Relational Language Influences Young Children's Number Relation Skills

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Author Contribution Statement

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Abstract

Relational language is shown to influence mathematical skills. This study examines the association between *relational language* and *number relation skills*—knowledge of cardinal, ordinal, and spatial principles—among 104 U.S. kindergartners (5.9 years; 44% boys; 37% White, 25% Black, 14% Asian, 24% other) in 2017-2018 academic year. Controlling for general verbal knowledge, executive function, and counting and number identification skills, relational language predicted later number relation skills, specifically number line estimation, β=.30. Relational language didn't differentially predict number line estimation performance in children with low or high number relation skills, likely due to the restricted ranges of data within subgroups. Number relation skills, specifically number line estimation and number ordering, may be a pathway between relational language and mathematical skills.

Keywords: mathematical language, early numeracy, mathematical development

Relational Language Influences Young Children's Number Relation Skills

Much of the research on the development of math skills focuses on early numerical predictors of later math achievement (e.g., Jordan et al., 2009; Mazzocco & Thompson, 2005) and the pathways through which these predictors operate. Yet non-numerical skills, such as language, may also be important for learning math. Researchers have found that a particular aspect of language, knowledge of relational terms, predicts later math skills (Purpura & Logan, 2015). In the present study, we investigate whether children's knowledge of relational language predicts their *number relation skills*, important aspects of early numeracy, and whether number relation skills may be a pathway through which relational language influences math skills.

Relational Language and Mathematical Skills

Language is a tool for communication, but it also supports reasoning and thought. Although the ability to form numerical representations, often measured as accuracy comparing sets of varying numerosities, is shared across species and is somewhat independent of language (Gelman & Butterworth, 2005), studies implicate influences of specific aspects of language on math skills (Brooks et al., 2011; Vukovic & Lesaux, 2013). For instance, preschoolers' expressive (e.g., naming objects) and receptive (e.g., pointing to pictures) vocabulary are both correlated with concurrent knowledge of number words (Negen & Sarnecka, 2012) and numeral identification skills two years later (LeFevre et al., 2010). When examining the pathways between general verbal knowledge and math skills, researchers have specifically found that children's knowledge of *relational language* fully mediates this association both in preschool (Purpura & Reid, 2016) and kindergarten (Toll & Van Luit, 2014b). These findings suggest that general verbal knowledge may help children learn number words across contexts, and that relational language in particular may be a potential pathway between language and math skills.

Relational language, defined as the vocabulary for connecting and describing relations between items, can be used to denote relations in terms of quantity (e.g., *many, most*), order (e.g., *last, before*), space (e.g., *left, below*), or magnitude or extent (e.g., *medium, full*). Relational language is, therefore, relevant to describing math concepts. For instance, words such as *long* and *heavy* reflect measurement; *near* and *far* describe distances between objects; *before* and *after* convey ordinal positions of numbers in the counting sequence and the spatial position on a number line; and *parts* and *whole* may be involved in learning about fractions and proportions. Researchers of young children's relational language focus on quantity and space, sometimes referring to these vocabularies as math language (Purpura & Logan, 2015; Toll & Van Luit, 2014b); however, other researchers use "math language" as a label for formal math terms, such as addition, equal sign, and subtraction (e.g., Powell & Driver, 2015). In this paper we use "relational language" to encompass vocabulary describing quantitative and spatial relations.

Correlational, longitudinal, and intervention studies provide some evidence to support the notion that relational language is linked to a broad set of math skills. For instance, preschoolers' knowledge of relational language in the fall semester predicts their general numeracy skills in the spring (Purpura, Day, et al., 2017). Kindergartners with higher vs. lower levels of relational language show more growth in their math skills six months later (Toll & Van Luit, 2014a). Although relational language predicts math skills, the results from relational language intervention studies are mixed. Purpura, Napoli, et al. (2017) randomly assigned preschoolers (N = 47) to either an eight-week intervention using small group relational language storybook reading (administered for 15- to 20-minutes two to three times per week) or a business-as-usual control condition. Children in the intervention condition showed more improvement on their relational language and general numeracy skills, compared to children in the control condition.

These researchers also examined the effects of a family-implemented relational language intervention on preschoolers' general numeracy skills (*N* = 84; Purpura et al., 2021). Families read picture books that either included or excluded relational language for a total of 12 reading sessions over four weeks. Children whose caregivers read them picture books with relational language content, as opposed to children whose caregivers read books without this content, scored higher on relational language and numeracy measures; and these positive effects on numeracy sustained eight weeks after the intervention.

Similarly, Jennings et al. (1992; *N* = 61) found that kindergartners who received a five-month intervention that replaced a traditional math curriculum with math book reading activities incorporating relational vocabulary showed more improvement on the Test of Early Mathematics Ability, and used more relational language during free play, compared to children who received only the traditional math curriculum. However, when Hassinger-Das et al. (2015; *N* = 124) implemented an intervention similar to that used by Jennings et al. for only two months, improvement emerged among low-achieving kindergartners' understanding of relational vocabulary but not their Woodcock-Johnson III Calculation and Applied Problems scores.

These inconsistent findings may be partly due to differences in sample characteristics, intervention intensities, the skills targeted by the interventions, and outcome measures used, which raises the need to further delineate the association between relational language and math skills. Despite the established link between relational language and general math skills above and beyond general language skills (e.g., Purpura & Reid, 2016; Toll & Van Luit, 2014b), only one study has examined the association between relational language and specific early numeracy skills. In that study, Hornburg et al. (2018) found concurrent associations between relational language and counting, cardinal knowledge, number comparison, and number ordering when controlling for children's general language skills. Two important extensions of these findings are to (1) investigate the longitudinal association between relational language and distinct aspects of early numeracy while controlling for related cognitive correlates, such as general verbal knowledge (LeFevre et al., 2010) and executive function skills (Gashaj et al., 2016), and (2) delineate the potential mechanisms underlying the association between relational language and math during early childhood. These extensions offer the additional value of replicating the finding of unique associations between relational language and aspects of numeracy skills.

Number Relation Skills

Number relation skills involve the knowledge of how two or more numbers are related to each other in terms of their cardinal values (e.g., more, big), ordinal positions in the counting sequence (e.g., after), and spatial positions on a number line (e.g., between, near, middle). Understanding the meanings of relational terms and using them accurately may be important skills for organizing number words in a meaningful way. Although relational language may be important for children's mathematical thinking in general, we propose that it may be particularly important for number relation skills. We therefore measure a variety of number relation skills and a contrasting aspect of numeracy, verbal counting and numeral identification skills. We test our hypothesis by examining the predictive association between relational language and number relation skills while controlling for other numerical (i.e., counting and number identification) and non-numerical (i.e., executive function and general verbal knowledge) factors shown to correlate with general mathematics (Cragg & Gilmore, 2014; LeFevre et al., 2010).

Relational Language and Number Relation Skills Across Performance Levels

Despite the positive correlation between relational language and math skills reported across studies, the effects of children's age or performance level on this association vary across studies. Some researchers report that relational language is a better classifier of later math performance than numerosity discrimination acuity or general numeracy skills, and that it is a consistent predictor of low math performance in preschoolers (Purpura, Day, et al., 2017). However, these researchers also find that the association between relational language and numeracy performance differs depending on children's general numerical ability, as measured by comprehensive batteries of numerical tasks. That is, among preschoolers with low numeracy scores, numerosity discrimination acuity is a stronger predictor of numeracy performance compared to relational language, whereas relational language is a stronger predictor than numerosity discrimination among preschoolers with high numeracy scores (Purpura & Logan, 2015). In contrast, among kindergartners with low numeracy scores, relational language is the strongest predictor of initial numeracy performance and growth in numeracy skills compared to numerosity discrimination acuity, number comparison, and working memory, yet it is one of the weakest predictors among kindergartners with average numeracy scores (Toll & Van Luit, 2014a). Together, these studies suggest that the association between relational language and math skills may vary with other aspects of children's math knowledge, although the inconsistent pattern of the results across studies may also suggest that other factors are involved. Building on this area of research, we examined the association between relational language and number relation skills over time in kindergartners with differing performance levels on our number relation tasks, based on sample-referenced composite scores in either the low or high range.

