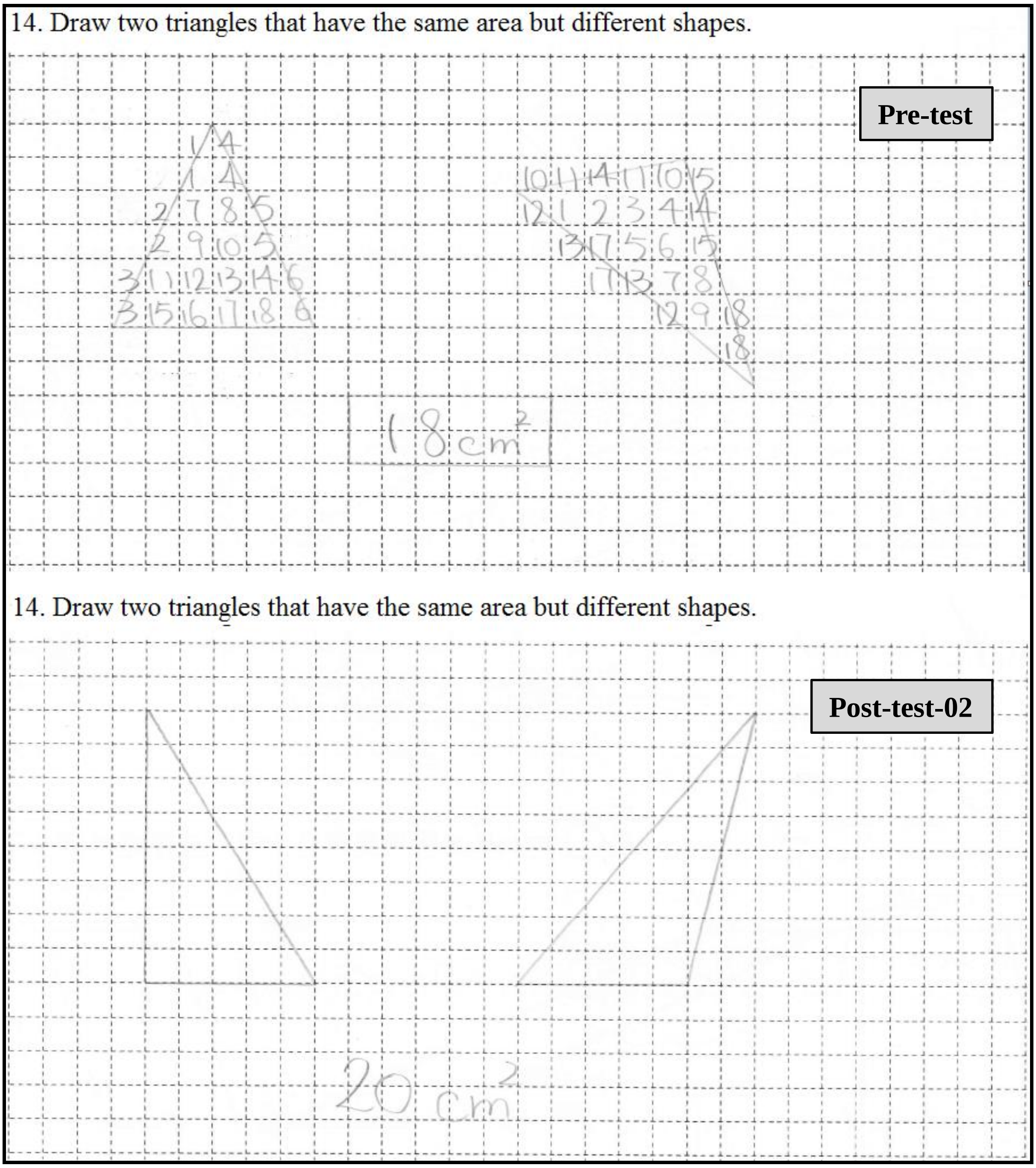

Figure 26: Answers that Student B provided for Questions 14 in pre-test and Post-test-02.

In Figure 26, it was obvious that, in the pre-test, Student B counted the unit squares occupied by the two triangles in order to ensure they had the same area. His drawings in Post-test-02 showed that he understood that as long as the triangles had the same heights and bases, their

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areas would always be the same. The drawings in Post-test-02 also showed that he could correctly identify the heights and bases of these two triangles.

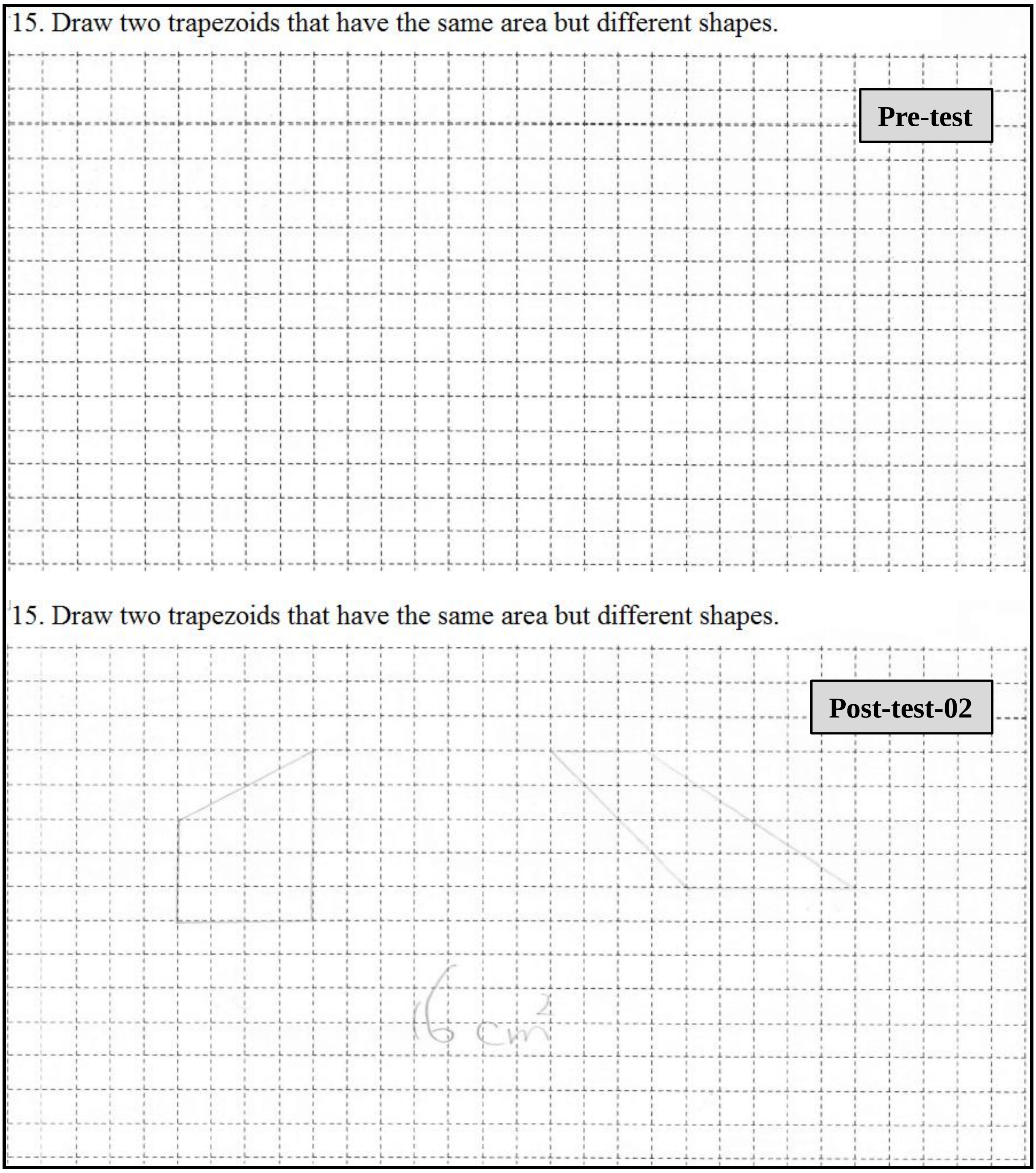

Figure 27: Answers that Student B provided for Questions 15 in pre-test and Post-test-02.

Similarly, in Figure 27, his drawings in Post-test-02 showed that he understood that as long

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as the trapezoids had the same heights, upper bases and lower bases, their areas would always be the same. The drawings in Post-test-02 also showed that he could correctly identify the heights, upper bases and lower bases of these two trapezoids. It would be reasonable to believe that the discovery sessions of this study could assist Student B to develop his capability in identifying the correct bases and heights of triangle and trapezoid.

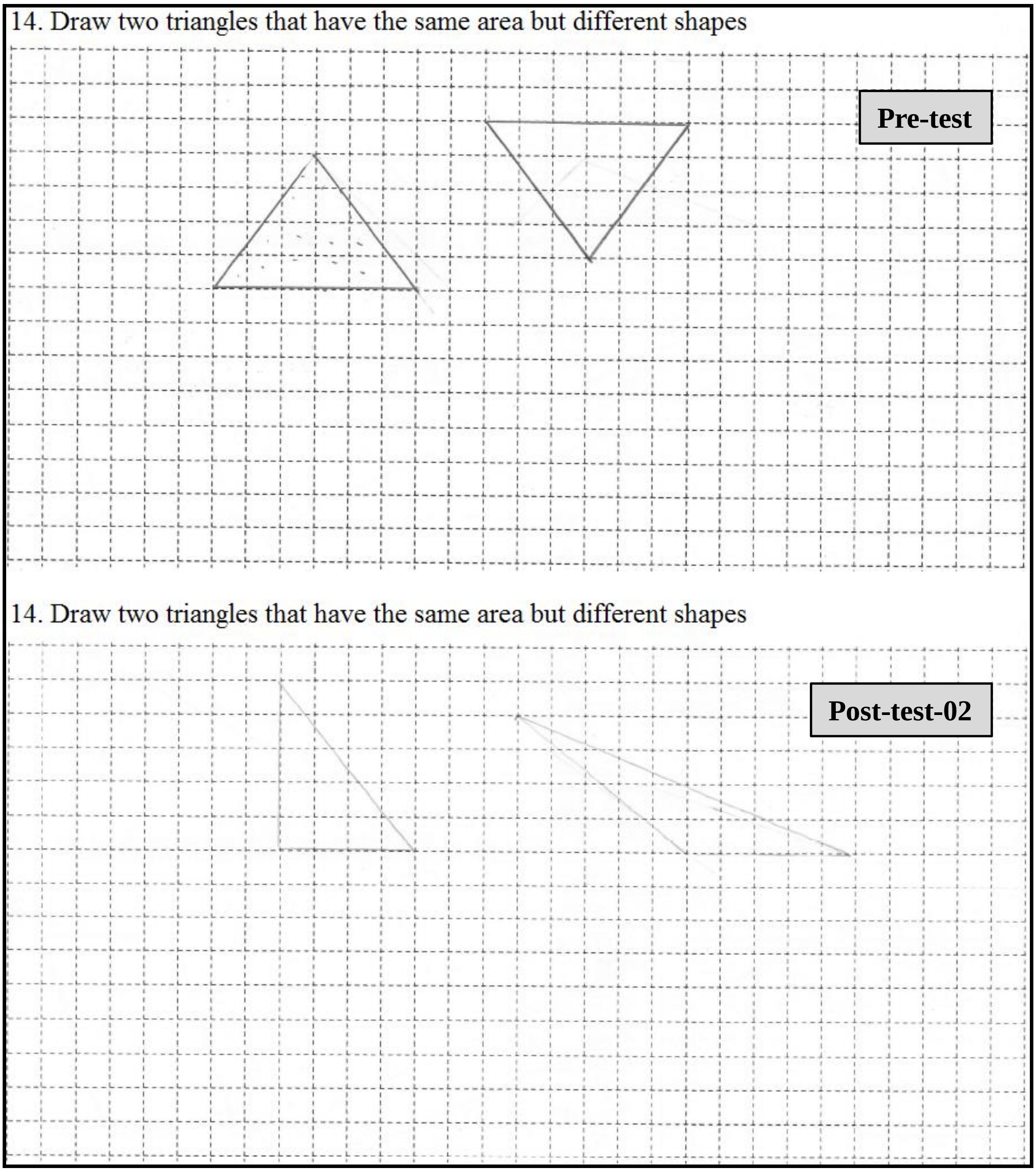

Figure 28: Answers to Questions 14 provided by Student C in pre-test and Post-test-02.

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Figure 28 and Figure 29 showed Student C's answers to Questions 14 and Question 15 respectively. His answers in the pre-test and Post-test-02 showed the similar phenomena of Student B discussed above.

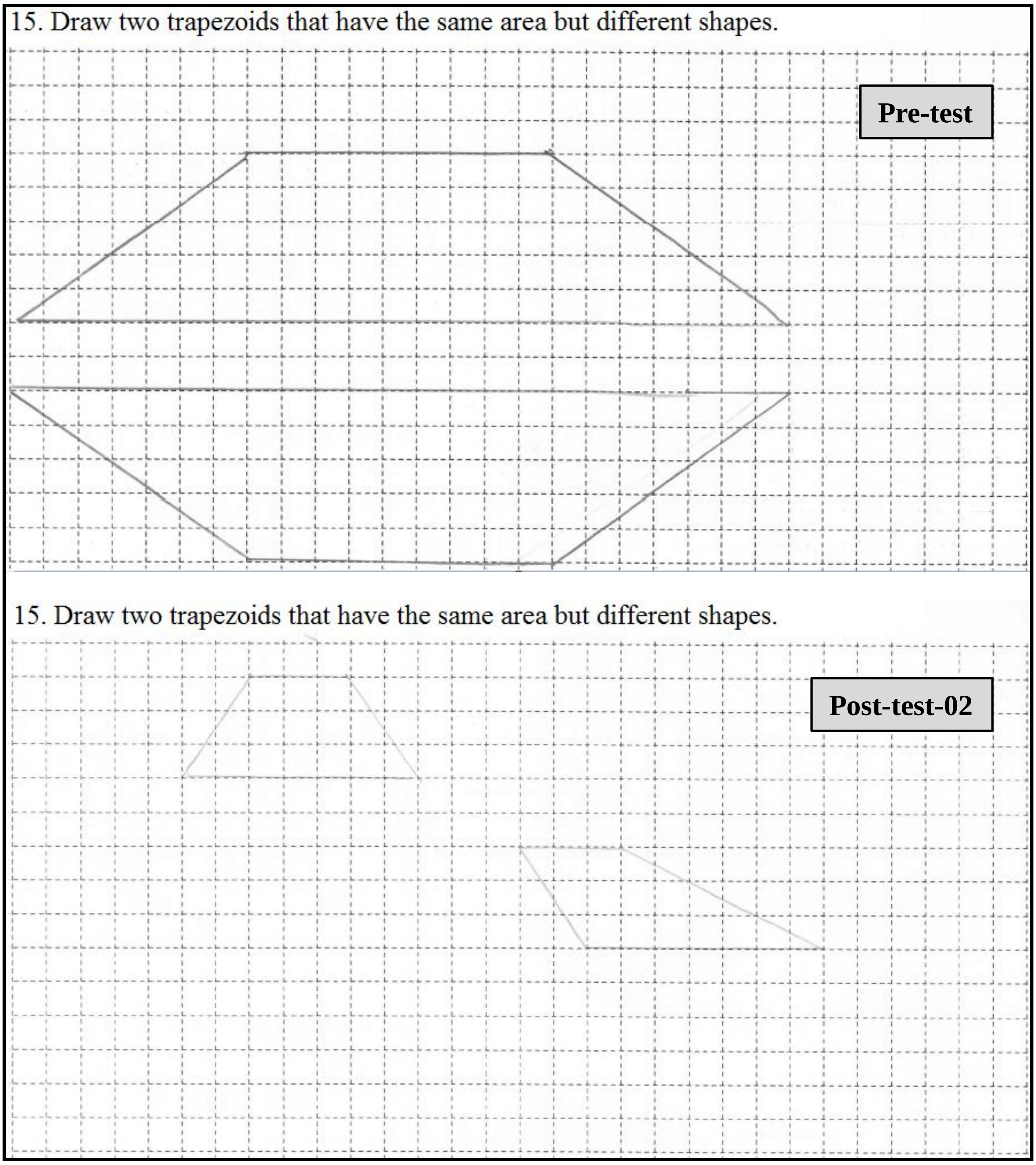

Figure 29: Answers to Questions 15 provided by Student C in pre-test and Post-test-02.