Potential Pathways between Relational Language and Mathematical Skills

If relational language is essential to the development of number relation skills, that specific role may account for the reported association between relational language and general math skills. Although previous studies demonstrate an influence of relational language (Purpura

& Reid, 2016; Purpura & Logan, 2015; Toll & Van Luit, 2014a, 2014b) and number relation skills (Booth & Siegler, 2008; De Smedt et al., 2009; Lyons et al., 2014) on general math skills, none explicitly focuses on number relation skills as a potential mediator in this association. To address this gap in the literature, we conducted mediation analyses to test whether number relation skills mediate between relational language and math skills.

The Role of Executive Function Skills

Children's relational language and number relation skills may be influenced also by their e*xecutive function (EF)* skills, the abilities to retain and manipulate information during a task, suppress habitual responses, and shift attention between multiple response options. When children compare items based on numerical or non-numerical dimensions and use language to describe these relations, children need to inhibit irrelevant information, determine and focus on relevant dimensions, shift attention between items being compared, and incorporate information from the comparisons. For instance, to understand the meaning of *medium* and identify the medium item from a set of objects, children need to conduct multiple pairwise comparisons and make iterative connections between these comparisons to order the objects based on size. Similarly, to compare four and five, children may need to remember the counting sequence, and focus on four and five separately while inhibiting irrelevant comparisons, such as four and three.

Prior studies implicate a close association between relational language, number relation skills, and EF skills. First graders' ability to hold information in working memory is correlated with their performance on seriating items based on size and sets based on quantity (Nunes et al., 2007). Preschoolers' ability to ignore irrelevant information while remembering relevant information is correlated with their concurrent performance on number comparison (Gashaj et al., 2016; Purpura, Schmitt et al., 2017). Four- to seven-year-olds' ability to hold information in working memory and flexibly shift between rules is correlated with their concurrent performance on number line estimation (Friso-van den Bos, et al., 2014; Gashaj et al., 2016). These findings show that EF skills are linked with children's relational language and number relations skills. In the current study, we estimate and statistically control for the influence of EF skills in the association between relational language and number relation skills.

The Current Study

In the present study, we examine the influence of relational language on number relation skills in kindergartners to address the following research questions:

RQa) Does relational language predict later number relation skills across different numerical tasks beyond other numerical (counting and number identification) and non-numerical skills (EF and general verbal knowledge)? Given the consistent association between relational language and math skills as well as their close connections to number relation skills, we hypothesize that relational language may predict later number relation skills beyond the covariates.

RQb) Is relational language a consistent predictor of later number relation skills among kindergartners with different initial number relation scores? Because of the mixed findings in the literature, we explore this research question and do not have a directional hypothesis.

RQc) Do number relation skills mediate the association between relational language and math skills in kindergartners? Based on prior empirical and theoretical work, we hypothesize that number relation skills may mediate the association between relational language and math skills.

Method

Participants

Kindergartners were recruited from 11 classrooms in four public schools within three school districts in a midwestern greater metropolitan area in the United States, during the 2017-

2018 academic year. At each school, the percentage of English Language Learners ranged from 9% to 57%, and the percentage of children eligible for free and reduced lunch (as a proxy of poverty) ranged from 22% to 89%. Children who spoke English, including English Language Learners, and who had no significant visual or auditory impairment or known developmental delay, were eligible and invited to participate. Recruitment occurred in person during parent teacher conferences held at each school, or by sending enrollment materials to parents through their child's teacher. On average, 50% of invited families agreed to enroll their child in the study. Enrollment rates across schools ranged from 27% to 66%.

A total of 104 typically developing kindergartners (46 boys, 58 girls) participated in the study. The participants were five or six years old (M = 5.9 years, SD = 0.34), and were of White (38), Black (26), Asian or Asian American (15), American Indian or Alaskan Native (2), or other races (23). In terms of ethnicity, 22 were Hispanic or Latino and 82 were not. Approximately half were regularly exposed to languages other than English in their home (22 Spanish, 14 Somali, 11 other). Five additional participants did not complete the study due to family relocation (n = 1) or opting out of some testing sessions (n = 4); these five children were excluded from analyses. The final sample included 104 children with varying levels of English language experience, providing opportunities to explore the associations between language and math skills among children from diverse backgrounds. The sample size affords 90% power to detect an effect of $f^2 \geq .08$ at $p < .05$. The conventional cut-offs for small, medium, and large effects are .02, .15, and .35 respectively (Cohen, 1992).

Procedure

The current study was part of a larger study in which participants completed a pretest, four training sessions in one of three conditions, and a posttest, all within six weeks. A 2 (Time:

Pretest versus Posttest) × 3 (Condition) repeated measures MANOVA on the number relation task scores revealed no effect of training condition or Time × Condition interaction, *p*s > .10 (Chan, 2019; see details in Appendix A). Thus, in the present study, we focused on children's performance at pretest and posttest only (hereafter Time 1 and Time 2, respectively). At Times 1 and 2, we measured relational language, number relations, counting and number identification, and EF skills. At Time 1, we also measured general verbal knowledge and general math skills. Children were tested individually, in a relatively quiet area at their school. Most children completed the Time 1 measures in two sessions, and the Time 2 measures in one session. Additional sessions occurred if children requested breaks or if teachers set time constraints.

Measures

Relational Language

The Boehm Test of Basic Concepts – 3rd Edition (hereafter Boehm Test; Boehm, 2001), a standardized picture vocabulary task, was used to measure children's receptive knowledge of relational language. The examiner presented an illustration with four or five response options (e.g., four horses lining up at a pond) and asked children to point to the target option (e.g., "the horse that is *last* in line"). The raw scores reflected the number of correct responses from 50 trials, and were scaled to age-referenced percentiles. We used the percentiles at Time 1 as the focal predictor in regression models and the exogenous variable in the mediation. The internal consistency reliability of this measure ranges from .88 to .90 (Boehm, 2001).

General Verbal Knowledge

The Verbal Knowledge subtest of the Kaufman Brief Intelligence Test – 2nd Edition (KBIT-2; Kaufman & Kaufman, 2004) was used to measure children's receptive English vocabulary at Time 1 only. Children viewed a set of six pictures per trial on a single page of a testing easel, while the examiner asked children to identify which picture corresponded to either the meaning of a spoken word (e.g., "point to, table") or the answer to a question (e.g., "what lives in the ocean?"). The number of correct picture selection responses was converted to a standard score based on an age-referenced mean of 100 and a standard deviation of 15, and this standard score was the covariate we used in this study. The internal-consistency reliability of this Verbal Knowledge subtest is .87 (Kaufman & Kaufman, 2004).

Number Relation Skills

Four tasks were administered to measure children's number relation skills at Times 1 and 2. The first two tasks focused on cardinal relations between numbers (number comparison, set relations); the third on ordinal relations (number ordering); the fourth on number-space mapping (number line estimation). All were adapted from prior research and had no overlap with items in the Test of Early Mathematics Ability–3rd Edition (TEMA-3; Ginsburg & Baroody, 2003).