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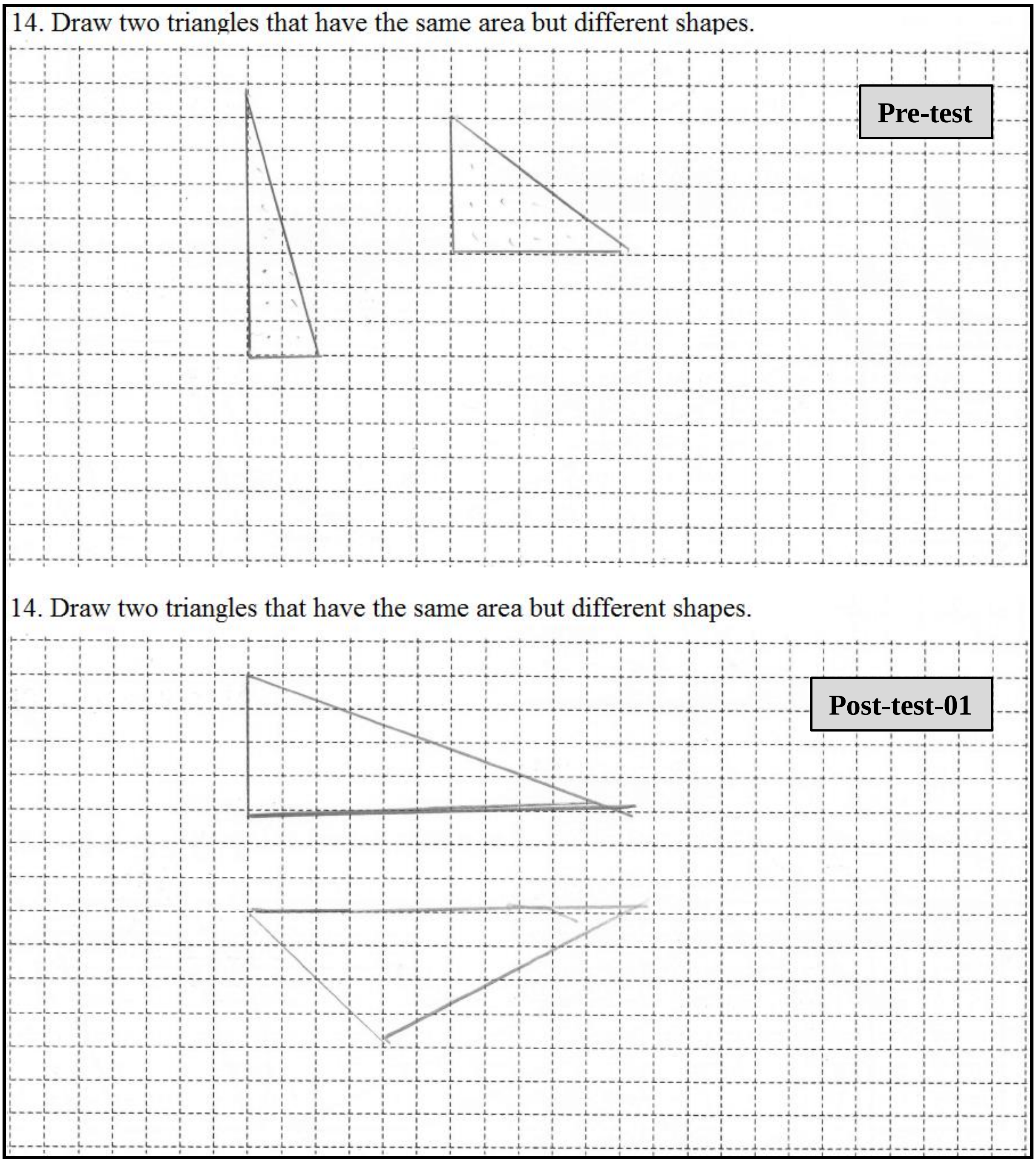

Figure 30: Answers to Questions 14 provided by Student D in pre-test and Post-test-01.

Figure 30 showed Student D’s answers to Questions 14 in the pre-test and Post-test-01. His answers showed the similar phenomena of Student A discussed above.

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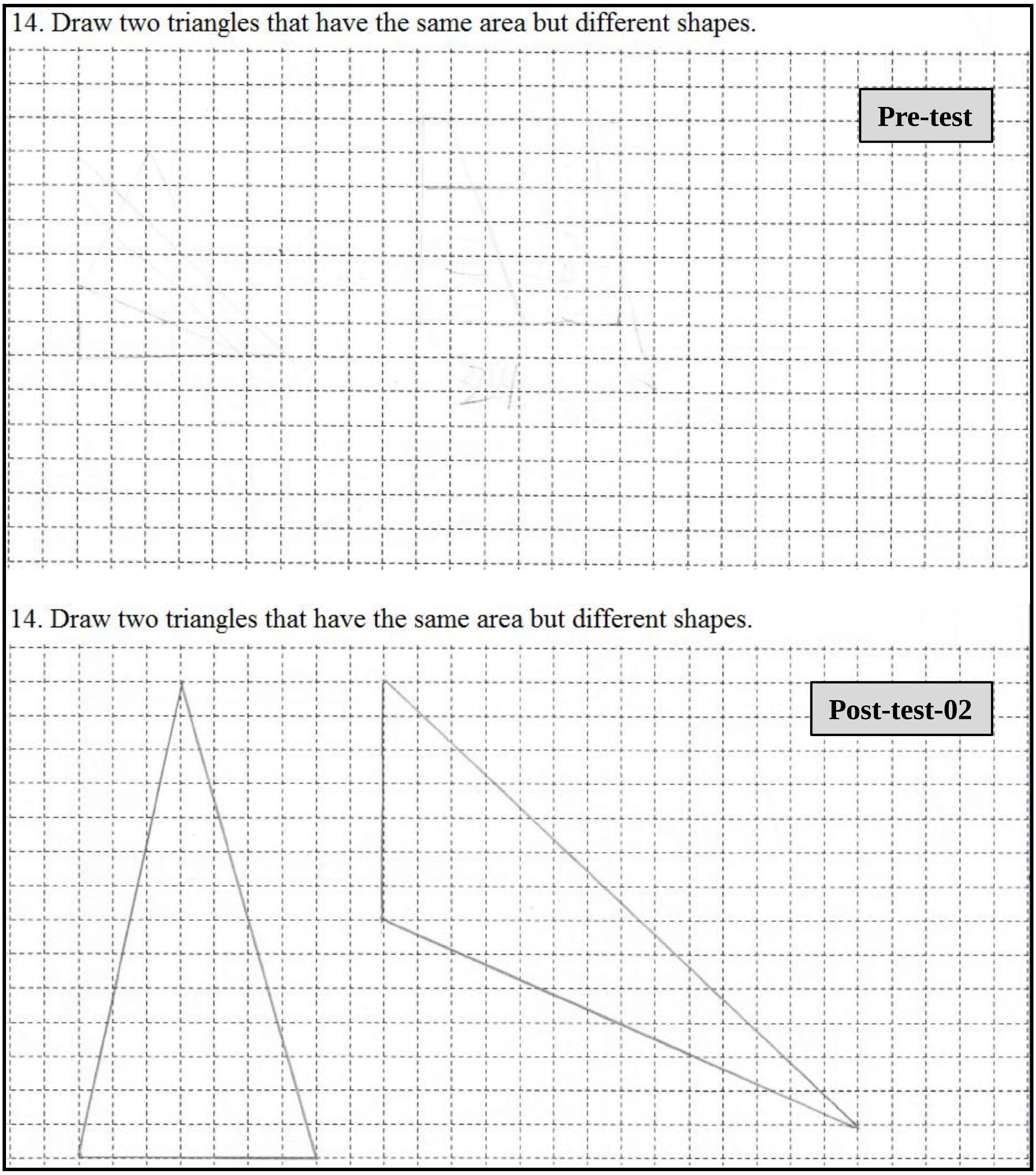

Figure 31: Answers to Questions 14 provided by the student in the control group in pre-test and Post-test-02.

Figure 31 showed the answers to Questions 14 provided by the student in the control group in the pre-test and Post-test-02. He was the only student in the control group who had produced these kinds of creative drawings after he was taught how to calculate the areas of the parallelogram, triangle, and trapezoid through the direct instructional approach. As shown in

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Figure 31, his drawings showed the similar phenomena of Student A and D discussed above.

The Chi-Square Test of Independence was conducted to find out whether the experimental group had a better improvement than the control group in these drawings. In particular, Question 14 and Question 15 were analyzed (Question 14 asked the students to draw two triangles with the same area but different shapes and Question 15 asked the students to draw two trapezoids with the same area but different shapes). For both of these questions, the marks that each student obtained in the pre-test and Post-test-02 were compared. Based on these comparisons, the students were put into one of the four categories, namely: (a) Fail-to-Fail, (b) Fail-to-Pass, (c) Pass-to-Fail, and (d) Pass-to-Pass. The “Fail-to-Fail” category consisted of students who did not answer the particular question correctly in both of the pre-test and Post-test-02. The “Fail-to-Pass” category consisted of students who did not answer the particular question correctly in the pre-test while they answered that question correctly in Post-test-02. The “Pass-to-Fail” category consisted of students who answered the particular question correctly in the pre-test while they did not answer that question correctly in Post-test-02. The “Pass-to-Pass” category consisted of students who answered the particular question correctly in both of the pre-test and Post-test-02. Table 22 and Table 23 summarized the number of students who fell into different categories for Question 14 and Question 15 respectively. For example, Table 22 showed that in the experimental group, there were 8 students who did not answer Question 14 correctly in both of the pre-test and Post-test-02, 10 students who did not answer Question 14 correctly in the pre-test but answered it correctly in Post-test-02, 1 student who answered Question 14 correctly in the pre-test but did not answer it correctly in Post-test-02, and 9 students who answered Question 14 correctly in both of the pre-test and Post-test-02.

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Table 22: The number of students in different categories for Question 14.

Group	Category				Total
	Fail-to-Fail	Fail-to-Pass	Pass-to-Fail	Pass-to-Pass	
Experimental	8	10	1	9	28
Control	11	6	3	5	25

Table 23: The number of students in different categories for Question 15.

Group	Category				Total
	Fail-to-Fail	Fail-to-Pass	Pass-to-Fail	Pass-to-Pass	
Experimental	8	9	2	9	28
Control	11	6	3	5	25

Table 22 and Table 23 showed that for both of Question 14 and 15, more students in the experimental group fell into the “Fail-to-Pass” category when comparing with the control group. On the other hand, less students in the experimental group fell into the “Pass-to-Fail” category when comparing with the control group. These were good indications showing that the experimental group might had a better improvement than the control group in terms of answering Question 14 and 15 in Post-test-02.

The Chi-Square Tests of Independence were conducted for Question 14 and 15 based on the numbers shown in Table 22 and Table 23 respectively. For Question 14, the Chi-Square Test showed that the control group and experimental group were not found to be significantly related at the 0.05 level of significance (Pearson χ^2 (3, N = 53) = 3.458, p = 0.326). For Question 15, the Chi-Square Test also showed that the control group and experimental group were not found to be significantly related at the 0.05 level of significance (Pearson χ^2 (3, N = 53) = 2.254, p = 0. 521). Based on these Chi-Square Tests, we could not say that the pedagogical approach and cognitive tools adopted in the experimental group had an influence on the improvement in Question 14 and 15.

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Nevertheless, the independent t-tests, the Mann-Whitney U tests, the feedback from student's questionnaire, and the creative drawings discussed earlier in this subsection still provided evidence to support that the students in the experimental group had developed better capability in identifying the correct bases and heights of the various shapes. Because the pedagogical approach adopted by the experimental group helped the students to overcome this conceptual difficulty, the experimental group students developed significantly better conceptual understanding than the control group students. Moreover, because improving conceptual understanding had positive effect on procedural knowledge, the experimental group students performed as well as the control group students in terms of the development of their procedural knowledge even though the experimental group did less in-class exercises and homework than the control group.

Overcame Difficulty "Misconception that Only Regular Shapes Had Measurable Area"

Another common difficulty that the students had when they were learning the mathematical topic on "area of the closed shapes" was the misconception that only regular closed shapes had measurable area that could be calculated by mathematical formulas (Chan et al., 2014). The cognitive tools adopted in this study were able to help the students to address this difficulty. The parallelogram cognitive tool allowed the students to interactively cut the parallelogram into two pieces and slide one of the pieces to turn the parallelogram into a rectangle. By visualizing that the parallelogram could be turned into a rectangle, the students could realize the mathematical formula used for calculating the area of the parallelogram. The triangle cognitive tool allowed the students to replicate an identical triangle and rotate the replicated triangle in order to form a parallelogram with these two identical triangles. By

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visualizing that two identical triangles could form a parallelogram, the students could apprehend the mathematical formula used for calculating the area of the triangle. Similarly, the trapezoid cognitive tool allowed the students to replicate an identical trapezoid and rotate the replicated trapezoid in order to form a parallelogram with these two identical trapezoids. By visualizing that two identical trapezoids could form a parallelogram, the students could understand the mathematical formula used for calculating the area of the trapezoid. Please refer to the subsection “GeoGebra Cognitive Tools” in Chapter 3 for details of these cognitive tools. Chan et al. (2014) suggested that, by doing so, the students “could then find that irregular closed shapes like triangles and trapezoids, the same as with other regular closed shapes like squares, had measurable area that could be calculated by mathematical formulas” (p. 351). As explained in the previous subsection, in addition to the manipulation of the cognitive tools, all the pedagogical activities carried out in the experimental group also contributed to helping students to address the common difficulty that the students had. For example, exploration prior to explanation provided opportunities for students to think hard in order to relate the area of a parallelogram to that of a rectangle (the knowledge on the area of rectangle was the students’ extant knowledge). Exploration of triangle and trapezoid allowed students to understand that these shapes could be transformed to parallelogram which in turn could be transformed to rectangle so as to facilitate the students to gradually induce the mathematical formulas for calculating the areas of triangle and trapezoid (i.e. allowed the students to relate the new knowledge on triangle and trapezoid to their extant knowledge on parallelogram and rectangle).

According to the feedback of the students’ questionnaire, 26 out of the 28 students in the experimental group agreed or strongly agreed that the cognitive tools helped them to understand that the parallelograms, triangles and trapezoids had measurable areas which

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could be calculated by mathematical formulas. Moreover, 25 out of the 28 students in the experimental group agreed or strongly agreed that the cognitive tools assisted them in understanding how to construct the mathematical formulas used for calculating the areas of the various shapes and helped them to learn the computation of the areas of the various shapes. For details of the students' questionnaire, please refer to the section "Student Questionnaire (for Experimental Group Only)" in this chapter.

In the teacher observer's interview, the teacher observer also said that the cognitive tools definitely helped the students in understanding that irregular shapes (such as trapezoids) had measurable areas which could be calculated by mathematical formulas. He noticed that, after the teaching cycles, most of the students were able to understand the meaning of the mathematical formulas used for calculating the areas of the various shapes. For details of the teacher observer's interview, please refer to the section "Interview of Teacher Observer" in this chapter.

Understanding that irregular closed shapes also had measurable area that could be calculated by mathematical formulas was important in helping students to develop their conceptual understanding and procedural knowledge. Because the pedagogical approach adopted by the experimental group helped the students to overcome this conceptual difficulty, the experimental group students developed significantly better conceptual understanding than the control group students. Moreover, because improving conceptual understanding had positive effect on procedural knowledge, the experimental group students performed as well as the control group students in terms of the development of their procedural knowledge even though the experimental group did less in-class exercises and homework than the control group.

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Student Interview

Based on the results of Post-test-02, six students from the experimental group (namely Student E1 to E6) and six students from the control group (namely Student C1 to C6) were selected for interviews. Each of the selected students were interviewed separately. These interviews aimed at providing supplementary information on whether the students possessed the conceptual understanding and procedural knowledge that the pre-test and post-tests intended to assess. The interviews were conducted within one week after the completion of Post-test-02. The students' interviews showed that, in general, the marks that the students obtained in Post-test-02 reflected their conceptual understanding and procedural knowledge on the topics.

Question 1 aimed at assessing students' procedural knowledge. This question asked the students to compute the area of a square. Eleven out of the 12 students interviewed answered this question correctly in Post-test-02. Among these 11 students who got the correct answers, only one of them was able to explain why the area of a square could be found by multiplying the length of one side by itself. She was the only one who could explain that multiplying the length of one side of the square by itself would find the number of "unit squares" that covered the square. The other 10 students, who answered this question correctly in Post-test-02, said that they did not know why. They simply said that the teachers taught them to use this formula to calculate the area of a square. The interviews showed that the majority of the students interviewed did not possess the conceptual understanding regarding the area of square even though they had gained the procedural knowledge on this topic.

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Question 2 aimed at assessing students' procedural knowledge. This question asked the students to compute the area of a rectangle. Ten out of the 12 students interviewed answered this question correctly in Post-test-02. Among these 10 students who got the correct answers, only one of them was able to explain that multiplying the length by the width of the rectangle would find the number of "unit squares" that covered the rectangle. The other 9 students said that they did not know why. They simply said that the teachers taught them to use this formula to calculate the area of a rectangle. The interviews showed that the majority of the students interviewed did not possess the conceptual understanding regarding the area of rectangle even though they had gained the procedural knowledge on this topic.

Question number 3 aimed at assessing students' conceptual understanding. This question showed three parallelograms and asked the students to identify which parallelograms had the same area. In fact, all the three parallelograms had the same area. Only 2 out of the 12 students interviewed answered this question correctly in Post-test-02. Both of these 2 students said that the three parallelograms had the same area as long as the lengths of their bases were the same and the lengths of their heights were also the same. The interviews showed that these two students had gained the conceptual understanding of this target topic.

Question number 4 aimed at assessing students' conceptual understanding. This question showed four parallelograms with height and base labels in each of them. Only two of these four parallelograms had correct height and base labels. The question asked the students to identify the parallelograms with correct labels. Only 7 out of the 12 students interviewed answered this question correctly in Post-test-02. All of these 7 students were able to give the correct justifications during the interview. They were able to tell that the height was the length of the line perpendicular to the base which extended from the base to the opposite side

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parallel to the base. The interviews showed that all of these 7 students had gained the conceptual understanding regarding the base and corresponding height of parallelogram.

Question number 5 aimed at assessing students’ procedural knowledge. This question asked the students to compute the area of a parallelogram. Nine out of the 12 students interviewed answered it correctly in Post-test-02. Among these 9 students who got the correct answers, only 5 of them were able to explain that the parallelogram could be transformed to a rectangle and the area of the parallelogram was equal to the area of the rectangle formed so that we could use the formula of rectangle to find the area of the parallelogram. The other 4 students, who answered this question correctly in Post-test-02, said that they did not know why. They simply said that the teachers taught them to use this formula to calculate the area of a parallelogram. The interviews showed that for the 9 students who got the correct answers, around half of them did not possess the conceptual understanding regarding the area of parallelogram even though they had gained the procedural knowledge on this topic.

Question number 6 aimed at assessing students’ conceptual understanding. This question showed four triangles and asked the students to identify which triangles had the same area. In fact, only two triangles had the same area. Only 8 out of the 12 students interviewed answered it correctly in Post-test-02. Among these 8 students who got the correct answers, two students said that the triangles had the same area as long as the lengths of their bases were the same and the lengths of their heights were also the same. Three out of these 8 students said that they calculated the area of the triangles using their bases and heights. The other 3 out of these 8 students were not able to give the correct justifications. Two of them said that they simply counted the small squares inside the triangles to come up with the correct answer. The other one student said that he just wild-guessed. The interviews showed

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that for this question, it was possible that not all the 8 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Question number 7 aimed at assessing students' conceptual understanding. This question showed a triangle with four dotted lines and asked the students to identify the two dotted lines which were the correct heights of the triangle. Only 4 out of the 12 students interviewed answered it correctly in Post-test-02. All of these 4 students were able to point out the corresponding bases of the correct heights that they had identified. The interviews showed that all of these 4 students had gained the conceptual understanding regarding the base and corresponding height of triangle.

Question number 8 aimed at assessing students' conceptual understanding. This question showed three triangles and their heights were indicated in the diagrams. The question asked the students to identify the corresponding bases of the triangles. Nine out of the 12 students interviewed answered this question correctly in Post-test-02. Among these 9 students, only 8 of them were able to tell that the height was perpendicular to the base. The remaining one student, who answered this question correctly, tried to explain by saying, "If you rotate this triangle, this was the base". However, he could not clearly explain how he could identify the corresponding bases of the heights. The interviews showed that for this question, it was possible that not all the 9 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Question 9 aimed at assessing students' procedural knowledge. This question asked the students to compute the area of a triangle. Only 7 out of the 12 students interviewed answered this question correctly in Post-test-02. Among these 7 students, only 5 of them were able to

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explain that two identical triangles could form a parallelogram so that the area of the triangle was half of the area of the parallelogram formed. Two out of these 7 students were not able to give the correct justifications. One of them said that he did not know why the area of a triangle could be calculated by multiplying the base and height and dividing that by 2. The other one offered a wrong justification. She said that the product of the base and height was the total area of the large and small triangle shown in the diagram. She explained that the area of the large triangle could be obtained by dividing this total area by 2. The interviews showed that not all of these 7 students had gained the conceptual understanding regarding the area of triangle even though they had gained the procedural knowledge on this topic.

Both Question 10 and Question 13 aimed at assessing students' procedural knowledge. Both of these questions asked the students to compute the area of a trapezoid. For both of these two questions, 9 out of the 12 students interviewed answered this question correctly in Post-test-02. Among these 9 students, only 7 of them were able to give the correct justifications. They were able to explain that two identical trapezoid could form a parallelogram so that the area of the trapezoid was half of the area of the parallelogram formed. Out of these 7 students, only 5 of them were also able to tell that the sum of the upper and lower bases of the trapezoid was equal to the base of the parallelogram formed. Two out of these 9 students, who answered this question correctly in Post-test-02, were not able to give any correct justifications. Both of them said they only knew that the area of a trapezoid could be calculated using the formula "(upper base + lower base) $\times$ height $\div$ 2". One of them even said that this formula did not relate to the area of the parallelogram. The interviews showed that not all of these 9 students had gained the conceptual understanding regarding the area of trapezoid even though they had gained the procedural knowledge on this topic.

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Question number 11 aimed at assessing students' conceptual understanding. This question showed four trapezoids with height and base labels in each of them. Only two of these four trapezoids had correct height and base labels. The question asked the students to identify the trapezoids with correct labels. Only 2 out of the 12 students interviewed answered this question correctly in Post-test-02. Both of them were able to tell that the height was the length of the line perpendicular to the upper and lower bases. The interviews showed that these two students had gained the conceptual understanding of this target topic.

Question number 12 aimed at assessing students' conceptual understanding. This question showed four trapezoids and asked the students to identify which trapezoids had the same area. In fact, only three trapezoids had the same area. Only 4 out of the 12 students interviewed answered it correctly in Post-test-02. Among these 4 students who got the correct answers, two students said that they calculated the area of the trapezoids using their bases and heights. The other 2 students were not able to give the correct justifications. They simply counted the small squares inside the trapezoids to come up with the correct answer. The interviews showed that for this question, it was possible that not all the 4 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Question number 14 aimed at assessing students' conceptual understanding. This question ask the students to draw two triangles that had the same area but different shape. Only 7 out of the 12 students interviewed answered it correctly in Post-test-02. Among these 7 students who got the correct answers, two students said that the triangles had the same area as long as the lengths of their bases were the same and the lengths of their heights were also the same. Two out of these 7 students said that they calculated the area of the triangles using their bases

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and heights. The other 3 out of these 7 students were not able to give the correct justifications. They simply counted the small squares inside the triangles to ensure that they had the same area after they drew the triangles. The interviews showed that for this question, it was possible that not all the 7 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Question number 15 aimed at assessing students' conceptual understanding. This question ask the students to draw two trapezoids that had the same area but different shape. Only 7 out of the 12 students interviewed answered it correctly in Post-test-02. Among these 7 students who got the correct answers, two students said that the trapezoids had the same area as long as their heights were the same and the sum of their upper and lower bases were also the same. One out of these 7 students said that she calculated the area of the trapezoids using their bases and heights. The other 4 out of these 7 students were not able to give the correct justifications. They simply counted the small squares inside the trapezoid to ensure that they had the same area after they drew the trapezoids. The interviews showed that for this question, it was possible that not all the 7 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Question number 16 aimed at assessing students' conceptual understanding. This question ask the students to draw two different parallelograms with the same area but one of them should have a base two times the length of the other one. Only 6 out of the 12 students interviewed answered it correctly in Post-test-02. Among these 6 students who got the correct answers, 3 students said that they calculated the area of the parallelograms using their bases and heights. The other 3 out of these 6 students were not able to give the correct justifications. They simply counted the small squares inside the parallelograms to ensure that they had the

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same area after they drew them. The interviews showed that for this question, it was possible that not all the 6 students who answered this question correctly in Post-test-02 possessed the conceptual understanding.

Interview of Teacher Observer

An interview with the original mathematics teacher of the “experimental group” class was conducted within one week after the completion of this study. He was present in every class session of the experimental group throughout this study, but he only acted as an observer without facilitating any of the sessions. Table 24 summarized the teacher observer’s feedback in the interview (these interview questions were adopted and modified from Kong and Li (2007).

Although the students had to be taught how to use the GeoGebra cognitive tools in the first place, the teacher observer agreed that these cognitive tools were easy for students to use and the students were able to manipulate the tools by themselves independently. He strongly agreed that the cognitive tools used in this study could foster student-student interactions. He said that, in terms of facilitating the discussions between students, these cognitive tools were much better than the interactive e-books. However, he expressed reservations regarding whether these tools were able to foster the teacher-student interactions because he did not see many bidirectional teacher-student discussions using these tools throughout this study.

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Table 24: Feedback provided by the teacher observer of the experimental group during the interview.

Question	Teacher observer's feedback
Were the cognitive tools easy for students to use?	Yes
Did the cognitive tools foster teacher-student interactions?	It was hard to tell because there were not many bidirectional teacher-student discussions using these tools
Could the students use the cognitive tools to discuss with their classmates?	Strongly agreed that the cognitive tools used in this study could foster student-student interactions
Could the cognitive tools help the students to:	
(a) understand the concept of area conservation?	It was hard to tell but it was obvious that the students knew the areas were conserved after cutting and re-arranging the shapes using the tools
(b) understand that irregular shapes (e.g. trapezoid) also had measurable areas which could be calculated by mathematical formulas?	Definitely helped the students in understanding this
(c) understand why the areas of the various shapes could be calculated using particular mathematical formulas?	Able to help most of the students to understand
(d) identify the height and base of the various shapes?	Might not be as effective as the traditional direct teaching
What were the main purposes of teaching area? Had the cognitive tools met these purposes?	Two main purposes: 1. Developed students' concept and understanding of the surface occupied by the particular shapes (the cognitive tools were able to meet this purpose) 2. Train the students to get good marks in the examinations (the cognitive tools did not meet this purpose)
Were there any aspects that needed to be improved regarding the cognitive tools?	– Add application-type questions to the cognitive tools – Asked students to calculate the base by giving them the area and the height (or to calculate the height by giving them the area and the base) – The cognitive tools were too slow when running in Samsung tablets
How would you evaluate this pedagogical approach?	This pedagogical approach was absolutely the right approach for teaching area, especially to the students of this generation
Presuming that you would use these cognitive tools next year, how would you make use of them?	Use exactly the same pedagogical approach of this study
Was this pedagogical approach worth promoting? If yes, how would you promote it?	It was worth promoting to others. He would first of all promote it to the teachers who were teaching the same grade. Then, promoted it to the whole school
What would be your overall comments regarding this pedagogical approach?	The approach might need to be adjusted for different students. Moreover, the University A should train the student teachers to use this pedagogical approach

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The teacher observer stated that it would be hard to tell whether the cognitive tools were able to help the students to understand the concept of area conservation. However, as the students had been cutting and re-arranging the shapes for many times using the cognitive tools, it was obvious that they definitely knew that the areas were conserved after the cutting and re-arranging. Nevertheless, the students might not be able to describe clearly the definition of area conservation but he believed that they understood this concept.

The teacher observer said the cognitive tools definitely helped the students in understanding that irregular shapes (such as trapezoids) had measurable areas which could be calculated by mathematical formulas. He noticed that, after the teaching cycles, most of the students were able to understand the meaning of the mathematical formulas used for calculating the areas of the various shapes. However, he pointed out that the cognitive tools might not be as effective as the traditional direct teaching in helping the students to identify the height and base. The teacher observer mentioned that some students were still unable to identify the correct height and base in their subsequent homework. He had a perception that the identification of height and base was not the focus of the cognitive tools.

The teacher observer mentioned that there were two main purposes of teaching the topic of area. The first main purpose was to develop students' concept and understanding of the surface occupied by the particular shapes. He strongly agreed that the cognitive tools were able to meet this purpose and help the students clearly understand the concept of area. The second purpose of teaching the topic of area was to train the students to get good marks in the examinations (i.e. being correct in answering the examination questions). In this regard, the cognitive tools did not meet the purpose, especially in helping the students to get the "application-type" questions correct. In order to meet the second purpose, he suggested to

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add some application-type questions to the cognitive tools or to the worksheet used in this study (e.g. questions that asked the students to calculate the cost of building a trapezoid-shaped lawn with a particular set of dimensions). He also suggested that the students should be asked to calculate the base by giving them the area and the height (or to calculate the height by giving them the area and the base) during the teaching cycles. The teacher observer noticed that the GeoGebra cognitive tools were too slow when they were run in the Samsung tablets. He suggested to improve the response time of these GeoGebra cognitive tools.

The teacher observer expressed a positive view on the pedagogical approach adopted in this study. He stated that it was absolutely the right approach for teaching the topic of area, especially to the students of this generation. In the subsequent homework submitted by the students, he noticed that the students had developed very good conceptual understanding. He quoted an example which he said he had never seen any students performed the calculation in such a way before. For that particular homework question, the students were given the area and the base of a triangle and they were asked to calculate the height. He said that, in the past, all the students would only memorize the formula (i.e. area x 2 ÷ base) taught by the teacher and they would directly apply the formula in answering this kind of questions. However, he was very surprised when he saw a student in the experimental group solved this question by doubling the area of the triangle "by heart" and then dividing the doubled area by base (i.e. doubled area ÷ base). The teacher observer said that, in his past ten years of teaching experience, he had never seen any students did that before. He concluded that this was due to the fact that the students in the experimental group were not taught the formula (i.e. area x 2 ÷ base) while the student had a very good conceptual understanding that he could find the height by using two identical triangle to form a parallelogram first. However, he commented

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that the whole period of this study did not have sufficient time for students to digest the material before they took the post-tests. He had the perception that the result of the post-tests would have been better if the students were given a few days to digest the material (even though he admitted that he did not know the results of the post-tests).

The teacher observer said that he preferred to use these cognitive tools in next year's teaching and he would use exactly the same pedagogical approach of this study. Nevertheless, he would balance between the development of students' conceptual understanding and the training of students' examination techniques. He thought this pedagogical approach was worth promoting to others. He would first promote it to the teachers who were teaching the same grade of students. Then, he would promote it to the whole school. One of the way to promote this pedagogical approach might be to produce a video and put the video onto the internet, for example YouTube. He suggested that the pedagogical approach might need to be adjusted depending on the characters and the quality of the students. He also suggested that University A should train the student teachers to use this pedagogical approach, so that all the student teachers could readily master this approach when they graduated from University A.

Student Questionnaire (for Experimental Group Only)

The students in the experimental group were asked to complete a questionnaire. Questions 1 to 16 of the questionnaire were devised to collect students' perceptions on the cognitive tools used in this study. Question 17 of the questionnaire was designed to understand the various ways through which the students learned the formula for calculating the area of trapezoid.

The students' questionnaire provided qualitative data for: (a) investigating the learning effect

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and the potential of using these cognitive tools to support the learning process of this subject topic; and (b) which activities or tools assisted the students' learning the target topics. The following two subsections summarized the findings.

Students' Perceptions on the Cognitive Tools

Questions 1 to 16 of the students' questionnaire were devised to collect students' perceptions on the GeoGebra cognitive tools used in this study. The Cronbach's alpha coefficient of the experimental group of this study was 0.87. According to Crano et al. (2015), if the Cronbach's alpha coefficient was 0.75 or higher, the degree of internal consistency reliability was acceptable. Therefore, question 1 to 16 of this questionnaire could be considered as having good internal consistency reliability.

In general, the students in the experimental group had positive perceptions on the cognitive tools. Table 25 summarized the students' feedback regarding the use of the GeoGebra cognitive tools in their learning of area. The students agreed that the cognitive tools were easy to use and they were able to operate the tools independently. They were able to understand the activities provided on the interface of the cognitive tools and the interface displays were compatible with those in the common learning materials they had encountered. The graphics and the interactive visual representation of the cognitive tools were able to assist the students in learning the key concepts of area and in identifying the bases and the corresponding heights of the various shapes.

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Table 25: Students' perception of the GeoGebra cognitive tools in their learning of area.

Evaluation item	Mean (S.D.)
Interface design of the cognitive tool	
I understand the activities provided on each computer interface	4.57 (0.57)
I can undertake the activities on each computer interface independently	4.54 (0.58)
The application is easy to use	4.43 (0.74)
The computer interface displays are compatible with those in common learning materials	4.21 (0.88)
Scaffold support of the cognitive tool	
The graphics assist me in learning the key concepts of area	4.75 (0.44)
I can use the cognitive tools to discuss with my classmates the ways of calculating areas of the shapes	4.57 (0.69)
The cognitive tools help me to understand the parallelograms, triangles and trapezoids have measurable areas which can be calculated by mathematical formulas	4.54 (0.74)
The interactive visual representation assists me in identifying the bases and the corresponding heights of the various shapes	4.50 (0.64)
The graphical manipulation assists me in understanding that the area is conserved after I cut the original shape into smaller pieces and re-combine them to form different shapes	4.46 (0.74)
The cognitive tools assist me in understanding how to construct the mathematical formulas which are used for calculating the areas of the various shapes	4.43 (0.69)
Overall perceptions of the cognitive tool	
The cognitive tools help me to learn the key concepts of area	4.54 (0.64)
I am confident of operating the cognitive tools independently	4.54 (0.64)
The cognitive tools help me to learn the computation of the areas of the various shapes	4.50 (0.69)
I like mathematics more after using the cognitive tools	4.07 (1.05)
Continued Usage of the cognitive tool	
I am interested in continuing to use the cognitive tools for learning	4.46 (0.88)
I am willing to introduce these cognitive tools to other schoolmates	4.11 (0.88)

Notes: 1 = strongly disagree; 2 = disagree; 3 = neutral; 4 = agree; 5 = strongly agree.

The students agreed that the cognitive tools helped them to understand that the parallelograms, triangles and trapezoids had measurable areas which could be calculated by mathematical formulas. They also agreed that the cognitive tools assisted them in understanding how to construct the mathematical formulas used for calculating the areas of the various shapes and helped them to learn the computation of the areas of the various shapes. They could use the cognitive tools to discuss with their classmates in the exploration.

The students' overall perceptions were that the cognitive tools could help them to learn the key concepts of area, and learn the computation of the areas of the various shapes. They were confident of operating the cognitive tool independently. Regarding the question which asked

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them whether they liked mathematics more after using the cognitive tools, 3 out of the 28 students' responses were "disagree" and 5 students' response were "neutral". Nevertheless, most of the students chose "agree" or "strongly agree" that they liked mathematics more after using these cognitive tools.

The vast majority of the students were interested in continuing to use the cognitive tools for learning. However, regarding the question which asked them whether they were willing to introduce these cognitive tools to other schoolmates, 9 out of the 28 students' responses were "neutral".

Ways Through Which Students Understood the Formula in Finding Area of Trapezoid

Question 17 of the questionnaire asked the students in the experimental group to indicate the various ways through which they learned and understood the area calculation formula of trapezoid. Table 26 summarized the feedback from the students. The students were allowed to select one or multiple ways through which they learned and understood the formula. Most of the students indicated that they learned and understood the formula through more than one way. For the more detailed breakdown of each student's feedback, please refer to Appendix F.