Number comparison was based on studies of verbal magnitude comparison (e.g., De Smedt et al, 2009). The examiner asked children, "which number means more," then stated two numbers. All number pairs differed by two (e.g., 3 versus 5), and the numbers ranged from 1 to 99. The examiner recorded the number of trials out of 16 on which children responded correctly. The internal reliability of these items was KR-20 = .74 and .77 at Times 1 and 2, respectively.

Set relations was adapted from Sarnecka and Carey's (2008) unit task. The examiner first showed children a small opaque bag and explained that there are *N* coins in the bag. N ranged from 1 to 20. Then the examiner added 1 or 2 coins, one at a time, to the bag, and asked children, "now are there [$N + 1$] or [$N + 2$] coins?". We used this task to assess children's mapping between set size and counting sequence, and if children understood that adding one more to a set meant increases of one in the counting sequence. The examiner recorded the number of correct responses out of 8 trials. The reliability of this task was KR-20 = .59 and .62 at Times 1 and 2, respectively. The relatively low reliability may be due to the limited number of items.

Number ordering was adapted from Lyons and colleagues' (2014) task. The examiner presented three Arabic numerals at a time, each printed on a separate card and presented face up. The examiner arranged the cards in a standardized way, out of numerical sequence, then asked children to rearrange the numbers in order from smallest (left) to largest (right). The numerals appearing on each card ranged from 1 to 50. Per trial, the numerical distance between adjacent cards was consistent within each triplet set and was either one (e.g., 1, 2, 3) or two (e.g., 1, 3, 5). The examiner recorded the number of correctly sequenced triplet sets out of 14 trials. The reliability of these items was KR-20 = .93 and .91 at Times 1 and 2, respectively.

Our *number line estimation* task was adapted from Siegler and Booth (2004). The examiner presented a number line and a target number on an 8.5" × 11" piece of paper in landscape orientation, and asked children to estimate the position of the target number on the number line. The length of the line was 10 inches, and each of two endpoints was marked with a 0.5-inch vertical line 2.5 inches from the nearest end of the line, resulting in a 5-inch span between the two points. A total of 26 trials were administered, comprised of 12 trials with the points labeled as 0 and 20 (3, 4, 6, 7, 8, 9, 11, 13, 14, 16, 17, 19), and 14 trials with the points labeled as 0 and 100 (7, 9, 15, 16, 26, 28, 51, 53, 64, 65, 76, 77, 89, 91). We selected these two number ranges because they have been used in prior studies of number line estimation skills in five- or six-year-olds (e.g., Berteletti et al., 2010; Ghazali et al., 2013; Hoard et al., 2008). Further, the correct midpoint (i.e., either 10 or 50) was marked with a 0.5-inch vertical line and labeled with the corresponding numeral on half of each set of trials, resulting four trial types (i.e., 0-20 without a midpoint, 0-20 with a midpoint, 0-100 without a midpoint, 0-100 with a midpoint). We included midpoint trials based on evidence suggesting selective benefits of an explicit, labeled midpoint on performance, possibly related to participants' familiarity with the number range presented in the task (Peeters et al., 2017). Together, the four trial types maximize variation in participants' number line estimation performance (see details in Appendix B).

Prior to giving children each type of the trials, the examiner gestured to the endpoints and, if applicable, the midpoint, while stating, for example, "0 goes here (point to 0), 10 goes here (point to 10), and 20 goes here (point to 20)." Per Siegler and Booth (2004), we calculated percent absolute error (|estimate – target number| / numerical range) for each trial. Children's percent absolute error across these four types of trials were correlated with each other ($.41 < r_s$s $< .76$, ps $< .001$), and loaded on one factor in exploratory factor analyses at both time points (.49 < loadings < .88). Thus, we used the average percent absolute error as an indicator of children's overall task performance in our primary analyses (see Appendix B for additional analyses by trial type). The test-retest reliability of this task was $r = .71$ in the current sample.

We also included a small subset of novel items as an additional exploratory assessment of children's conceptual understanding of number line representations. The six trials in this subset involved placing the numbers outside the depicted number line endpoints, specifically 22, 24, 31, and 33 on the 0-20 number line, and 105 and 120 on the 0-100 number lines. Importantly, the number lines presented across all trials continued beyond the endpoints for 2.5 inches, as a way to indicate that numbers continued beyond these endpoints. Children received one point for every item to which they responded correctly, defined as placing the number to the right, instead of left, of the right-most (numerically largest) endpoint on the number line, for a maximum score of 6. The reliability of these six items was KR-20 = .91 and .92 at Times 1 and 2, respectively. Because these trials differed qualitatively from traditional number line assessments, we examined them separately from other number relation tasks and the traditional number line assessment reported previously. We used children's scores on these novel items to provide some construct validity for the qualitative differences between children with low versus high number relation skills.

Counting and Number Identification Skills

We used four tasks to measure children's verbal counting skills (counting aloud, counting backwards, and counting after) and their mapping of number words to numerals (numeral identification). During the *counting aloud* task, the examiner asked children to count as high as possible from 1 to determine their counting span (i.e., the highest number they verbally count up to without error). If children stopped counting, the examiner prompted, "what comes next?" The task ended when children indicated that they did not know what came next or reached 130. The examiner recorded the highest number that children counted up to correctly. In the *counting backwards* task, the examiner first demonstrated counting backwards from three (i.e., "three, two, one"), then asked children to count backwards from 10 and 20. Children received a score based on the highest number from which they could count backwards without error. For instance, they received a score of 12 if they could count backwards from 12, or a score of 0 if they could not count backwards correctly at all. In the *counting after* task, the examiner said a number, and asked children to name the subsequent number by asking, "what comes next?" The examiner stated a number between 1 and 99, noted children's response, and recorded the number of trials out of 14 on which children responded correctly. The reliability of these items was KR-20 = .91 and .90 at Times 1 and 2, respectively. In the *numeral identification* task, the examiner presented 20 printed Arabic numerals in a random sequence (1 to 9 and 11 numbers between 10 and 99), asked children to name the numerals, and recorded how many they named correctly. The reliability of these items was KR-20 = .91 and = .90 at Times 1 and 2, respectively.

We note that it was impossible to avoid overlap between our counting and number identification tasks and several TEMA-3 items that tapped the same skills. However, each of those TEMA-3 items was based on one to three trials per item, with a binary pass (1) or fail (0) score per item to be added to a much larger composite raw score with a maximum of 72. Two TEMA-3 items tapped whether children could count aloud to 21 or 42, respectively, yielding a maximum score of 2 points. Children had to count as high as they could for our more precise measure of their counting span. Likewise, two TEMA-3 items tapped counting backwards from 10 or 20, with a binary score applied to each (maximum score of 2), and our measure included a counting backwards span from 20. Three TEMA-3 items tapped counting after, with each item yielding a binary score from three trials each (maximum score of 3); and another three TEMA-3 items tapped numeral identification of three one-digit and six two-digit numbers (with a maximum score of 3). Our assessments were more precise, with more granular scoring yielding larger ranges, compared to this set of TEMA-3 items. Moreover, on average, participants completed an additional 25 items on the TEMA-3 assessment that tapped additional skills to inform a single general math score, as reported next.

Mathematical Skills

The Test of Early Mathematics Ability – 3rd Edition Form A (TEMA-3; Ginsburg & Baroody, 2003) was administered at Time 1 only. It is a comprehensive assessment of children's general formal and informal math skills normed for three- to eight-year-olds. In this test, the examiner provided pictures, manipulatives, or paper and pencil; and children either pointed to a picture or gave a verbal or written response. TEMA-3 items tapped skills such as verbal counting, set counting, story problems, and arithmetic fact retrieval. Similar types of items were grouped for the ease of test administration but we followed the standard procedure, with all children starting on the entry item for five-year-olds and stopping after failing five consecutive items. We used the age-referenced standard score as the endogenous variable in the mediation. The internal-consistency reliability of the TEMA-3 is α = .94 (Ginsburg & Baroody, 2003).