Nine students indicated that they had already learned the formula before they attended the teaching sessions of this study. Out of these 9 students, 5 students also indicated that the pedagogical activities in this study helped them further understand the formula for calculating the area of trapezoid. These 5 students indicated that the parallelogram and triangle teaching sessions, the trapezoid cognitive tool, the discussions with fellow students in the class, or the

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teacher's teaching in this study helped them further understand the formula for trapezoid.

Table 26: Ways through which students learned the area calculation formula of trapezoid.

Ways through which students learned the area calculation formula of trapezoid	No. of students
I had learned the formula before I attended the teaching sessions of this study.	9
After attending the parallelogram and triangle teaching sessions of this study, I realized that the area of trapezoid could be calculated using this formula.	16
I was able to construct the area calculation formula for trapezoid by myself using the "trapezoid" cognitive tool provided by this study.	7
I learned the formula through discussions with my classmates in this study.	14
I learned the formula through teacher's teaching in this study.	10
Teacher taught me according to my individual learning ability during this study.	9
Parents or tuition teacher taught me.	1

Note: students were allowed to indicate more than one way through which they learned the formula

It was interesting to see that 21 out of the 28 students (i.e. 75% of the students) indicated that the combination of multiple pedagogical activities of this study helped them to understand the formula for calculating the area of trapezoid. They indicated that the combination of the parallelogram and triangle teaching sessions, the trapezoid cognitive tool, the discussions with fellow students in the class, the teacher's teaching in this study, or the teacher's individual coaching during the study helped them further understand the formula for trapezoid. This indicated the fact that the cognitive tools alone might not be effective in helping them to understanding this formula. Instead, it was the cognitive tools together with the combination of the pedagogical activities which contributed to the students' understanding of this formula.

Discussions

The findings mentioned in the previous sections were consistent with the extant literature which showed that inquiry-based learning was an effective way to foster students' conceptual

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understanding (Haury, 1993; Boaler, 1998a; Boaler, 1998b; Bybee, 2009; Furtak et al., 2012). The findings of this study were also consistent with the research findings of Boaler (1998a, 1998b) which showed that the students under the traditional direct instructional approach developed the procedural knowledge while students under the inquiry-based learning approach developed both of the procedural and conceptual knowledge.

There was a paucity of literature examining the effect of integrating cognitive tools with inquiry-based learning on students' conceptual understanding and procedural knowledge. De Jong (2006) suggested that computer-supported cognitive tools might solve the problems encountered by the students who were learning under the inquiry-based learning environment. The findings of this study provided empirical evidence which supported De Jong's (2006) suggestions and showed the positive effect of integrating cognitive tools with inquiry-based learning on students' conceptual understanding and procedural knowledge.

Many researches found that the students in the middle school, high school or college were able to derive different ways or formulas to calculate the area of trapezoid (Manizade & Mason, 2014; Peterson & Saul, 1990; Wanko, 2005). However, there had been little investigation on the primary school students regarding their exploration and construction of the formulas for the calculation of the areas of closed shapes. This study found that almost all of the primary school students in the experimental group were able to derive the formula to calculate the area of parallelogram and triangle through the individual exploration or group exploration activities without teacher's assistance. On the other hand, most of them were not able to derive the formula to calculate the area of trapezoid through the individual exploration or group exploration activities without teacher's assistance. It was due to the fact that it was difficult for the primary school students to figure out by themselves that the *sum of the upper*

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and lower bases of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. The teacher had to make use of the cognitive tools and guided the students through this particular difficulty, step by step. This particular finding of this research (i.e. the difficulty and the step-by-step guidance) could be regarded as the pedagogical content knowledge for the topic of “area of trapezoid” that the teachers should be aware of prior to teaching this topic.

Chan et al. (2014) identified three common difficulties, that the students had when they were learning the mathematical topic of “area of closed shapes”, namely: (a) the lack of the concept of area conservation; (b) the failure to identify a base and its corresponding height for area calculation; and (c) the misconception that only regular closed shapes had measurable area and corresponding mathematical formulas for area calculation. Chan et al. (2014) had also proposed several cognitive tools developed on the GeoGebra platform to support students to explore the mathematical formulas for calculating the areas of parallelogram, triangle and trapezoid so as to address these three common difficulties that the students had. This research had adopted the cognitive tools proposed by Chan et al. (2014). The findings of this study provided empirical evidence to support that these cognitive tools could at least help the students to overcome two of these three common difficulties in learning the concepts of area, namely: a) the failure to identify a base and its corresponding height for area calculation; and b) the misconception that only regular closed shapes had measurable area and corresponding mathematical formulas for area calculation.

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CHAPTER 5 – Implications and Conclusion

Implications

Four implications surfaced from this study. The first implication revealed that integrating carefully designed cognitive tools with the appropriate pedagogical approach adopted in this study was a promising way to teach the concept of area. In this study, while the learning time of both groups was the same (please refer to Table 1 for details of learning time, which referred to the number of class sessions the experimental and control groups spent on each of the target topics), the experimental group completed fewer practices in terms of in-class exercises and homework than the control group. Nevertheless, just as for the control group students, the experimental group students developed significant procedural knowledge on the target topics. Most importantly, the experimental group students developed better conceptual understanding of the target topic than those students in the control group. Although the cognitive tools helped the students to learn the key concepts and computation of the areas of the various shapes, it was the cognitive tools together with the combination of the pedagogical activities that contributed to the students' understanding of the target topics. In the students' questionnaire, they also indicated that the combination of the (a) parallelogram and triangle teaching sessions, (b) trapezoid cognitive tool, (c) in-class discussions with fellow students, (e) teacher's whole-class explanation, and (f) teacher's individual coaching in the class helped them further understand the formula for calculating the area of trapezoid.

The second implication regarded the teachers' understanding of what made learning these particular topics easy or difficult. Shulman (1986) regarded this understanding as pedagogical content knowledge. When the students were learning how to calculate the area of the

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trapezoid, it was difficult for them to understand that the base of the parallelogram formed by two identical trapezoids was equal to the *sum of the upper and lower bases* of the trapezoid. The teachers should be sensitive to the fact that many students experienced this difficulty. Also, the teachers should not presume that it would be easy for the students to understand the concepts and formula for finding the area of a trapezoid even after having already learned similar concepts for finding the area of a triangle. The teachers should spend more time patiently guiding the students through this difficulty, step by step. The teachers could follow the steps detailed in the subsection “Pedagogical Content Knowledge Regarding the Topic on Area of Trapezoid” in Chapter 4. Subsequently, the teachers should evaluate whether the students had understood that the *sum of the upper and lower bases* of the trapezoid equaled the base of the parallelogram.

The third implication regarded the future improvement of the trapezoid cognitive tool. This was related to the technological pedagogical content knowledge (TPCK) as these proposed future enhancements were imbued with the “knowledge of what made concepts difficult or easy to learn and how technology could help redress some of the problems that students faced” (Mishra & Koehler, 2006, p. 1029). With these enhancements, the cognitive tool could provide more scaffolds to assist students to construct by themselves the conceptual understanding and procedural knowledge regarding the area of trapezoid. The aim was to further minimize the teacher’s mediations during students’ exploration activities. Please refer to the section “Future Development” in this chapter for details of the suggested future enhancements to the trapezoid cognitive tool.

The fourth implication concerned the potential of utilizing the cognitive tools and the pedagogical approach adopted in this study to other mathematical topics. In view of the

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finding of this study that most of the students indicated they liked mathematics more after using these cognitive tools (and that these tools helped the students develop better conceptual understanding and procedural knowledge), it was worthwhile to explore the potential of the cognitive tools developed on the GeoGebra platform and the pedagogical approach adopted in this study. These tools could transcend the mathematical topic of “area of closed shapes.” They could be applicable to other mathematics topics (such as perimeters of the various shapes) to help students to develop better procedural knowledge and concept understanding. Further studies could be conducted to investigate the potential of these cognitive tools and their effect on other mathematics topics.

Conclusion

The aim of this study was to investigate whether primary school students could develop better conceptual understanding and procedural knowledge on the topic of “area of closed shapes” when they explored mathematical formulas for calculating the areas of the shapes using the inquiry-based learning approach scaffolded by related cognitive tools developed on the GeoGebra platform. Fifty-three students participated in this study. All of them were primary students studying in Grade 5. There were 28 students in the experimental group and 25 students in the control group. The students in the experimental group used the cognitive tools developed on the GeoGebra platform to explore how to calculate the areas of the parallelogram, triangle, and trapezoid. The BSCS 5E Instructional Model, which was an inquiry-based instructional model, was used in the experimental group. The students in the control group learned how to calculate the areas of the parallelogram, triangle, and trapezoid through the direct instructional approach. The control group students used neither the GeoGebra cognitive tools nor the inquiry-based learning approach. Each student took one

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pre-test and two post-tests, namely, Post-test-01 and Post-test-02. Both groups of participants attended their respective parallelogram and triangle classes first. Then, they took the Post-test-01. After that, both groups continued the study and attended their respective trapezoid classes. Finally, they took the Post-test-02. Students in both the experimental and control groups spent the same amount of learning time on the target topics.

In terms of in-class exercises and homework, the experimental group students completed 54 questions in total, and the control group students finished 69 questions in total. The control group students completed 28% more in-class exercises and homework than the experimental group students. Moreover, before taking Post-test-01, the experimental group students did not do any in-class exercises or homework questions on the topic of the triangle, while the control group completed 10 triangle questions. Before taking Post-test-02, the experimental group did not answer any questions on the topic of trapezoid while the control group answered 16 trapezoid questions. Therefore, in terms of in-class exercises and homework, the students in the control group were more prepared for the post-tests than the students in the experimental group. The differences in exercises between the two groups were mainly due to the teachers' preferences and their considerations on the progress of the classes.

This study first wanted to find out whether the experimental group students would develop better conceptual understanding and procedural knowledge than the control group students. The pre-test results showed that there was no significant difference between the two groups before this study. The overall student achievement in Post-test-02 showed that the students in the experimental group performed significantly better than those in the control group after both groups had attended all of their respective classes for this study. Further analyses showed that the students in the experimental group had developed significantly better

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conceptual understanding than those in the control group, while there was no significant difference between the two groups in terms of the development of their procedural knowledge. This provided evidence that the pedagogical approach adopted by the experimental group was as effective as the direct instructional approach in developing students' procedural knowledge even though students in the experimental group had practiced less than the control group in terms of in-class exercises and homework. Most importantly, this provided empirical evidence showing that the pedagogical approach used in the experimental group was more effective than the direct instructional approach regarding the development of students' conceptual understanding on the topic of "area of closed shapes."

This study also wanted to find out how the pedagogical approach used in the experimental group helped the students develop better conceptual understanding. Further analyses showed that, after both groups attended all of their respective classes in this study, the students in the experimental group had developed significantly better conceptual understanding of the area of trapezoid while the students in the control group did not develop significantly better conceptual understanding on this topic. Analyses indicated that the pedagogical approach adopted by students in the experimental group was more effective than the direct instructional approach in helping students to develop their conceptual understanding of the area of trapezoid. This was related to a particular difficulty that many students had when they were learning this topic. In fact, during the trapezoid discovery sessions of this study, many students in the experimental group approached the teacher and asked how to work out the mathematical formula for calculating the area of trapezoid. In the discussions with these students during the trapezoid discovery sessions, the researcher of this study noticed that it was difficult for students to figure out by themselves that the *sum of the upper and lower*

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bases of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. Initially, both the teacher observer and the researcher of this study expected it would be easy for the students to understand that the *sum of the upper and lower bases* of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. That was because in the previous triangle sessions of the experimental group, (a) the students had learned that two identical triangles could form a parallelogram, and (b) almost all of them had been able to figure out, without assistance, that the base of the triangle was equal to the base of the parallelogram formed by two identical triangles. Thus, when they noticed that the students were not able to easily transfer this understanding from the triangle to the trapezoid sessions, they were surprised. To overcome this difficulty and help the students to construct their understanding of the area of a trapezoid, the researcher of this study made use of the cognitive tools and guided the students through this particular difficulty, step by step. For details, please refer to the subsection “Pedagogical Content Knowledge Regarding the Topic on Area of Trapezoid” in Chapter 4. This phenomenon showed that for the learning of this content knowledge, inquiry-based learning scaffolded by cognitive tools should be supplemented by the teacher’s step-by-step guidance to help students through this particular difficulty (i.e., the difficulty of understanding that the *sum of the upper and lower bases* of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids). This particular difficulty and the step-by-step guidance could be regarded as the pedagogical content knowledge for the topic of area of trapezoid.

Many teachers presumed it would be easy for the students to understand that the *sum of the upper and lower bases* of the trapezoid was equal to the base of the parallelogram formed by two identical trapezoids. This incorrect presumption was one of the factors contributing to the relative ineffectiveness of the direct instruction approach adopted in the control group

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compared to the pedagogical approach adopted in the experimental group. Other factors included the lack of exploration activities and the lack of student-to-student and student-to-teacher interactive discussions to help clarify doubts of control group students.

The pedagogical approach and the cognitive tools adopted in this research made the concepts of the target topics easier for the students to learn. The analyses of the pre-test, post-tests, student questionnaires, and the experimental group teacher observer's interview revealed that these tools also helped the students to redress some of the problems they faced.

First, the pedagogical approach and the cognitive tools adopted in this research helped the students to overcome the failure to identify a base and its corresponding height for area calculation. The visual display in the cognitive tools helped the students to envision the various sets of base and height of the shapes. For example, the parallelogram cognitive tool displayed one of the two sets of heights and bases when the students clicked the corresponding "height and base" button so that they could see that each parallelogram had two sets of base and height. In the cognitive tool, the students could also drag the vertices of the parallelogram to turn it into different shapes of parallelograms. By doing so, the students could observe how this movement dramatically changed the corresponding height and base of the shape. This allowed the students to explore and understand these relationships, especially the fact that the height and base were perpendicular to each other. In addition to the manipulation of the cognitive tools, all the pedagogical activities carried out in the experimental group also contributed to helping students to address this difficulty. Pedagogical activities included exploration prior to explanation, student-to-student interaction, teacher-to-student interaction, and whole-class discussions. Exploration prior to explanation provided opportunities for students to challenge the shortcomings in their prior knowledge or

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existing conceptions as well as options for them to connect new knowledge to their extant knowledge. Student-to-student interaction provided opportunities for students to interact with their peers to discuss, explain, and justify their solutions and interpretations so as to promote self-reflection and help construct their knowledge. Teacher-to-student interaction and whole-class discussions provided chances for the teacher to guide the students toward the correct conceptions.

Second, the pedagogical approach and the cognitive tools adopted in this research helped the students to overcome the misconception that only regular closed shapes had a measurable area that could be calculated by mathematical formulas. For example, the parallelogram cognitive tool allowed the students to interactively cut the parallelogram into two pieces and slide one of the pieces to turn the parallelogram into a rectangle. By visualizing that the parallelogram could be turned into a rectangle, the students could realize the mathematical formula used for calculating the area of the parallelogram. The triangle cognitive tool allowed the students to replicate an identical triangle and rotate the replicated triangle in order to form a parallelogram with these two identical triangles. By visualizing that two identical triangles could form a parallelogram, the students could apprehend the mathematical formula used for calculating the area of the triangle. As mentioned in the previous paragraph, all the pedagogical activities carried out in the experimental group also contributed to helping students to address this difficulty.

Overcoming the above two difficulties helped the students to develop a better conceptual understanding of the target topics. This was very critical in the development of students' procedural knowledge on the target topics as well. There was extensive evidence indicating that conceptual understanding and procedural knowledge developed iteratively (i.e.,

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improving conceptual understanding had a positive effect on procedural knowledge and vice versa). As the experimental group students significantly developed their conceptual understanding of the target topics, their conceptual understanding exerted a positive effect on their procedural knowledge.

The students' interviews showed that, in general, the marks that the students obtained in Post-test-02 reflected their conceptual understanding and procedural knowledge on the topics. The teacher observer's interview revealed that the teacher highly appreciated the pedagogical approach adopted in the experimental group together with the cognitive tools utilized in this study. The teacher observer indicated that he would promote these cognitive tools and the pedagogical approach to other teachers in the school. The feedback in the students' questionnaire indicated that the experimental group students liked to learn the target topics through the cognitive tools used in this study. The students also indicated that the cognitive tools helped them to learn the key concepts and computation of the target topics.

This study added empirical evidence on the importance of integrating an appropriate pedagogical approach (which included the pedagogical content knowledge) with the prudently designed cognitive tools for the development of students' conceptual understanding and procedural knowledge of the mathematical topic of "area of closed shapes." These positive results confirmed that it was a promising way to teach the topic of "area of closed shapes" through the use of the cognitive tools and the pedagogical approach adopted in this study. Therefore, it was worthwhile for educators and teachers to consider applying these tools in their instructional strategy design.

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Limitations

This study had several limitations. Firstly, the sample size of this study was small. There were only 28 students in the experimental group and 25 students in the control group. Although this study found that there were significant differences between students in the experimental and control group in various aspects, the sample size was too small to make any generalizations.

Second, as all the students came from the same school, they were similar in terms of their academic backgrounds, intellectual abilities, and physical characteristics. One could argue, therefore, that the study findings were only applicable to the students in this particular school and might not be generalizable to schools in other settings and with different students.

Third, there were confounding influences of teachers. As two different people administered the experimental group and the control group, their characteristics and profiles might exert confounding influences on the study findings. For instance, the researcher of this study was responsible for facilitating the experimental group, and he did not have any prior experience in teaching mathematics in primary schools. On the other hand, the control group teacher had around 10 years of experience instructing primary school students in mathematics. Their differences in pedagogy experience might exert a negative influence on the results of the experimental group. However, the enthusiasm of the researcher might exert a positive influence on the results of the experimental group. These characteristics and profiles were unmeasured factors that might have affected the outcome of this study.

Fourth, as this was a quasi-experimental study in which the two groups of participants were

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not randomly assigned to groups for different treatment, the outcome of this study might be affected by the composition of the groups other than the treatment.

Future Development

This study revealed that the teacher's mediations were required to assist the students to develop conceptual understanding and procedural knowledge of the area of a trapezoid. Therefore, the trapezoid cognitive tool could be further improved to provide additional scaffoldings for students to construct their knowledge with minimal mediations from teachers. The first improvement to the trapezoid cognitive tool was to use different colors for the upper base and lower base. The trapezoid cognitive tool adopted in this study used green lines to indicate both of the lower and upper bases (please refer to Figure 15). Using different colors—for example, green for upper base and red for lower base—could highlight for the students that the base of the parallelogram formed by two identical trapezoids consisted of two parts: (a) the upper base of the trapezoid (green), and (b) the lower base of the trapezoid (red). The second improvement to the trapezoid cognitive tool was to put the labels "upper base" and "lower base" on the replicated trapezoid as well. In the trapezoid cognitive tool adopted in this study, the labels "upper base" and "lower base" only appear once on the original trapezoid (please refer to Figure 16). Adding these labels to the replicated trapezoid could provide sufficient hints for the students to understand that the base of the parallelogram formed by two identical trapezoids consisted of two parts: (a) the upper base of the trapezoid, and (b) the lower base of the trapezoid.

Researchers could conduct longitudinal studies to further investigate the effect of this pedagogical approach on students' future academic results. Also, researchers could

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investigate the effect of the pedagogical approach on different types of students so as to further fine-tune it to suit students of different academic backgrounds, intellectual abilities, and physical characteristics. Moreover, further studies could improve the pedagogical approach and the cognitive tools.

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Appendix

Appendix A – Worksheet

Appendix A1 – Parallelogram Worksheet (Part A)

Click the "parallelogram A" button in the lower left corner of your screen

1. Can you measure the area of this "parallelogram A"?

Perform the following steps

a) Click the "base & height 1" button in the upper left corner of your screen

b) Click the "transform to rectangle" button in the upper right corner of your screen

c) Drag the red slider to transform the parallelogram to a rectangle

2. Measure the area of the rectangle you have created by counting the number of small squares in the rectangle.

Let's say the length of the side of the small square is 1 cm.
That means the area of the small square is 1 cm^2.

The area of the rectangle is ______________________________ cm^2

3. What is the relationship between the length and width of the rectangle and the area of the rectangle?

area of the rectangle = length _____ width

$+ - \times \div$?

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4. Is the area of the rectangle the same as the area of the original parallelogram?

yes / no

5. Is the length of the rectangle the same as the base of the original parallelogram?

yes / no

6. Is the width of the rectangle the same as the height of the original parallelogram?

yes / no

7. Can you suggest a mathematical formula to find the area of a parallelogram?

Area of Parallelogram = ________________________________

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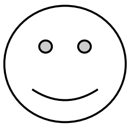
<u>Appendix A2 – Parallelogram Worksheet (Part B)</u>

Click the "parallelogram A" or "parallelogram B" button in the lower left corner of your screen

1. How many pairs of bases and heights does a parallelogram have?

   A parallelogram has _________ pairs of bases and heights



<u>Hints</u>

a) Click the "base & height 1" button in the upper left corner of your screen

b) Change the shape of the parallelogram by dragging its vertices, observe the changes of the base and height

c) Observe to see if the base and height are always perpendicular to each other

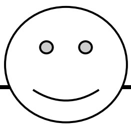
d) Click the "base & height 2" button in the upper left corner of your screen and repeat the above actions

2. Are the base and the height of the parallelogram always perpendicular to each other?

   <u>yes / no</u>

3. Can you use any pair of base and height of a parallelogram to calculate its area?

   <u>yes / no</u>



<u>Hints</u>

a) Click the "parallelogram A" or "parallelogram B" button

b) Click the "base & height 1" button and transform the parallelogram to a rectangle

c) Click the "base & height 2" button and transform the parallelogram to a rectangle

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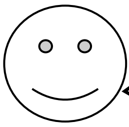
4. Does “parallelogram A” have the same area as “parallelogram B”?

yes / no



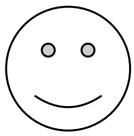
Hints

a) Click the “base & height 1” or “base & height 2” button
b) Click the “parallelogram A” button
c) Transform the parallelogram to a rectangle
d) Count the number of small squares in the rectangle
e) Click the “parallelogram B” button and repeat the above actions

5. Calculate the areas of “parallelogram A” and “parallelogram B” using their bases and heights

Hints

a) Click the “base & height 1” button
b) repeatedly press the “parallelogram A” and “parallelogram B” buttons in alternation
c) Observe to see if these two parallelograms have the same base
d) Observe to see if their heights are the same
e) Let’s say the length of the side of the small square is 1 cm.



The area of “parallelogram A” is:

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_________________________________ = ____________cm^2

The area of “parallelogram B” is:

_________________________________ = ____________cm^2

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6. Does "parallelogram C" have the same area as "parallelogram D"?

   __yes / no__

7. Calculate the areas of "parallelogram C" and "parallelogram D" using their bases and heights

   The area of "parallelogram C" is:

   ______________________________ = ____________cm^2

   The area of "parallelogram D" is:

   ______________________________ = ____________cm^2

8. When two parallelograms have the same base and same height, are their areas always the same?

   __yes / no__

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Appendix A3 – Triangle Worksheet

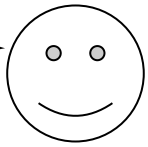
A) **Calculate the Area of Triangle**

Click the "triangle A" button in the lower left corner of your screen

1. Can you find the area of this "triangle A"?

Hint

Drag the slider to replicate the triangle to form a parallelogram



2. Is the height of the triangle the same as the height of the parallelogram formed?

yes / no

Hints

a) Click the "base & height 1" button in the upper left corner of your screen

b) Drag the slider back and forth, and observe to see if the height of the triangle is the same as the height of the parallelogram formed

3. Is the base of the triangle the same as the base of the parallelogram formed?

yes / no

4. Can you suggest a mathematical formula to find the area of a triangle?

Area of Triangle = ________________________________

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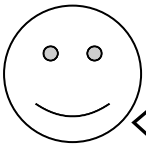
B) Base and Height of Triangle

5. How many pairs of bases and heights does a triangle have?

A triangle has _________ pairs of bases and heights



Hints

a) Click the “triangle A” or “triangle B” button in the lower left corner of your screen

b) Click the “base & height 1”, “base & height 2” or “base & height 3” button at the top of your screen

c) Change the shape of the triangle by dragging its vertices, observe the changes of the base and height

d) Observe to see if the base and height are always perpendicular to each other

6. Are the base and the height of the triangle always perpendicular to each other?

__yes / no__

7. Can you use any pair of base and height of a triangle to calculate its area?

__yes / no__

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C) Compare the Areas of Different Triangles

8. Does "triangle A" have the same area as "triangle B"? ___yes / no___

Hints

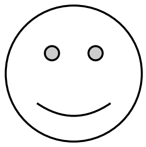
f) Click the "base & height 1" button

g) repeatedly press the "triangle A" and "triangle B" buttons in alternation

h) Observe to see if these two triangles have the same base

i) Observe to see if their heights are the same

j) Let's say the length of the side of the small square is 1 cm.



9. The area of "triangle A" is: ________________________________ = ________ cm^2

10. The area of "triangle B" is: ________________________________ = ________ cm^2

11. Does "triangle C" have the same area as "triangle D"? ___yes / no___

12. The area of "triangle C" is: ________________________________ = ________ cm^2

13. The area of "triangle D" is: ________________________________ = ________ cm^2

14. When two triangles have the same base and same height, are their areas always the same?

___yes / no___

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Appendix A4 – Trapezoid Worksheet

A) Calculate the Area of Trapezoid

Click the "trapezoid A" button in the lower left corner of your screen

1. Can you find the area of this "trapezoid A"?

Hint

Drag the slider to replicate the trapezoid to form a parallelogram

2. Is the sum of the upper base and lower base of the trapezoid the same as the base of the parallelogram formed?

yes / no

Hints

c) Click the "base & height " button in the upper left corner of your screen

d) Drag the slider back and forth, and observe to see if the sum of the upper base and lower base of the trapezoid is the same as the base of the parallelogram formed

3. Is the height of the trapezoid the same as the height of the parallelogram formed?

yes / no

4. Can you suggest a mathematical formula to find the area of a trapezoid?

Area of Trapezoid = ________________________________

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B) Base and Height of Trapezoid

5. How many pairs of upper base, lower base, and height does a trapezoid have?

A trapezoid has __________ pair of upper base, lower base, and height

Hints

e) Click the "trapezoid A" or "trapezoid B" button in the lower left corner of your screen

f) Click the "base & height" button at the top of your screen

g) Change the shape of the trapezoid by dragging its vertices, observe the changes of the bases and height

h) Observe to see if the height is always perpendicular to the upper base and lower base

6. Is the height of the trapezoid always perpendicular to its upper base and lower base respectively?

yes / no

C) Compare the Areas of Different Trapezoids

7. Does "trapezoid A" have the same area as "trapezoid B"? yes / no

Hints

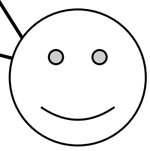
k) Click the "base & height " button

l) repeatedly press the "trapezoid A" and "trapezoid B" buttons in alternation

m) Observe to see if these two trapezoids have the same upper base and lower base

n) Observe to see if their heights are the same

o) Let's say the length of the side of the small square is 1 cm.



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8. The area of “trapezoid A” is: ______________________________ = ________ cm^2

9. The area of “trapezoid B” is: ______________________________ = ________ cm^2

10. Does “trapezoid C” have the same area as “trapezoid D”? ____yes / no____

11. The area of “trapezoid C” is: ______________________________ = ________ cm^2

12. The area of “trapezoid D” is: ______________________________ = ________ cm^2

13. When two trapezoids have the same lower base, the same upper base and the same height, are their areas always the same?

____yes / no____

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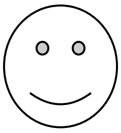
Appendix A5 – Combined and Simplified Worksheet

A) Area of Parallelogram

Click the "parallelogram A" button in the lower left corner of your screen

1. Can you measure the area of this "parallelogram A"?



Perform the following steps

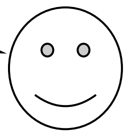
d) Click the "base & height 1" button in the upper left corner of your screen

e) Click the "transform to rectangle" button in the upper right corner of your screen

f) Drag the red slider  to transform the parallelogram to a rectangle

2. Measure the area of the rectangle you have created by counting the number of small squares in the rectangle.

Let's say the length of the side of the small square is 1 cm.
That means the area of the small square is 1 cm^2.



The area of the rectangle is ______________________________ cm^2

3. What is the relationship between the length and width of the rectangle and the area of the rectangle?

area of the rectangle = length ____ width

$+ - \times \div$?

4. Is the area of the rectangle the same as the area of the original parallelogram?

__yes / no__

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5. Is the length of the rectangle the same as the base of the original parallelogram?

yes / no

6. Is the width of the rectangle the same as the height of the original parallelogram?

yes / no

7. Can you suggest a mathematical formula to find the area of a parallelogram?

Area of Parallelogram = ________________________________

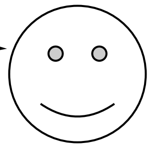
B) <u>Area of Triangle</u>

Click the "triangle A" button in the lower left corner of your screen

8. Can you find the area of this "triangle A"?

<u>Hint</u>

Drag the slider  to replicate the triangle to form a parallelogram



9. Is the height of the triangle the same as the height of the parallelogram formed?

yes / no

<u>Hints</u>

e) Click the "base & height 1" button in the upper left corner of your screen

f) Drag the slider  back and forth, and observe to see if the height of the triangle is the same as the height of the parallelogram formed

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10. Is the base of the triangle the same as the base of the parallelogram formed?

 yes / no

11. Can you suggest a mathematical formula to find the area of a triangle?

 Area of Triangle = ________________________________

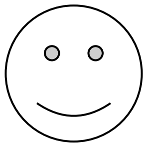
C) Area of Trapezoid

Click the "trapezoid A" button in the lower left corner of your screen

12. Can you find the area of this "trapezoid A"?

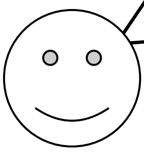
Hint

Drag the slider  to replicate the trapezoid to form a parallelogram



13. Is the sum of the upper base and lower base of the trapezoid the same as the base of the parallelogram formed?

 yes / no



Hints

g) Click the "base & height " button in the upper left corner of your screen

h) Drag the slider  back and forth, and observe to see if the sum of the upper base and lower base of the trapezoid is the same as the base of the parallelogram formed

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14. Is the height of the trapezoid the same as the height of the parallelogram formed?

    yes / no

15. Can you suggest a mathematical formula to find the area of a trapezoid?

    Area of Trapezoid = ________________________________

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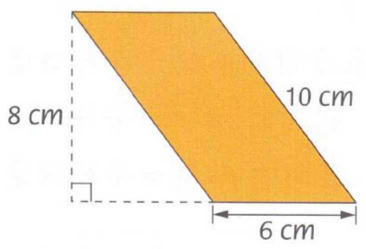
Appendix B – Sample In-class Exercises and Homework Questions

Sample Question 1

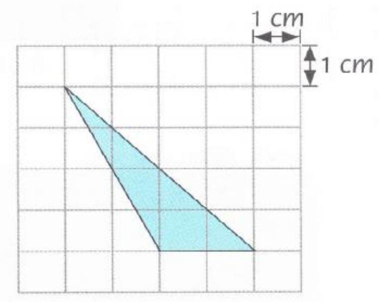


The area is __________ cm^2

Sample Question 2

Draw three parallelogram that have the same area but different shape.

Sample Question 3



The area is __________ cm^2

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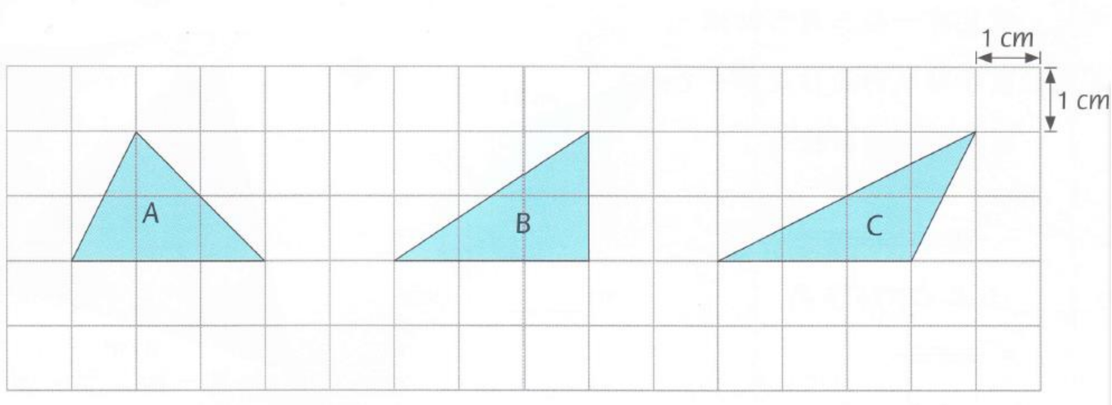
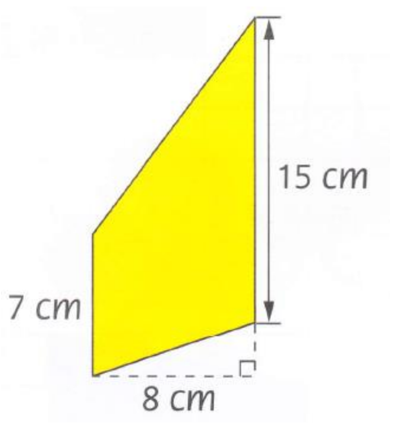
Sample Question 4



Fill in the following table:

Triangle	Base (cm)	Height (cm)	Area (cm^2)
A			
B			
C			

Sample Question 5



The area is：

____

____

= __________________ cm^2

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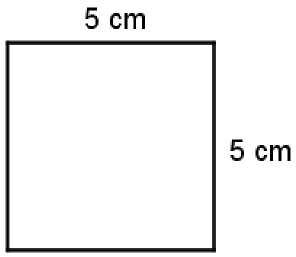
Appendix C – Pre-test / Post-tests

Note that the pre-test and the post-tests consisted of the same set of questions.