Executive Function Skills

We administered Head Toes Knees Shoulders task and Minnesota Executive Function Scale to measure children's EF skills. Both tasks engaged children's inhibitory control, working memory, and cognitive flexibility, and were considered composite measures of EF skills.

Head Toes Knees Shoulders task (HTKS; Cameron Ponitz et al., 2008) is a standardized activity in which children are instructed to execute the opposite of what the examiner asks. For instance, children were taught to touch their toes when the examiner said, "touch your head." The task included practice and test trials across three levels presented in increasing difficulty, which collectively captured variability of EF skills. Level 1 was comprised of one pair of rules: (a) "touch your head" meant touch your toes, and (b) "touch your toes" meant touch your head. Level 2 was similar to Level 1 but with an additional pair of rules: (c) "touch your knees" meant touch your shoulders, and (d) "touch your shoulders" meant touch your knees. Rules were changed in Level 3: (e) head was paired with knees, and (f) shoulders were paired with toes. The examiner provided practice trials with up to three reminders at each level. We administered 47 trials (17 practice and 30 test trials), and children received 0 (*incorrect*), 1 (*self-correct*), or 2 (*correct*) points per trial, with a maximum score of 94. The internal consistency reliability ranges from α = .87 to .92 (Cameron Ponitz et al., 2008).

Minnesota Executive Function Scale (MEFS; Carlson & Zelazo, 2014) is an adaptive iPad version of the Dimensional Change Card Sort task (Zelazo, 2006). Children sorted virtual cards into two virtual boxes according to examiner stated rules. Children had to remember the rules, ignore conflicting features, and sort cards flexibly when the rules changed. For instance, if the examiner revealed a virtual card depicting a green lion, children should drag the card to the box illustrated by a green monkey when playing “the color game” or to the box illustrated with an orange lion when playing “the shape game.” MEFS had seven levels of difficulty and the entry level was determined by the participant’s age. The difficulty increased with the level as the examiner introduced additional rules (in Levels 1 to 3), changed the rules (in Levels 4 and 5), or did both (Levels 6 and 7). The examiner provided practice trials with feedback at the beginning of each level, and children progressed forward or backward depending on their performance. Total scores were computed based on highest level passed, highest level attempted, errors, and reaction time. We used an age-referenced standard score as the indicator of children’s performance on this task. The test-retest reliability for MEFS is *ICC* = .93 (Beck et al., 2011).

Results

Analytic Approach

Prior to addressing our main research questions, we conducted descriptive and correlation analyses to identify the variables that were most appropriate for our primary analyses. We report the results on the distribution of, and the interrelations between, scores on each task in the Descriptive and Correlation Analyses section. For our principal numerical tasks, two sets of four tasks comprised the separate *number relation* and *counting and number identification* scores; therefore, we conducted confirmatory factor analyses to confirm these intended constructs. Confirmatory factor analyses revealed that all four number relation tasks loaded highly on one factor at both time points (.63 < loadings < .93, *p*s < .001); and that all four counting and number identification tasks also loaded highly on one factor at both time points (.67 < loadings < .93, *p*s < .001). Because of our focus on the association between relational language and number relation skills, we examined the four number relation task scores separately in the primary analyses. We used the composite scores to address our research question on subgroup differences, using a median split based on the average *Z* score across the four tasks, resulting in two subgroups of children with low or high number relation skills. We note that although the factor loadings and the correlations between four number relation tasks were moderate to high at both time points for the full sample (.41 < $|r_s\text{s}|$ < .77; Table S1), the correlations were lower in our two subgroups of children (low number relation group: .24 < $|r_s\text{s}|$ < .63; high number relation group: .06 < $|r_s\text{s}|$ < .65), likely due to the restricted ranges of the data within each subgroup since scores were limited to the lower or higher range, respectively. We used the average *Z* score strategy to create the counting and number identification composite score for use as a covariate in the analyses. EF skills were measured with only two tasks, so factor analysis was not indicated. HTKS raw score and MEFS standard scores were correlated with each other at Times 1 (r_s = .44) and 2 (r_s = .52), so we used the average *Z* score of the two tasks as an EF composite score in analyses.

To inform primary analyses on number relation skills at the task level, we conducted a repeated measures MANOVA with Time 1 versus Time 2 as the independent variable, and the four number relation task scores as the dependent variables, to identify the tasks in which children showed significant gains. On tasks for which significant gains emerged, we conducted regression analyses to examine the effects of relational language on *later* number relation performance. We considered the nesting structure of children within school; however, since the number of schools was small (4) and we did not have predictors at the school level, we included school as a dummy-coded variable and proceeded with one-level linear regressions.

We focused on the number relation task scores that showed significant gains from Time 1 to Time 2, and tested whether relational language predicted number relation skills in the overall sample (RQa). In a regression model, we tested the effect of relational language at Time 1 on number relation task performance at Time 2 while controlling for Time 1 number relation task score, counting and number identification composite, general verbal knowledge, and EF composite scores. We multiplied the percent absolute error on the number line estimation task by -1 prior to conducting the regression models so that larger values represented better estimation. We note that one child had missing data on the number line estimation task at Time 2, therefore, the child was omitted in the analyses on children's later number line estimation performance.

To explore whether gains in number relation and effects of relational language on these gains varied as a function of initial number relation skills (RQb), we repeated the MANOVA and regression analyses with two subgroups of children whose initial number relation score was low or high, based on the composite score median split. Although underpowered, the analyses may still provide unbiased estimates of the effect of relational language on number relation skills.

To examine whether number relation skills may be a potential pathway between relational language and math skills (RQc), we conducted mediation analyses with scores at Time 1. We used the relational language score as the exogenous variable, the number relation task scores as the mediators, the TEMA score as the endogenous variable, and general verbal knowledge and EF scores as covariates on TEMA score. We conducted the analyses with the full sample, then explored the pathways with the two subgroups. We used *lavaan* package (Rosseel, 2012) with maximum likelihood in R to conduct the mediation analyses.

Descriptive and Correlation Analyses

Through a review of the descriptive analyses (Table 1), we found that none of the tasks was subject to ceiling or floor effects, as reflected by the mean and range of scores. Moreover, scores on our novel number line estimation trials were widely distributed and bimodal. Interestingly, 84% of the children who scored 0 or 1 out of 6 on the novel number line estimation items were in the subgroup with low initial number relation skills (M = 1.77, SD = 2.18) whereas 81% of the children who scored 5 or 6 on these items were in the subgroup with high initial number relation skills (M = 4.31, SD = 2.09), suggesting that these novel items captured qualitative differences in how children with low or high number relation skills interpreted number lines. The sample represented a wide range of performance levels across all the tasks administered for this study, which afforded investigating the associations between relational language, early numeracy, and math skills. We used Spearman correlations due to non-normal distributions of scores on counting backwards, counting aloud, and the relational language tasks. All correlations were significant, $.24 < |r_s\text{s}| < .78$, ps < .05, and the associations between all pairs of task scores were moderate or strong at both time points (Table S1). [Insert Table 1 Here]

Predictors of Number Relation Skills

First, we conducted a one-way repeated measures MANOVA comparing individual number relation task scores at Time 1 and Time 2, and found that the effect of time was significant, $F(4,99) = 20.50$, $p < .001$, $\eta_p^2 = .453$. ANOVAs revealed that children improved on number ordering, $F(1,102) = 12.99$, $p < .001$, $\eta_p^2 = .113$; and reduced percent absolute error on number line estimation, $F(1,102) = 60.23$, $p < .001$, $\eta_p^2 = .372$ (Table 1). The children did not improve on number comparison or set relation tasks from Time 1 to Time 2, ps > .05.

Informed by the MANOVA results, we conducted two sets of regressions to examine whether relational language score at Time 1 predicted later performance on number ordering and number line estimation tasks, respectively. The baseline model predicting Time 2 number ordering performance revealed that children with higher counting and number identification skills at Time 1 scored higher on number ordering task compared to children with lower counting and number identification skills, $B = 0.94$, $p = .014$. Children's relational language score at Time 1 did not significantly predict their later performance on number ordering task above and beyond the covariates, $p = .497$ (Table 2, Model 1; Figure 1). [Insert Table 2 and Figure 1 Here]

The baseline model predicting Time 2 number line estimation performance revealed that counting and number identification, general verbal knowledge, and EF skills did not predict number line estimation above and beyond Time 1 number line estimation, *p*s > .10. Adding relational language score as a predictor revealed that relational language at Time 1 positively predicted number line estimation at Time 2, $B = 0.06$, $p = .029$, ΔR =2.3% (Table 2, Model 2).

Predictors of Number Relation Skills Vary by Children's Initial Number Relation Scores

We conducted parallel MANOVAs and regressions to examine whether improvement on number relation tasks from Time 1 to Time 2, and predictors of task scores at Time 2, vary by children's initial number relation scores. Two MANOVAs comparing number relation task scores at Time 1 versus Time 2 revealed that both subgroups of children with low or high number relation composites improved on number relation tasks, *p*s < .001, η_p^2s > .419. The follow-up ANOVAs revealed that, (a) *children with low number relation composite* improved on the number ordering (Time 1: $M = 6.04$, $SD = 3.87$; Time 2: $M = 8.02$, $SD = 3.93$), $F(1,50) = 32.79$, $p < .001$, $\eta_p^2 = .396$; and number line estimation tasks (Time 1: $M = 18\%$, $SD = 5\%$; Time 2: $M = 14\%$, $SD = 6\%$), $F(1,50) = 27.11$, $p < .001$, $\eta_p^2 = .352$; (b) *children with high number* *relation composite* improved on only the number line estimation task (Time 1: M = 13%, SD = 8%; Time 2: M = 9%, SD = 5%), $F(1, 51) = 32.84$, $p < .001$, η_p^2 =.392 (Table S2).

We conducted three regression models to test the influence of counting and number identification skills, general verbal knowledge, EF, and relational language at Time 1 on Time 2 performance on (a) number ordering and (b) number line estimation in children with low number relation composite, and (c) number line estimation in children with high number relation composite. Among children with low number relation composite, counting and number identification skills predicted performance on number ordering, B = 1.10, p = .022 (Table 3, Model 1); none of the numerical or non-numerical skills significantly predicted later number line estimation performance beyond the Time 1 number line estimation performance, *p*s > .05 (Table 3, Model 2). Among children with high number relation composite, none of the variables significantly predicted children's later number line estimation performance beyond their earlier number line estimation performance, *p*s > .05 (Table 3, Model 3; Figure 1). [Insert Table 3 Here]

Mediation of Number Relation Skills

The first step of the mediation analysis revealed that, after controlling for general verbal knowledge and EF, relational language significantly predicted children's concurrent math skills and accounted for 4.2% of variance in TEMA-3 scores (c = 0.15, p = .017; Figure 2a). Next, when we added concurrent number ordering (a = 0.09, b = 1.16, *p*s < .001) and number line estimation scores (d = 0.14, e = 0.77, *p*s < .001) as two mediators, the two scores fully mediated the association between relational language and math skills (c' = -0.05, p = .342; Figure 2b). Next, we explored the pathways between relational language and math skills in children with low versus high number relation skills. In both models, relational language did not significantly predict math skills after controlling for general verbal knowledge and EF skills (low number relation group: $c = 0.10$, $p = .333$; high number relation group: $c = 0.05$, $p = .579$); we did not, therefore, further explore mediations in these two subgroups of children. [Insert Figure 2 Here]

Discussion

We began this work with three questions. The first was whether relational language predicts later number relation skills above and beyond other numerical skills—such as counting and number identification—and non-numerical skills, such as EF and general vocabulary. We found that relational language did, in fact, predict later number line estimation performance above and beyond prior number skills of counting and identification, EF, and general vocabulary. By documenting these effects longitudinally, our results extend those of Hornburg et al. (2018) who found concurrent associations between relational language and aspects of numeracy skills. Our results offer additional evidence that relational language may play a unique role in the development of number line estimation skills.

Our second question was whether relational language was a consistent predictor of later number relation skills among kindergartners, including children with varying levels of initial number relation skills. To answer this question, we compared the results from the full sample with the results from subgroups of the sample with low or high levels of initial number relations skills. Although the analysis on the full group revealed that relational language predicted performance on number line estimation, it was not a significant predictor of later number line estimation performance in either subgroup of children, potentially owing to the relatively small group size and the restricted range of scores. Further, counting and number identification skills predicted number ordering performance in children with a below median number relation composite score, and none of the numerical or non-numerical skills predicted number line estimation performance in children with an above median number relation composite.

Our third question asked *how* relational language impacts math skills. Is its impact on math skills direct, or indirect via number relation skills? To answer this question, we conducted a series of analyses to test the potential mediating effects of number ordering and number line estimation performance on the association between relational language and math skills. Our mediation analyses indicated that both number ordering and number line estimation performance mediated the effect of relational language on math skills, and this mediating effect was independent of the contribution of EF and general verbal knowledge. Thus, our findings suggest that relational language may influence math skills through number relation skills.

Taken together, these results illustrate that an understanding of the development of early math skills concerns more than just understanding how the components of math skills are related to each other, but also involves identifying the period of development when the different effects are likely to emerge. Understanding these nuanced effects is relevant to developing interventions that are appropriately sequenced, timed, and sensitive to the level of children's math skills in a way that considers aptitude by instruction interactions that may influence learning (Connor et al., 2018). We discuss these findings in more detail below.

Relational Language's Unique Role in Number Relation Skills

Prior work revealed that relational language positively predicted math skills, and might be one pathway to improving math skills in early childhood (Hornburg et al. 2018; Jennings et al., 1992; Purpura et al., 2021; Toll & Van Luit, 2014a). However, the role of potential factors that co-vary with the development of relational language were not examined simultaneously in past work. When considering these covariates together, we found that relational language predicted later number relation skills above and beyond other numerical and non-numerical factors known to influence general math skills, and that the predictive relation was moderate and specific to number line estimation performance (β = .30). Importantly, number line estimation performance by young children is one of the best predictors of later math achievement (Booth & Siegler, 2008; Schneider et al., 2018). It predicts kindergartners' risk for future math learning difficulties (Bull et al., 2020) and differentiates intellectually precocious first graders from their peers with average math achievement (Hoard, et al., 2008). Number lines provide a spatial representation of numerical magnitudes, with the direction and distance between numbers reflecting both ordinal and cardinal relations, and both types of knowledge influence the nature and precision of performance on the number line estimation task. These aspects of understanding number lines likely provide an important foundation for later math learning and development, and reflect the depth of a child's number relation understanding.

It is unlikely that this finding was an artifact of our data distribution, as none of the number relation task scores was subject to ceiling or floor effects. If this close association between relational language and number line estimation reflects a genuine connection between these two skills, it may be that number line estimation requires multiple aspects of relational knowledge and concepts. Estimating the location of a number in relation to the endpoints may require children to think about the order of numbers in the counting sequence and how ordinal and cardinal relations between numbers can be represented in space. In contrast, number comparison and set relation tasks require a focus on cardinal relations, whereas the number ordering task relies mostly on ordinality of the counting sequence. This hypothesis warrants future investigation, but the current finding provides a starting point for exploring how relational language may influence later math skills through specific aspects of early numeracy.