1. Calculate the area of this square.

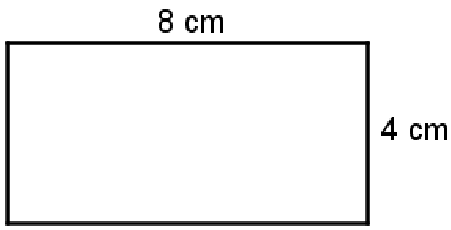


The area of the square is：

____

= ____________________ cm^2

2. Calculate the area of this rectangle.

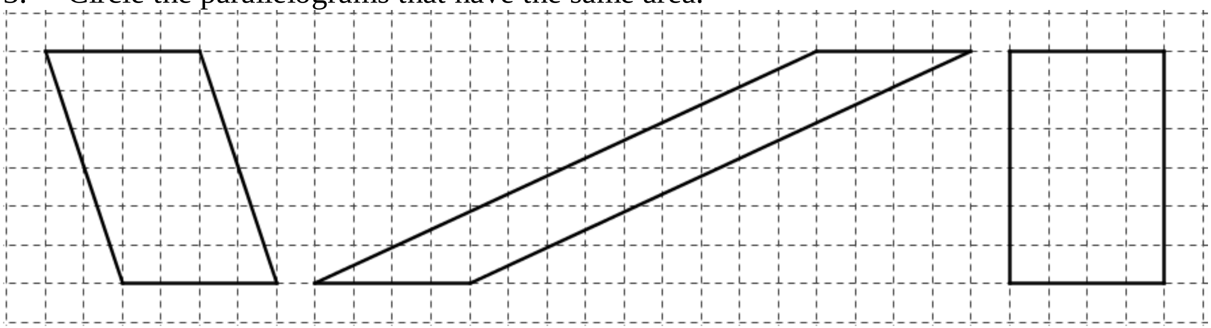


The area of the rectangle is：

____

= ____________________ cm^2

3. Circle the parallelograms that have the same area.



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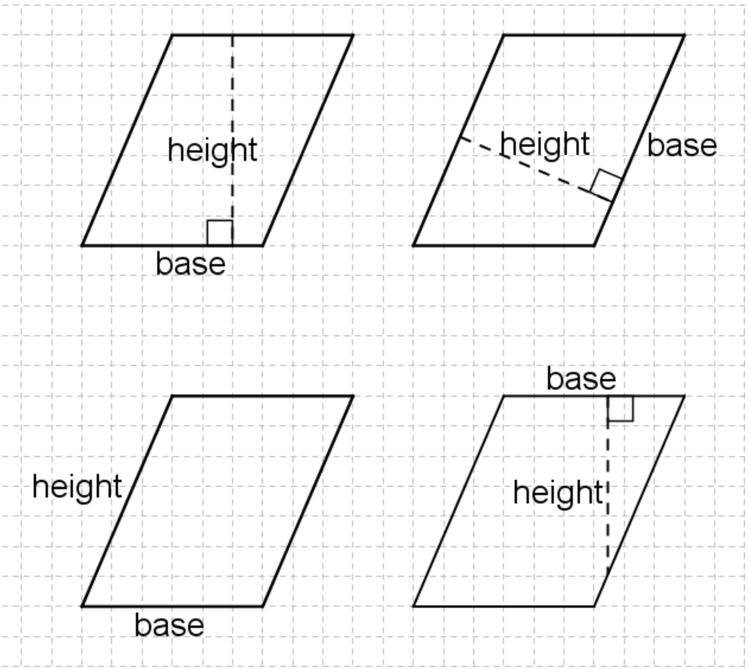
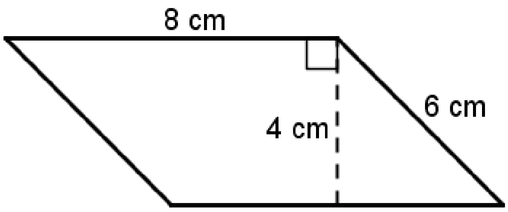
4. There are four parallelograms. Their bases and heights are indicated in the diagram. Some of them are correctly indicated while some are incorrectly indicated. Circle the parallelograms that have correct bases and heights indicated in the diagram.



5. Calculate the area of this parallelogram.

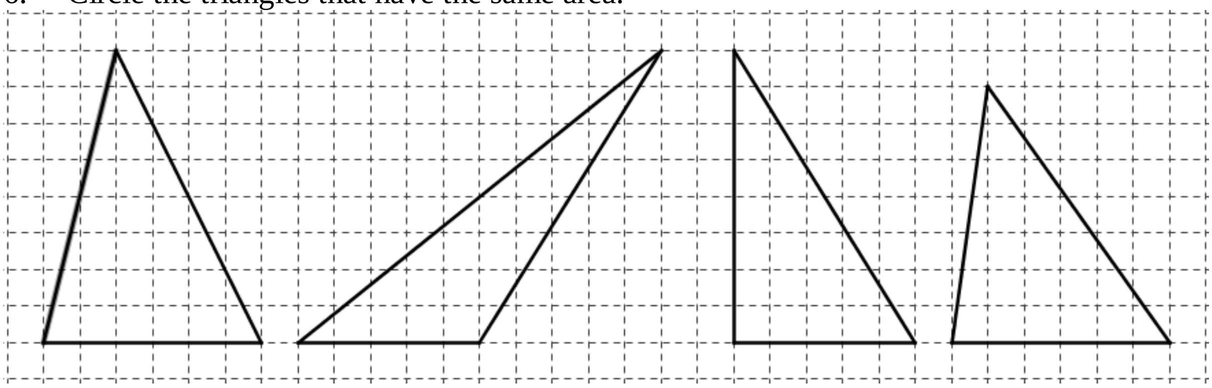


The area of the parallelogram is：

____

= ________________ cm^2

6. Circle the triangles that have the same area.



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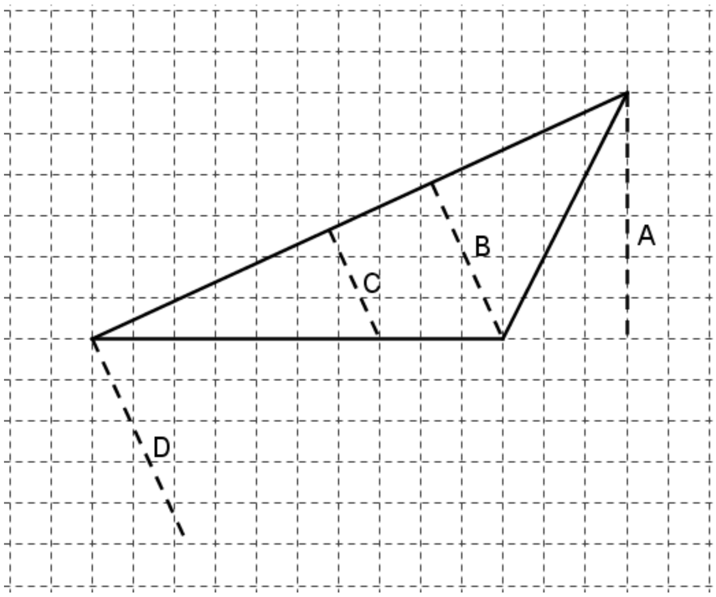
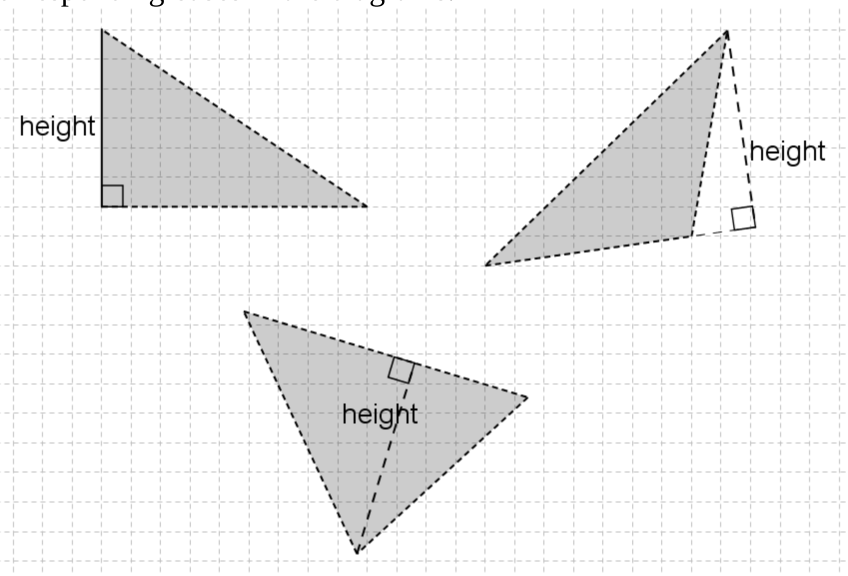
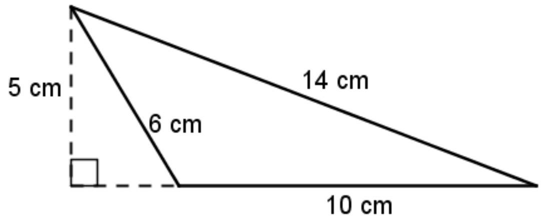
7. This diagram shows four dotted lines. Circle the two dotted lines which are the "heights" of the triangle.



8. There are three triangles and their heights are indicated in the diagrams. Darken the correct corresponding bases in the diagrams.



9. Calculate the area of this triangle.



The area of the triangle is：

____

= __________________ cm^2

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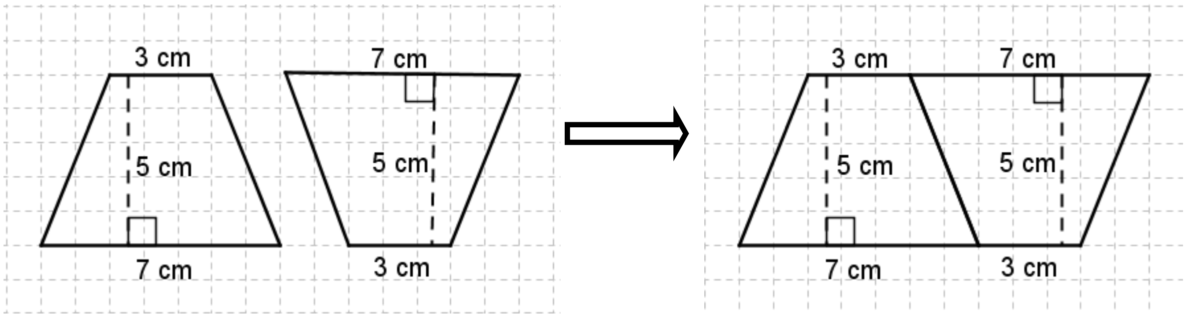
10. There are two identical trapezoids. They can be put together to form a parallelogram.



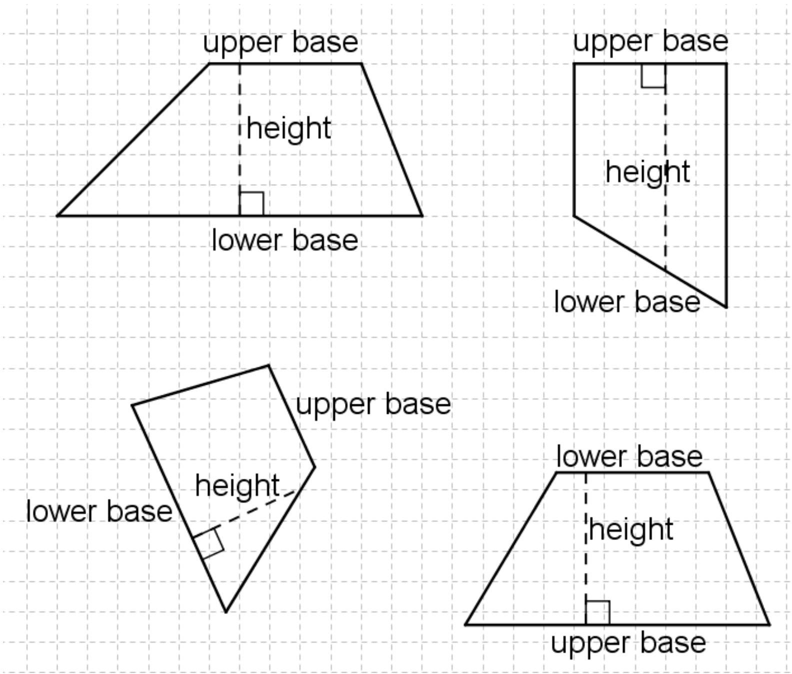
Calculate the area of one of these trapezoids.

The area of the trapezoid is：

____

= ____________________ cm^2

11. There are four trapezoids. Their bases and heights are indicated in the diagrams. Some of them are correctly indicated while some are incorrectly indicated. Circle the trapezoids that have correct bases and heights indicated in the diagrams.



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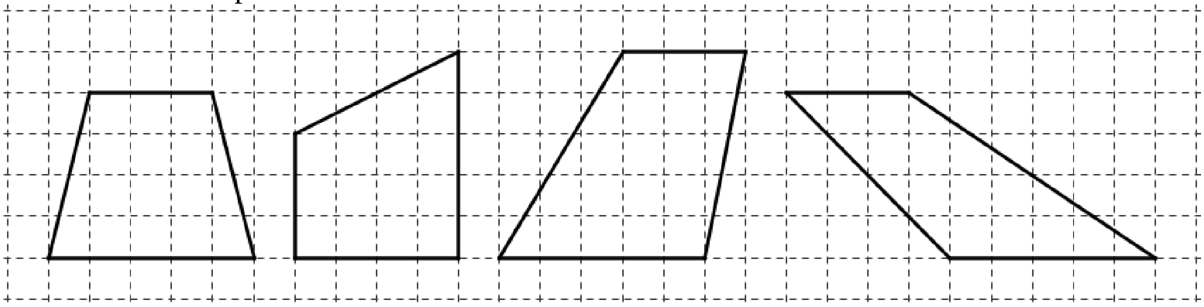
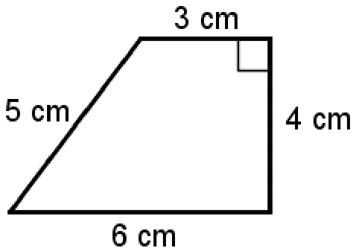
12. Circle the trapezoids that have the same area.



13. Calculate the area of this trapezoid.

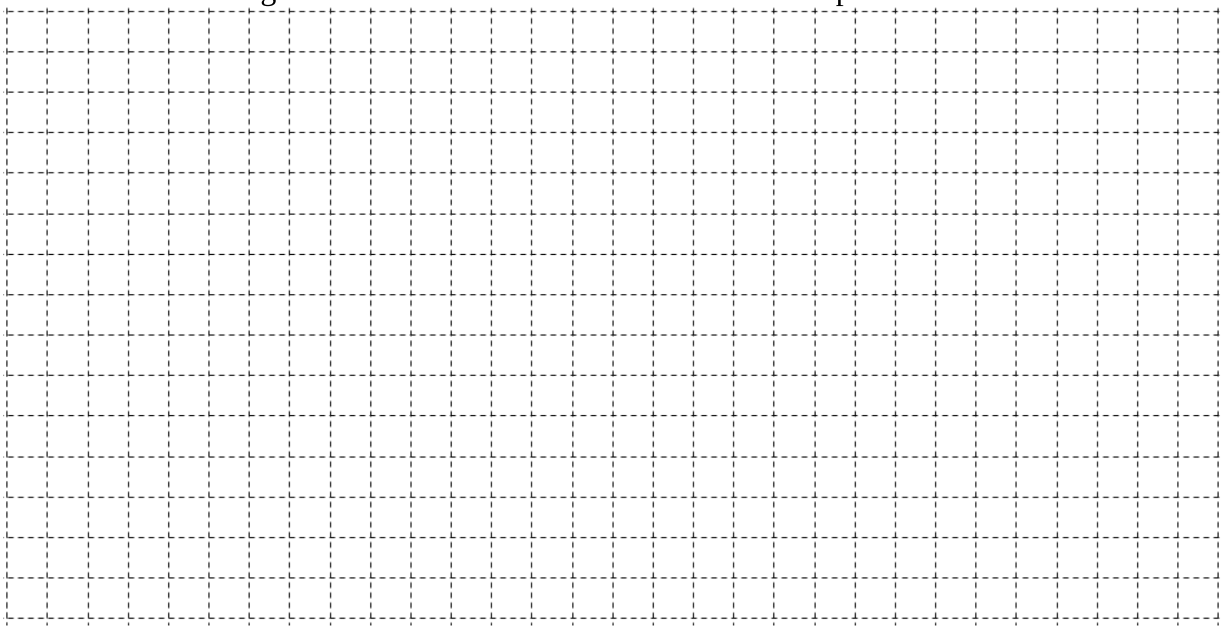


The area of the trapezoid is：

____

= ________________ cm^2

14. Draw two triangles that have the same area but different shapes.



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15. Draw two trapezoids that have the same area but different shapes.

16. Draw two different parallelograms with the same area but one of them should have a base two times the length of the other one.

** End **

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Appendix D – Student's Questionnaire

The evaluation items were adopted and modified from Kong and Li (2007) and Kong (2011).

Evaluation item		strongly agree	agree	neutral	disagree	strongly disagree
Interface design of the cognitive tool		please tick (✓) the appropriate box				
1.	I understand the activities provided on each computer interface					
2.	I can undertake the activities on each computer interface independently					
3.	The computer interface displays are compatible with those in common learning materials					
4.	The application is easy to use					
Scaffold support of the cognitive tool		please tick (✓) the appropriate box				
5.	The graphics assist me in learning the key concepts of area					
6.	The graphical manipulation assists me in understanding that the area is conserved after I cut the original shape into smaller pieces and re-combine them to form different shapes					
7.	The interactive visual representation assists me in identifying the bases and the corresponding heights of the various shapes					
8.	The cognitive tools help me to understand the parallelograms, triangles and trapezoids have measurable areas which can be calculated by mathematical formulas					
9.	The cognitive tools assist me in understanding how to construct the mathematical formulas which are used for calculating the areas of the various shapes					
10.	I can use the cognitive tools to discuss with my classmates the ways of calculating areas of the shapes					
Overall perceptions of the cognitive tool		strongly agree	agree	neutral	disagree	strongly disagree
11.	The cognitive tools help me to learn the key concepts of area					
12.	The cognitive tools help me to learn the computation of the areas of the various shapes					
13.	I like mathematics more after using the cognitive tools					
14.	I am confident of operating the cognitive tools independently					
Continued Usage of the cognitive tool		strongly agree	agree	neutral	disagree	strongly disagree
15.	I am interested in continuing to use the cognitive tools for learning					
16.	I am willing to introduce these cognitive tools to other schoolmates					

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17. Through which of the following ways did you learn and understand this area calculation formula of trapezoid：(upper base + lower base) × height ÷ 2 ?