Another gap in previous findings that linked relational language to math skills concerns differences in children's EF skills, which may have accounted for the association between relational language and math skills. As expected, we observed positive correlations between relational language and EF tasks at both time points in our study (.57 < r_s < .65), the magnitudes of which were comparable to those reported in a previous study with preschoolers (.50 < r < .52; Schmitt et al., 2019). Because of the significant correlations, we statistically controlled for the influences of EF skills in all analyses to estimate the independent influences of relational language on number relation skills. We found that EF skills did not significantly predict later number ordering or number line estimation performance after controlling for earlier performance on these tasks, counting and number identification skills, and general verbal knowledge, whereas relational language remained a significant predictor of later number line estimation performance.

Finally, the pathways between relational language and math skills were examined with the concurrent data using mediation analysis. In the first step of the mediation analysis (Figure 2a), relational language significantly predicted math skills above and beyond general verbal knowledge and EF skills (*ß* = .33) and accounted for 4.2% of the variance in math skills. Although small, this effect is comparable to previous findings (Purpura & Reid, 2016) where relational language accounts for 4% of the variance in early numeracy performance above and beyond demographic variables, general verbal knowledge, and Rapid Automatized Naming of pictures and colors—subtests that are sensitive to individual differences in information processing skills (Denckla & Rudel, 1976). In our full mediation model (Figure 2b), relational language was a positive and significant predictor of concurrent number ordering and number line estimation performance, and these relations accounted for the concurrent association between relational language and math skills. Together, these findings suggest that relational language may have unique positive influences on general math skills through number ordering and number line estimation, and these influences are independent of general verbal knowledge and EF skills.

Individual Differences in the Impact of Relational Language on Number Relation Skills

Extending beyond the overall sample, we explored the association between relational language and later number relation skills in smaller subgroups of children with low or high initial number relation skills. Of interest was that children rarely placed values outside the endpoints except on novel trials, as appropriate, and yet correct placement outside the right-most endpoint was evident among only a subset of participants. Children's responses on the novel number line estimation trials thus provided some convergent validity for how these two subgroups varied qualitatively in their understanding of number lines. Children with low number relation skills tended to incorrectly place numbers greater than the endpoints to the left of the endpoints, whereas children with high number relation skills tended to correctly place these large numbers to the right of the endpoints. Although children with low or high number relation skills varied qualitatively in their understanding of number lines, relational language knowledge was not a significant predictor of later number line estimation performance in either subgroup. The results differed from previous findings in preschoolers where relational language predicted later math performance in children with high general numeracy skills (Purpura & Logan, 2015) and in kindergartners where relational language predicted general numeracy performance in children with low numeracy scores (Toll & Van Luit, 2014a).

One interpretation of our findings is that the influence of relational language on number line estimation performance that emerged from the sample as a whole is not driven by either subgroup alone; instead, it may be important for kindergarten children regardless of their initial number relation skills. By separating children into low or high initial number relation skills, however, we restricted the range of scores on relational language and number line estimation tasks, and this may have limited the possibility of finding significant associations. Alternatively, the small subgroups may be underpowered to detect significant effects. The β coefficient for the effect of relational language on number line estimation performance among children with low initial number relation skills was comparable to that of the overall sample, suggesting that the effect observed in the overall sample may potentially be driven by the children with low initial number relation skills, consistent with Toll and Van Luit's (2014a) findings. Finally, differences in outcome variables examined between the current study and prior research may contribute to different results; whereas we focused on number relation skills as outcomes of interest, prior researchers focused on broader numeracy skills (Purpura & Logan, 2015; Toll & Van Luit, 2014a). Future studies should examine the association between relational language and number relation skills among a larger sample of children with different initial number relation skills.

The predictive association that did emerge between counting and number identification skills and later number ordering performance in children with low initial number relation composite extends previous findings that the acuity of numerosity discrimination predicted later math performance in preschoolers with low general numeracy skills (Purpura & Logan, 2015), and that numerical and non-numerical correlates of math are subject to both developmental and individual differences (Geary et al., 2007; Mazzocco et al., 2011; Murphy et al., 2007). The current finding provides evidence for a progression of numerical abilities that may underlie math development in early childhood, one that may account for some of the inconsistencies reported in earlier research (Hassinger-Das et al., 2015; Jennings et al., 1992; Purpura, Napoli, et al., 2017). Through this progression, specific numerical skills or concepts may differ in the extent to which they are necessary or sufficient for further math development. Specifically, counting and number identification skills appear essential for the development of number ordering skills early on. Yet once counting and number identification knowledge is established, it may not further support the subsequent development of number ordering skills.

These findings have important implications for exploring individualized math instruction that is sensitive to children's developmental level. The effects of individualized early childhood reading instruction are well substantiated (e.g., Connor et al., 2009), and more recent extension of that work to math show that second grade instruction tailored to students' math aptitude is more effective at improving math achievement compared to math instruction that is not individualized (Connor et al., 2018). Whether and when such findings emerge may depend on the nature of the instruction and children's developmental level. In a study that involved supplementing first graders' math curriculum with individualized peer-assisted math learning activities, the effectiveness of supplemental activities varied with first graders' math achievement in the fall (Wood et al., 2020). Among first graders scoring in the upper quartile on math, children who received supplemental math activities showed greater gains relative to children who did not receive these activities; by contrast, among first graders scoring in the lower quartile on math, children who did *not* receive the supplemental activities showed greater gains compared to children who did. Children in the lower quartile may not have had the essential knowledge needed to support benefitting from peer-assisted learning activities.

Likewise, children with low number relation skills may rely on basic counting and number identification skills when learning about number ordering. Individualizing instruction based on initial number relation skills may be more effective at supporting numeracy development for all kindergartners compared to one-size-fits-all approaches. Specifically, children with low number relation skills may benefit more from instruction that draws explicit connections between counting, number identification, and number ordering.

Mediation between Relational Language and Mathematical Skills

The current study also adds to past work by identifying potential mechanisms underlying the association between relational language and math skills. We found that number ordering and number line estimation performance fully mediated the concurrent association between relational language and math skills, even after controlling for general verbal knowledge and EF skills. Extending previous longitudinal studies on the association between relational language and math skills (e.g., Toll & Van Luit, 2014a), the current findings suggest that number relation skills, specifically number ordering and number line estimation, are potential pathways through which relational language influence math skills, although this does not preclude additional roles of other numerical or non-numerical skills.

The association between relational language and number relation skills supports our hypothesis that children's knowledge of relational language influences their understanding of number relations, which in turn influences their math skills. But there are several ways in which this association may manifest. One potential way that relational language may influence number relation skills is that relational terms can be applied to different instances of number word uses (e.g., two more cookies, two more minutes) and highlight the quantitative relations between number words and different kinds of objects, just as they have been shown to unify different instances of spatial relations (e.g., above, below) leading to more efficient and abstract spatial concepts (Scott et al., 2015; Scott & Sera, 2018). Although this hypothesis as to how relational language influences number relation skills may seem counterintuitive, it is supported by the work showing an association between children's understanding of relational language and number words. Barner et al. (2009) found that preschoolers who were able to identify and produce sets of objects described by quantitative terms (e.g., give more than one coin when asked to give *some*, and give all available coins when asked to give *all*) were also more likely to identify and produce sets described by exact number words (e.g., *five* coins).

It is also possible that the association between relational language and number relation skills may be bidirectional. In particular, children with higher number relation skills may elicit interactions with their caregivers that include more relational language compared to children with lower number relation skills. We could not, however, infer the directionality of influences studied with mediation analyses based on concurrent associations. The reverse mediations with relational language as a mediator between number relation skills and math skills revealed a significant association between number relation skills and relational language, but the path between relational language and math skills was not significant (see Appendix C). These findings suggest that relational language may not mediate the association between number relation skills and math skills, yet do not exclude the possibility of the bidirectional effect between relational language and number relation skills.

Limitations and Future Directions

This study had several limitations. First, we used pre- and post-test data from a larger training study that spanned only six weeks. However, we did not find training effects on the skills of interest, the time of year when pre-testing occurred did not fully account for children's level of initial number relation skills, and children improved on some number relation tasks even within six weeks. As we demonstrated, number relation skills were developing rapidly among our kindergartners, so our data were appropriate for examining the influences of relational language on number relation skills among children with varying levels of math skills.

A second limitation is that we observed some differences in children's performance on the four types of number line estimation trials (see Appendix B), raising the question of whether the impact of relational language on number line estimation performance is limited to trials of intermediate difficulty for children, such as trials that lack a midpoint or involve a larger and less familiar range of numbers. Such a finding may suggest that the impact of relational language is not on understanding the relations among a well-known set of small numbers but instead affects a more advanced understanding of the relations relevant to emerging concepts that apply to numbers within smaller and familiar ranges (e.g., knowing the “middle” of a set of numbers), or emerging knowledge about the ordering of numbers in larger and less familiar ranges. Although this explanation offers a potential mechanism underlying the association between relational language and number relation skills, our explanation is speculative. The number of trials within each trial type was too small (6 or 7 trials) to afford reliable analyses, and the patterns of results were statistically equivalent regardless of whether the trial types were combined. Because the focus of the current study was on the association between relational language and various number relation skills, and based on the statistical relations observed between these four types of number line trials, we reported only the findings from the four trial types combined in the manuscript. Future studies should include additional types of the number line estimation task, a sufficient number of trials per trial type, and a broader age range of children to further investigate the interaction of number range and midpoints on children’s number line estimation performance, as well as how these variations of the task reveal nuanced associations between children’s relational language and number knowledge.

A third limitation is that we did not include a measure of general cognitive ability or IQ which may be a stable predictor of math learning throughout development (Bailey et al., 2014). However, our measure of general verbal knowledge is a subtest from a standardized abbreviated IQ measure, and it is highly correlated with the full-scale IQ score on the Wechsler Intelligence

Scale for Children – 3rd Edition (Canivez, 1995). Future studies may benefit from accounting for the potential influences of general cognitive ability or IQ, exploring the influences of relational language on the number line estimation tasks that vary on key dimensions (e.g., midpoint, number range), utilizing longitudinal datasets with multiple time points across development to examine mediations and long-term influences of relational language on number relation skills in children with varying levels of math skills, as well as further investigating potential influences of EF skills on the association between relational language and number relation skills.

Conclusions

The current findings on the influences of relational language on number relation skills have implications for future research and practice. The findings suggest a close connection between relational language and later number relation skills and demonstrate potential pathways between relational language and math skills. Together, our findings provide a starting point to examine the longitudinal association between relational language and aspects of early numeracy skills, delineate this association across development and performance levels, and further investigate the correlation between relational language and EF skills in the context of math learning. The current study suggests the unique influences of relational language on number relation skills, by providing evidence for potential ways in which relational language may influence math skills. If the associations between relational language and math skills depend on aspects of math, it is important to consider these aspects of math when designing instruction that meets children's learning needs, and when conducting interventions that aim to promote early math development through relational language-based instruction.

References

Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2014). State and trait effects on individual differences in children's mathematical development. *Psychological Science*, *25*, 2017-2026. https://doi.org/10.1177/0956797614547539

Barner, D., Chow, K., & Yang, S.-J. (2009). Finding one's meaning: A test of the relation between quantifiers and integers in language development. *Cognitive Psychology*, *58*, 195–219. https://doi.org/10.1016/j.cogpsych.2008.07.001

Beck, D. M., Schaefer, C., Pang, K., & Carlson, S. M. (2011). Executive function in preschool children: Test–retest reliability. *Journal of Cognition and Development, 12*, 169–193. https://doi.org/10.1080/15248372.2011.563485

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S. & Zorzi, M. (2010): Numerical estimation in preschoolers. *Developmental Psychology*, 46, 545–551. doi:10.1037/a0017887

Boehm, A. E. (2001). *Boehm Test of Basic Concepts* (3rd ed.). San Antonio, TX: Pearson.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*, 1016–1031. doi: 10.1111/j.1467-8624.2008.01173.x

Brooks, N., Pogue, A., & Barner, D. (2011). Piecing together numerical language: Children's use of default units in early counting and quantification. *Developmental Science*, *14*, 44–57. https://doi.org/10.1111/j.1467-7687.2010.00954.x

Bull, R., Lee, K., & Muñez, D. (2020). Numerical magnitude understanding in kindergartners: A specific and sensitive predictor of later mathematical difficulties? *Journal of Educational Psychology*, *113*, 911–928. https://doi.org/10.1037/edu0000640

Cameron Ponitz, C. E., McClelland, M. M., Jewkes, A. M., Connor, C. M., Farris, C. L., & Morrison, F. J. (2008). Touch your toes! Developing a direct measure of behavioral regulation in early childhood. *Early Childhood Research Quarterly*, *23*, 141–158. https://doi.org/10.1016/j.ecresq.2007.01.004

Canivez, G. L. (1995). Validity of the Kaufman Brief Intelligence Test: Comparisons with the Wechsler Intelligence Scale for Children—Third edition. *Assessment*, *2*, 101-111.

Carlson, S. M., & Zelazo, P. D. (2014). *Minnesota executive function scale: Test Manual*. St Paul, Minnesota: Reflection Sciences.

Chan, J. Y. C. (2019) *The influences of executive function and relational language on number relation skills* [Doctoral dissertation, University of Minnesota-Twin Cities]. University of Minnesota Digital Conservancy. https://hdl.handle.net/11299/219330.

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155.

Connor, C. M. D., Mazzocco, M. M. M., Kurz, T., Crowe, E. C., Tighe, E. L., Wood, T. S., & Morrison, F. J. (2018). Using assessment to individualize early mathematics instruction. *Journal of School Psychology*, *66*, 97–113. https://doi.org/10.1016/j.jsp.2017.04.005

Connor, C. M., Piasta, S. B., Fishman, B., Glasney, S., Schatschneider, C., Crowe, E., Underwood, P., Morrison, F. J. (2009). Individualizing student instruction precisely: Effects of child × instruction interactions on first graders' literacy development. *Child Development*, *80*, 77–100. https://doi.org/10.1111/j.1467-8624.2008.01247.x

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, *3*, 63–68. https://doi.org/10.1016/j.tine.2013.12.001

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, *103*, 469–479. https://doi.org/10.1016/j.jecp.2009.01.010

Denckla, M. B., & Rudel, R. G. (1976). Rapid "automatized" naming (R.A.N.): Dyslexia differentiated from other learning disabilities. *Neuropsychologia*, *14*, 471–479.

Friso- van den Bos, I., Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. M. (2014). Explaining variability: Numerical representations in 4- to 8-year-old children. *Journal of Cognition and Development*, *15*, 325–344. https://doi.org/10.1080/15248372.2012.742900

Gashaj, V., Uehlinger, Y., & Roebers, C. M. (2016). Numerical magnitude skills in 6-years-old children: Exploring specific associations with components of executive function. *Journal of Educational and Developmental Psychology*, *6*, 157–172. doi: 10.5539/jedp.v6n1p157

Geary, D., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, *78*, 1343–1359. doi: 10.1111/j.1467-8624.2007.01069.x

Gelman, R., & Butterworth, B. (2005). Number and language: How are they related? *TRENDS in Cognitive Sciences*, *9*, 6–10. https://doi.org/10.1016/j.tics.2004.11.004

Ghazali, M., Ayub, A., & Othman, A. R. (2013). Preschool children's representation of numbers on a linear number line: Implications to teaching and learning of number concepts. *IOSR Journal Of Humanities And Social Science*, *14*, 87-92.