(Note：you are allowed to indicate more than one way)

		tick (✓) the appropriate box
a.	I had learned the formula before I attended the teaching sessions of this study	
b.	After attending the parallelogram and triangle teaching sessions of this study, I realized that the area of trapezoid could be calculated using this formula	
c.	I was able to construct the area calculation formula for trapezoid by myself using the "trapezoid" cognitive tool and worksheet provided by this study	
d.	I learned the formula through discussions with my classmates in this study	
e.	I learned the formula through teacher's teaching in this study	
f.	Teacher taught me according to my individual learning ability during this study	
g.	I learned the formula through other ways: (please specify)	

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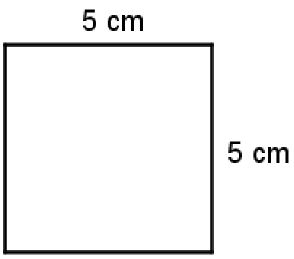
Appendix E – Marking Scheme for Pre-test and Post-tests

1. Calculate the area of this square. **Procedural Question** (1 mark)

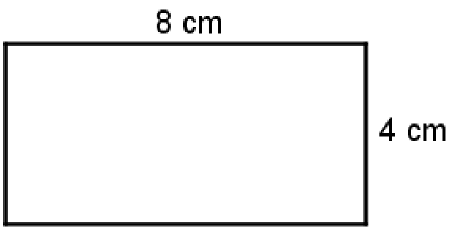


The area of the square is:

5 cm × 5 cm

= 25 cm^2

2. Calculate the area of this rectangle. **Procedural Question** (1 mark)

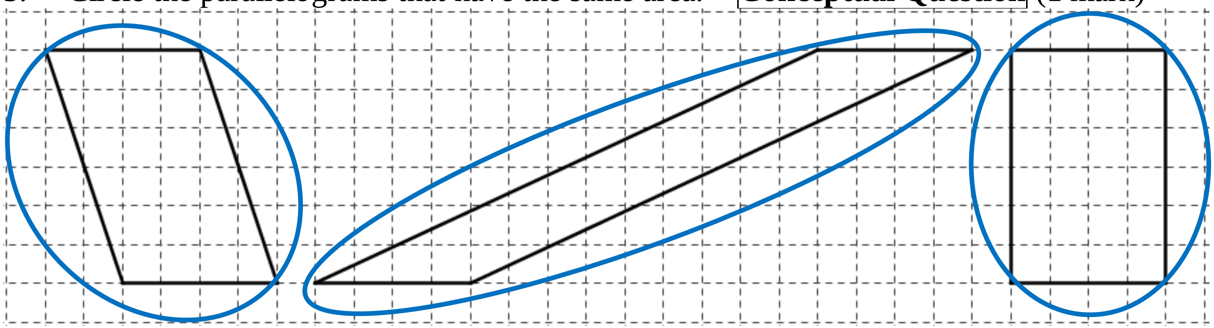


The area of the rectangle is:

8 cm × 4 cm

= 32 cm^2

3. Circle the parallelograms that have the same area. **Conceptual Question** (1 mark)



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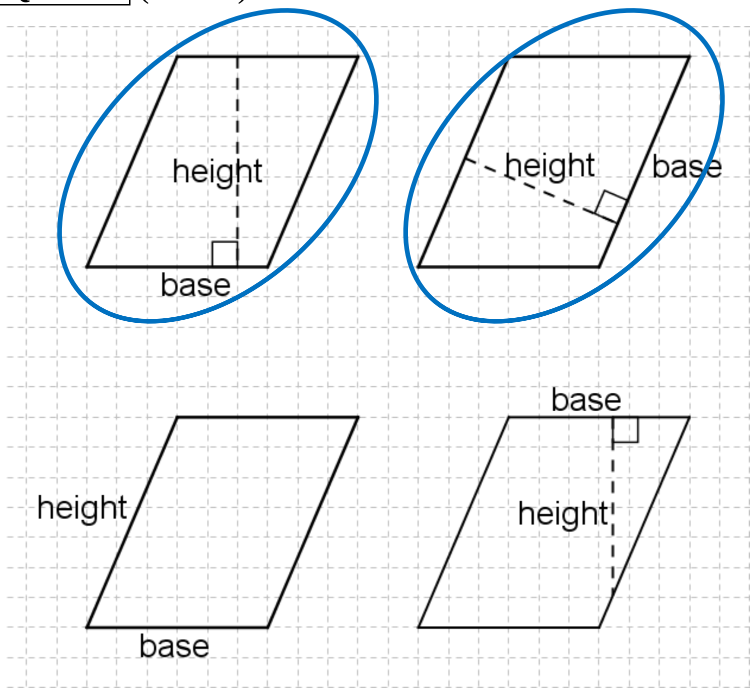
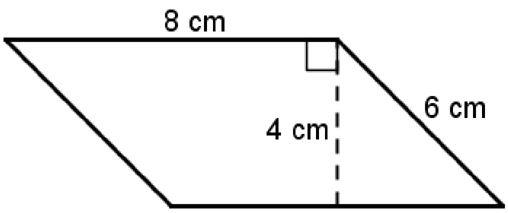
4. There are four parallelograms. Their bases and heights are indicated in the diagram. Some of them are correctly indicated while some are incorrectly indicated. Circle the parallelograms that have correct bases and heights indicated in the diagram. **Conceptual Question** (1 mark)



5. Calculate the area of this parallelogram. **Procedural Question** (1 mark)

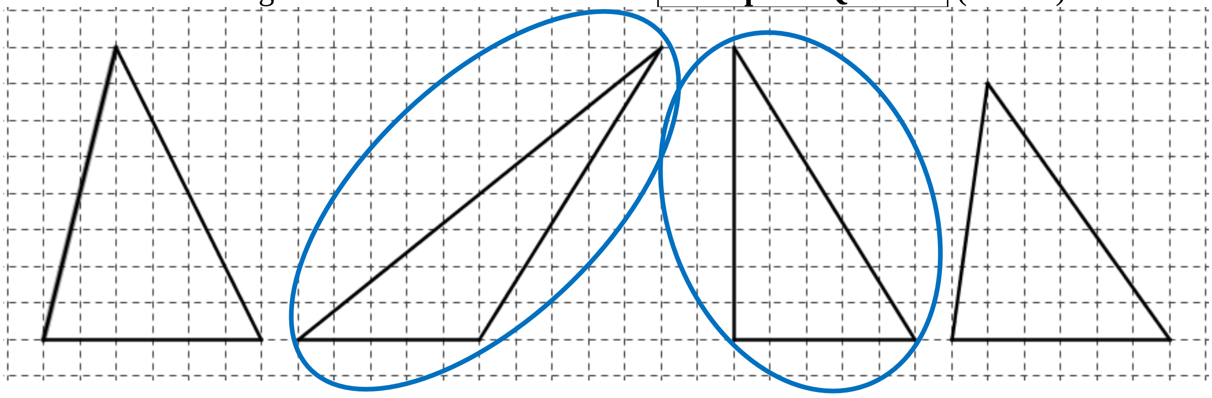


The area of the parallelogram is：

8 cm × 4 cm

= 32 cm^2

6. Circle the triangles that have the same area. **Conceptual Question** (1 mark)



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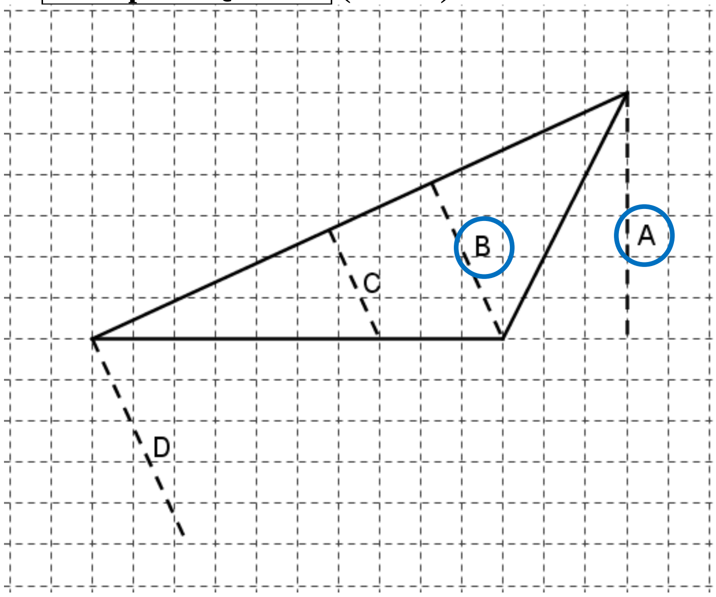
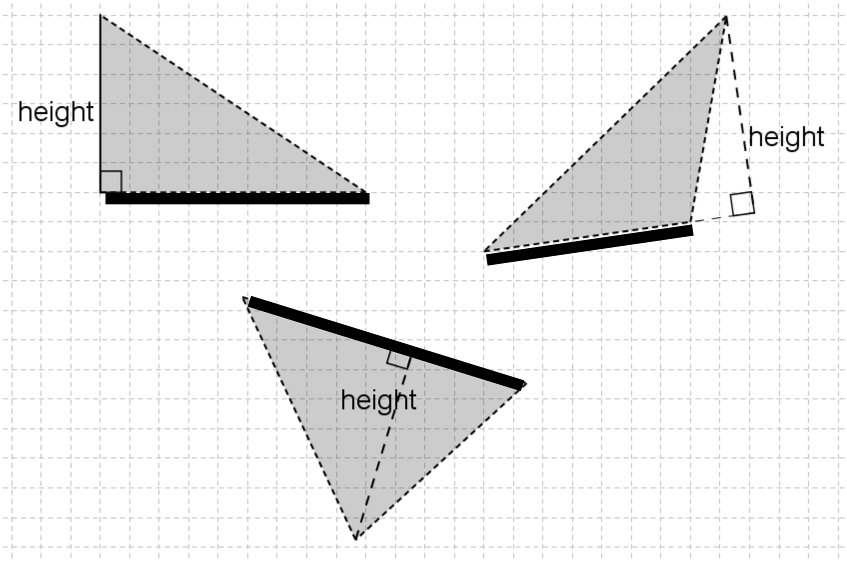
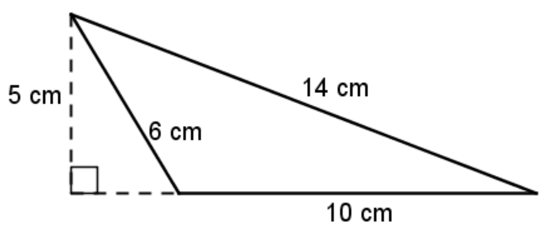
7. This diagram shows four dotted lines. Circle the two dotted lines which are the "heights" of the triangle. **Conceptual Question** (1 mark)



8. There are three triangles and their heights are indicated in the diagrams. Darken the correct corresponding bases in the diagrams. **Conceptual Question** (1 mark)



9. Calculate the area of this triangle. **Procedural Question** (1 mark)



The area of the triangle is：

5 cm × 10 cm ÷ 2

= 25 cm^2

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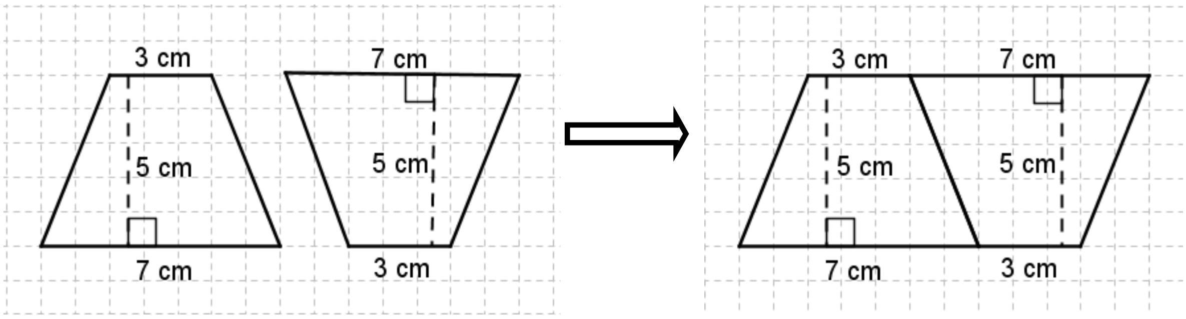
10. There are two identical trapezoids. They can be put together to form a parallelogram. **Procedural Question** (1 mark)



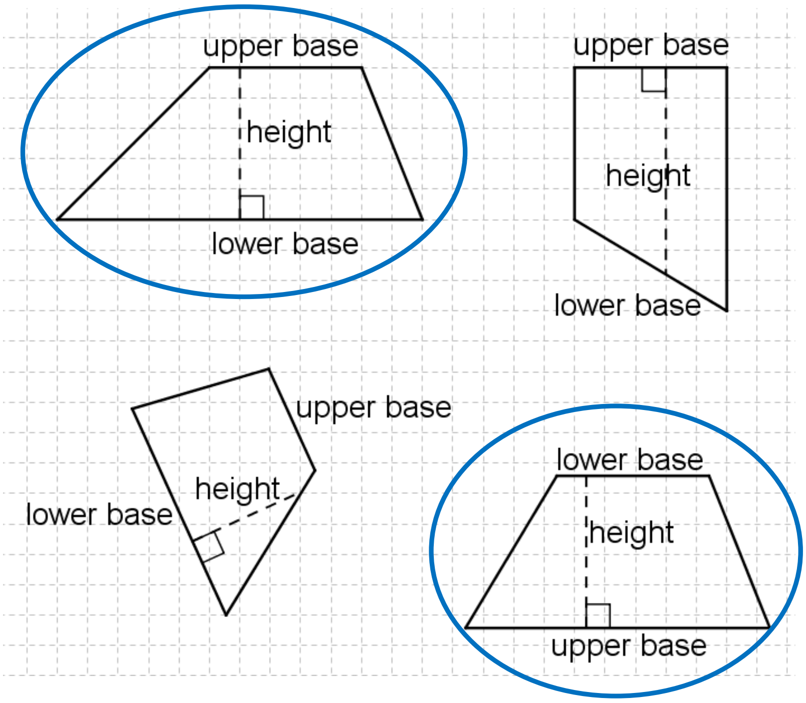
Calculate the area of one of these trapezoids.

The area of the trapezoid is：

$(3\text{ cm} + 7\text{ cm}) \times 5\text{ cm} \div 2$

= 25 cm^2

11. There are four trapezoids. Their bases and heights are indicated in the diagrams. Some of them are correctly indicated while some are incorrectly indicated. Circle the trapezoids that have correct bases and heights indicated in the diagrams. **Conceptual Question** (1 mark)



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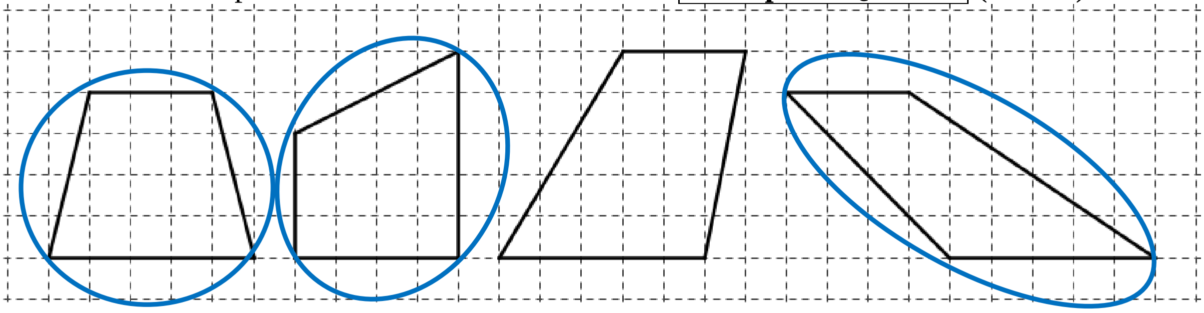
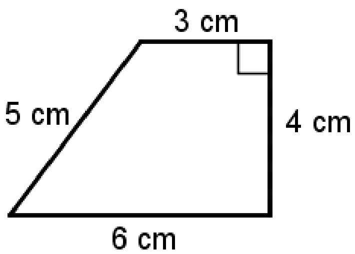
12. Circle the trapezoids that have the same area. **Conceptual Question** (1 mark)



13. Calculate the area of this trapezoid. **Procedural Question** (1 mark)

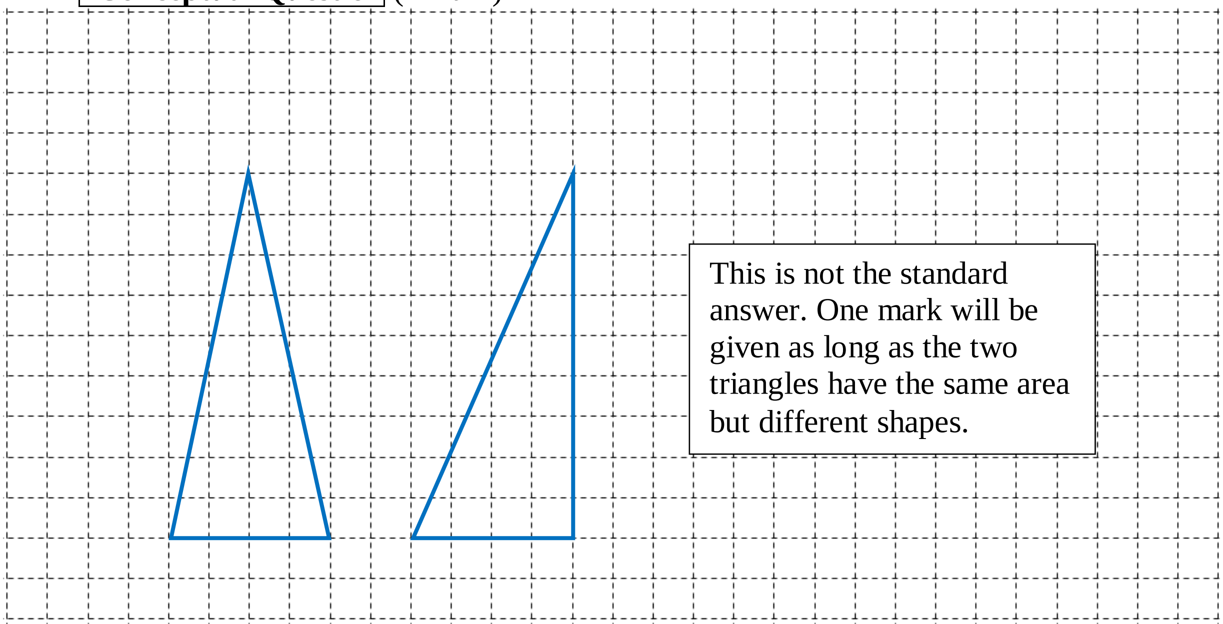


The area of the trapezoid is：

$(3\text{ cm} + 6\text{ cm}) \times 4\text{ cm} \div 2$

$=$ 18 cm^2

14. Draw two triangles that have the same area but different shapes.
Conceptual Question (1 mark)



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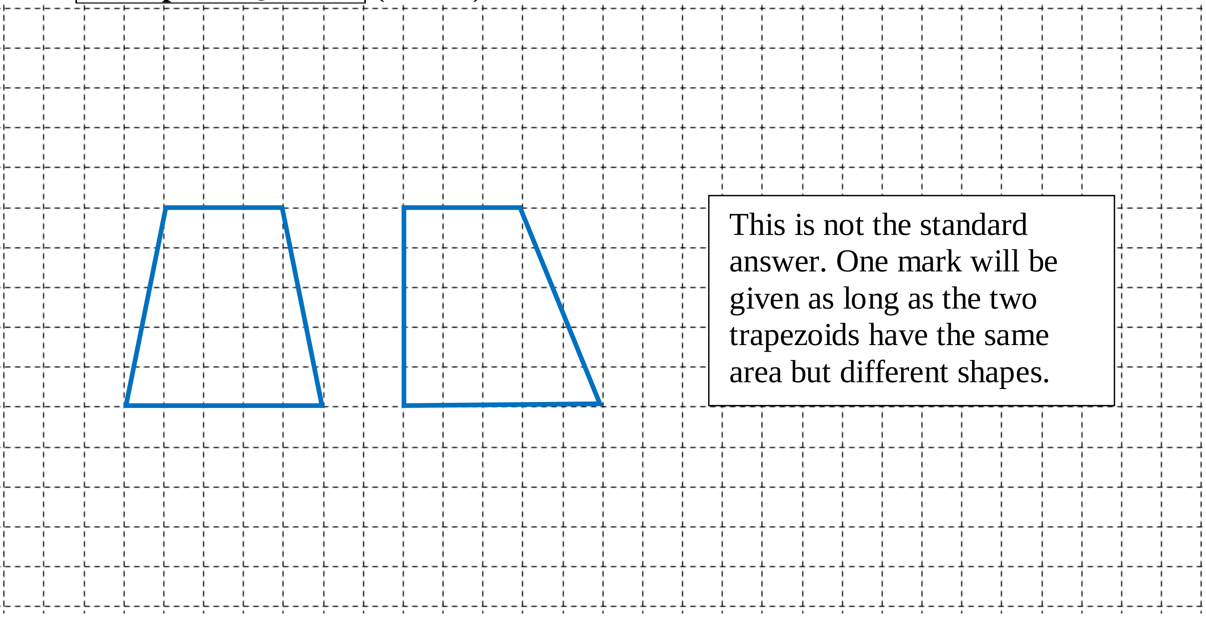
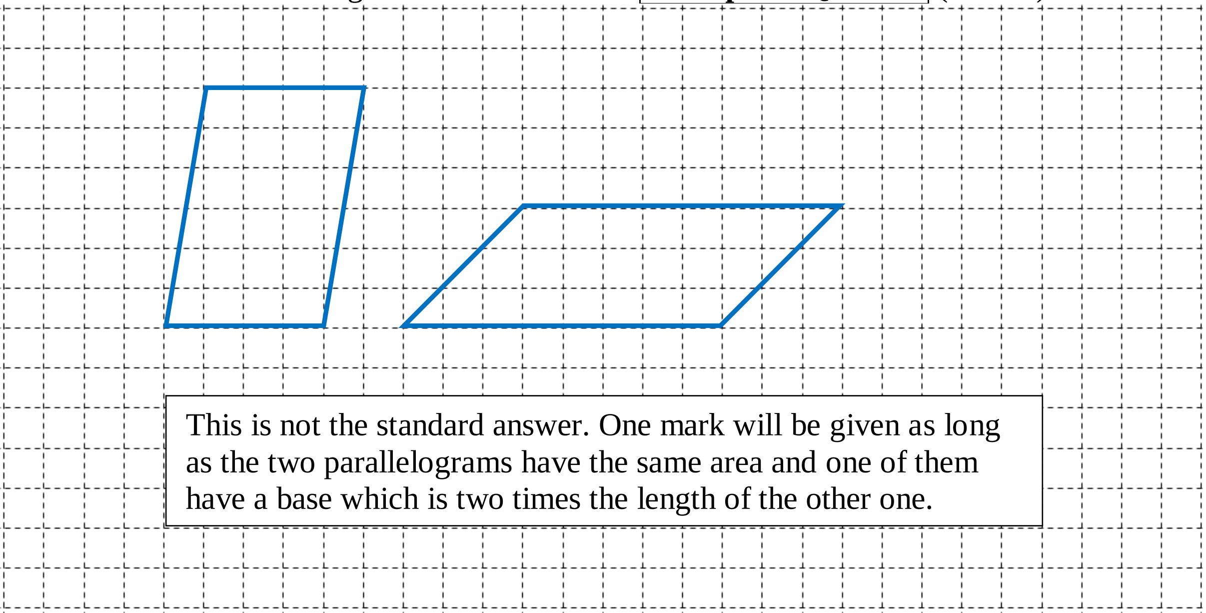
15. Draw two trapezoids that have the same area but different shapes.
Conceptual Question (1 mark)



This is not the standard answer. One mark will be given as long as the two trapezoids have the same area but different shapes.

16. Draw two different parallelograms with the same area but one of them should have a base two times the length of the other one. **Conceptual Question** (1 mark)



This is not the standard answer. One mark will be given as long as the two parallelograms have the same area and one of them have a base which is two times the length of the other one.

** End **

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Appendix F – Students' Feedback on Question 17 of the Student Questionnaire

Question 17 of the student questionnaire asked students to indicate the various ways through which they learned and understood the formula for the calculation of the area of trapezoid. The students were allowed to select one or multiple ways through which they learned the formula. The following table summarized the feedback of each of the students:

Student	*Ways through which students learned the area calculation formula of trapezoid						
	A	B	C	D	E	F	G
1	X	X	X				
2				X			
3		X	X		X		
4	X						
5	X	X					
6	X	X	X				
7		X					
8				X		X	
9				X		X	
10				X			
11	X	X		X	X		
12	X	X					
13		X	X	X	X	X	
14		X			X		
15				X	X	X	
16	X						
17	X						
18		X		X	X		
19	X						
20		X		X			
21		X		X		X	
22		X			X	X	
23				X	X	X	X
24		X	X				
25				X		X	
26			X	X	X	X	
27		X		X			
28		X	X		X		
Total	**9**	**16**	**7**	**14**	**10**	**9**	**1**

* Ways through which students learned the area calculation formula of trapezoid:

A – I had learned the formula before I attended the teaching sessions of this study

B – After attending the parallelogram and triangle teaching sessions of this study, I realized that the area of trapezoid could be calculated using this formula

C – I was able to construct the area calculation formula for trapezoid by myself using the "trapezoid" cognitive tool and worksheet provided by this study

D – I learned the formula through discussions with my classmates in this study

E – I learned the formula through teacher's teaching in this study

F – Teacher taught me according to my individual learning ability during this study

G – Others: Parents or tuition teacher taught me

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Appendix G – School's Consent Form

THE EDUCATION UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS AND INFORMATION TECHNOLOGY

CONSENT TO PARTICIPATE IN RESEARCH (FOR SCHOOL)

Development of Conceptual Understanding and Procedural Knowledge of Mathematics through Inquiry Based Learning Scaffolded by Cognitive Tools

My school hereby consent to participate in the captioned project supervised by Prof. Kong Siu Cheung and conducted by Lau Kam Sun, who is a student of Doctor of Education in the Education University of Hong Kong.

I understand that information obtained from this research may be used in future research and may be published. However, our right to privacy will be retained, i.e., the personal details of my students will not be revealed.

The procedure as set out in the **attached** information sheet has been fully explained. I understand the benefits and risks involved. My students' participation in the project are voluntary.

I acknowledge that we have the right to question any part of the procedure and can withdraw at any time without negative consequences.

I agree that the captioned research project can be carried out at this school.

Signature: ____________________
Name of Principal/Delegate*: (Prof/Dr/Mr/Mrs/Ms/Miss*) ____________________
Post: ____________________
Name of School: ____________________
Date: ____________________
(please delete as appropriate)*

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INFORMATION SHEET

Development of Conceptual Understanding and Procedural Knowledge of Mathematics through Inquiry Based Learning Scaffolded by Cognitive Tools

Your school is invited to participate in a project supervised by Prof. Kong Siu Cheung and conducted by Lau Kam Sun, who is a student of Doctor of Education in the Education University of Hong Kong.

The aim of this study is to investigate appropriate pedagogical approach for primary school students to develop conceptual understanding and procedural knowledge in mathematics learning with support from digital learning resources. Around 60 students studying in Primary 5 will be selected for the research. They will be divided into two groups, namely the experimental group and the control group. Each group will consist of around 30 students. The experimental group will use the cognitive tools in GeoGebra to explore how to calculate the areas of the parallelogram, triangle, and trapezoid. The control group will be taught how to calculate the areas of the parallelogram, triangle, and trapezoid using direct instruction approach.

The study will be conducted during regular school classes. Upon the completion of these classes, it is hypothesized that a significant percentage of students in the experimental group will be able to construct their own conceptual understanding and procedural knowledge to compute the areas of the various shapes, and be able to apply these newly constructed knowledge to closely related but new situations. The control group will provide data for comparing the effectiveness of the proposed pedagogical approach with that of the traditional direct instruction approach.

Please understand that your students' participation are voluntary. They have every right to withdraw from the study at any time without negative consequences. All information related to your students will remain confidential, and they will be identifiable by codes known only to the researcher. This research involves no potential risks. The results of this research will be published in the form of thesis, journal articles, books, presentations, and on-line web based reports. Permission will be obtained in advance from participants before any videos are used for public dissemination.

If you would like to obtain more information about this study, please contact Lau Kam Sun at telephone number xxxx xxxx or his supervisor Prof. Kong Siu Cheung at telephone number xxxx xxxx. If you have any concerns about the conduct of this research study, please do not hesitate to contact the Human Research Ethics Committee by email at xxxx@ied.edu.hk or by mail to Research and Development Office, The Education University of Hong Kong. .

Thank you for your interest in participating in this study.

Lau Kam Sun
Principal Investigator

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<u>Appendix H – Parents' Consent Form</u>

THE EDUCATION UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS AND INFORMATION TECHNOLOGY

CONSENT TO PARTICIPATE IN RESEARCH

Development of Conceptual Understanding and Procedural Knowledge of Mathematics through Inquiry Based Learning Scaffolded by Cognitive Tools

I ______________________________ hereby consent to my child participating in the captioned research supervised by Prof. Kong Siu Cheung and conducted by Lau Kam Sun.

I understand that information obtained from this research may be used in future research and may be published. However, our right to privacy will be retained, i.e., the personal details of my child will not be revealed.

The procedure as set out in the **<u>attached</u>** information sheet has been fully explained. I understand the benefits and risks involved. My child's participation in the project is voluntary.

I acknowledge that we have the right to question any part of the procedure and can withdraw at any time without negative consequences.

Name of participant ______________________________

Signature of participant ______________________________

Name of Parent or Guardian ______________________________

Signature of Parent or Guardian ______________________________

Date ______________________________

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INFORMATION SHEET

Development of Conceptual Understanding and Procedural Knowledge of Mathematics through Inquiry Based Learning Scaffolded by Cognitive Tools

You are invited to participate with your child in a project supervised by Prof. Kong Siu Cheung and conducted by Lau Kam Sun, who is a student of Doctor of Education in the Education University of Hong Kong.

The aim of this study is to investigate appropriate pedagogical approach for primary school students to develop conceptual understanding and procedural knowledge in mathematics learning with support from digital learning resources. Around 60 students studying in Primary 5 will be selected for the research. They will be divided into two groups, namely the experimental group and the control group. Each group will consist of around 30 students. The experimental group will use the cognitive tools in GeoGebra to explore how to calculate the areas of the parallelogram, triangle, and trapezoid. The control group will be taught how to calculate the areas of the parallelogram, triangle, and trapezoid using direct instruction approach.

The study will be conducted in regular school classes during 13 - 27 Oct 2015. Upon the completion of these classes, it is hypothesized that a significant percentage of students in the experimental group will be able to construct their own conceptual understanding and procedural knowledge to compute the areas of the various shapes, and be able to apply these newly constructed knowledge to closely related but new situations. The control group will provide data for comparing the effectiveness of the proposed pedagogical approach with that of the traditional direct instruction approach.

Your child's participation in the research is voluntary. You and your child have every right to withdraw from the study at any time without negative consequences. All information related to your child will remain confidential, and they will be identifiable by codes known only to the researcher. This research involves no potential risks. The results of this research will be published in the form of thesis, journal articles, books, presentations, and on-line web based reports. Permission will be obtained in advance from participants before any videos are used for public dissemination.

If you would like to obtain more information about this study, please contact Lau Kam Sun at telephone number xxxx xxxx or his supervisor Prof. Kong Siu Cheung at telephone number xxxx xxxx. If you or your child have/ has any concerns about the conduct of this research study, please do not hesitate to contact the Human Research Ethics Committee by email at xxxx@ied.edu.hk or by mail to Research and Development Office, The Education University of Hong Kong.

Thank you for your interest in participating in this study.

Lau Kam Sun
Principal Investigator

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Important Notes

1. The results of the pre-test and post-tests of this research will **NOT** have any impact to the students' academic results in school.

2. In order to ensure the accuracy of the research, we earnestly request parents or guardians not to teach the participants how to calculate the areas of the parallelogram, triangle, and trapezoid prior to the completion of this research.

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Appendix I – Ratings Given By the Two Raters on Students' Interviews

Ratings Given by Rater-01

Question no.	Student											
	C1	C2	C3	C4	C5	C6	E1	E2	E3	E4	E5	E6
1	P	B	P	P	P	P	P	P	P	P	N	P
2	P	B	P	P	N	N	P	P	P	P	P	P
3	N	N	N	N	N	N	C	C	N	N	N	N
4	C	C	C	N	C	N	C	C	C	C	N	C
5	P	B	P	P	N	N	B	B	B	B	P	B
6	C	C	N	N	N	N	C	C	C	N	N	N
7	C	N	N	N	N	N	C	C	N	C	N	N
8	C	C	C	N	C	N	C	C	C	N	N	C
9	B	B	B	N	N	N	B	B	B	P	P	B
10	B	B	B	N	C	N	B	B	B	P	P	B
11	C	N	N	N	N	N	C	C	C	N	N	N
12	N	N	N	N	N	N	C	C	N	N	N	N
13	B	B	B	N	B	N	B	B	B	P	P	B
14	N	C	N	N	N	N	C	C	C	N	N	N
15	N	N	N	N	N	N	C	C	C	N	N	N
16	N	C	C	N	N	N	C	N	N	N	N	N

**please refer to Table 9 for the descriptions of the coding used in the above table*

Ratings Given by Rater-02

Question no.	Student											
	C1	C2	C3	C4	C5	C6	E1	E2	E3	E4	E5	E6
1	P	B	P	P	P	N	P	P	P	P	N	P
2	P	B	P	N	N	N	P	P	P	N	P	P
3	N	N	N	N	N	N	C	C	N	N	N	N
4	C	C	C	N	N	N	C	C	C	C	N	N
5	P	B	P	P	N	N	B	B	B	B	P	N
6	C	C	N	N	N	N	C	C	C	N	N	N
7	C	N	N	N	N	N	C	C	N	C	N	N
8	C	C	C	N	C	N	C	C	C	N	N	C
9	B	B	B	N	N	N	B	N	B	P	P	N
10	B	B	B	N	N	N	B	B	B	P	P	B
11	N	N	N	N	N	N	C	C	N	N	N	N
12	N	N	N	N	N	N	C	C	N	N	N	N
13	B	B	B	N	B	N	B	B	B	P	P	N
14	N	C	N	N	N	N	C	C	C	N	N	N
15	N	N	N	N	N	N	C	C	C	N	N	N
16	N	C	C	N	N	N	C	N	N	N	N	N

**please refer to Table 9 for the descriptions of the coding used in the above table*

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