Ginsburg, H. P., & Baroody, A. J. (2003). Test of Early Mathematics Ability-Third Edition. Austin, TX: Pro-Ed.

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *The Elementary School Journal*, *116*, 242–264. doi:10.1086/683986

Hoard, M. K., Geary, D. C., Byrd-Craven, J., & Nugent, L. (2008). Mathematical cognition in intellectually precocious first graders. *Developmental Neuropsychology*, *33*, 251–276. https://doi.org/10.1080/87565640801982338

Hornburg, C. B., Schmitt, S. A., & Purpura, D. J. (2018). Relations between preschoolers' mathematical language understanding and specific numeracy skills. *Journal of Experimental Child Psychology*, *176*, 84–100. https://doi.org/10.1016/j.jecp.2018.07.005

Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly*, *7*, 263–276. https://doi.org/10.1016/0885-2006(92)90008-M

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*, 850–867. https://doi.org/10.1037/a0014939

Kaufman, A. S., & Kaufman, N. L. (2004). *Kaufman Brief Intelligence Test* (2nd ed.). Bloomington, MN: Pearson.

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*, 1753–1767. doi: 10.1111/j.1467-8624.2010.01508.x

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1-6. *Developmental Science*, 1–13. doi: 10.1111/desc.12152

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development*, *82*, 1224–1237. https://doi.org/10.1111/j.1467-8624.2011.01608.x

Mazzocco, M. M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice*, *20*, 142–155. https://doi.org/10.1111/j.1540-5826.2005.00129.x

Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities*, *40*, 458–478. https://doi.org/10.1177/00222194070400050901

Negen, J., & Sarnecka, B. W. (2012). Number-concept acquisition and general vocabulary development. *Child Development*, *83*, 2019–2027. doi: 10.1111/j.1467-8624.2012.01815.x

Nunes, T., Bryant, P., Evans, D., Bell, D., Gardner, S., Gardner, A., & Carraher, J. (2007). The contribution of logical reasoning to the learning of mathematics in primary school. *British Journal of Developmental Psychology*, *25*, 147–166. doi: 10.1348/026151006X153127

Peeters D, Sekeris E, Verschaffel L, & Luwel K. (2017) Evaluating the effect of labeled benchmarks on children's number line estimation performance and strategy use. *Frontiers in Psychology*, *8*, 1082. doi: 10.3389/fpsyg.2017.01082.

Powell, S. R., & Driver, M. K. (2015). The influence of mathematics vocabulary instruction embedded within addition tutoring for first-grade students with mathematics difficulty. *Learning Disability Quarterly*, *38*, 221–233. https://doi.org/10.1177/0731948714564574

Purpura, D. J., & Logan, J. A. R. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology*, *51*, 1717–1724. https://doi.org/10.1037/dev0000055

Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly*, *36*, 259–268. https://doi.org/10.1016/j.ecresq.2015.12.020

Purpura, D. J., Day, E., Napoli, A. R., & Hart, S. A. (2017). Identifying domain-general and domain-specific predictors of low mathematics performance: A classification and regression tree analysis. *Journal of Numerical Cognition*, *3*, 365–399. doi: 10.5964/jnc.v3i2.53

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness*, *10*, 116–137. https://doi.org/10.1080/19345747.2016.1204639

Purpura, D. J., Schmitt, S. A., & Ganley, C. M. (2017). Foundations of mathematics and literacy: The role of executive functioning components. *Journal of Experimental Child Psychology*, *153*, 15–34. https://doi.org/10.1016/j.jecp.2016.08.010

Purpura, D. J., Schmitt, S. A., Napoli, A. R., Dobbs-Oates, J., King, Y. A., Hornburg, C. B., Westerberg, L., Borriello, G. A., Bryant, L. M., Anaya, L. Y., Kung, M., Litkowski, E., Lin, J., & Rolan, E. (2021). Engaging caregivers and children in picture books: A family-implemented mathematical language intervention. *Journal of Educational Psychology*. https://doi.org/http://dx.doi.org/10.1037/edu0000662

Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, *48*, 1–36.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*, 662–674. doi:10.1016/j.cognition.2008.05.007

Schmitt, S. A., Purpura, D. J., & Elicker, J. G. (2019). Predictive links among vocabulary, mathematical language, and executive functioning in preschoolers. *Journal of Experimental Child Psychology*, *180*, 55–68. https://doi.org/10.1016/j.jecp.2018.12.005

Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, *89*, 1467–1484. https://doi.org/10.1111/cdev.13068

Scott, N. M., & Sera, M. D. (2018). Language unifies relational coding: The roles of label acquisition and accessibility in making flexible relational judgments. *Journal of Memory* *and Language*, *101*, 136–152. https://doi.org/10.1016/j.jml.2018.04.002

Scott, N., Sera, M. D. & Georgopoulos, A. (2015). An information theory analysis of spatial decisions in cognitive development, *Frontiers in Neuroscience*, *9*, 14. doi: 0.3389/fnins.2015.00014.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*, 428–444. https://doi.org/10.1111/j.1467-8624.2004.00684.x

Toll, S. W. M., & Van Luit, J. E. H. (2014a). Explaining numeracy development in weak performing kindergartners. *Journal of Experimental Child Psychology*, *124*, 97–111. https://doi.org/10.1016/j.jecp.2014.02.001

Toll, S. W. M., & Van Luit, J. E. H. (2014b). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children*, *8*, 1–31. https://doi.org/10.1177/0014402914532233

Vukovic, R. K., & Lesaux, N. K. (2013). The relationship between linguistic skills and arithmetic knowledge. *Learning and Individual Differences*, *23*, 87–91. https://doi.org/10.1016/j.lindif.2012.10.007

Wood, T. S., Mazzocco, M. M. M., Calhoon, M. B., Crowe, E. C., & Connor, C. M. (2020). The effect of peer-assisted mathematics learning opportunities in first grade classrooms: What works for whom? *Journal of Research on Educational Effectiveness*, *13*, 601-624. https://doi.org/10.1080/19345747.2020.1772422

Zelazo, P. D. (2006). The Dimensional Change Card Sort (DCCS): A method of assessing executive function in children. *Nature Protocols*, *1*, 297–301. doi: 10.1038/nprot.2006.46

Figure Captions

Figure 1. The predictors of later number relation task performance in (a) full sample, (b) children with low initial number relation composite scores, and (c) children with high initial number relation composite scores.
Note. The number relation task scores at Time 1 are controlled for in all models. Only those number relation tasks on which children demonstrate significant growth from Time 1 to Time 2 are illustrated in the figure.
$^{*}p < .05$

Figure 2. Panel (a) shows the concurrent association between relational language and mathematical skills at Time 1. Panel (b) shows the concurrent mediation between relational language and mathematical skills through number ordering and number line estimation task scores.
Note. Both models include general verbal knowledge and EF skills as covariates of mathematical skills. The direct path from relational language to math skills is labeled as c in panel (a) and c' in panel (b). In panel (b), the indirect paths through number ordering are labeled as a and b; ab represents the indirect effect of number ordering. The indirect paths through number line estimation are labeled as d and e; de represents the indirect effect of number line estimation. The values represent non-standardized estimates. $^{*}p < .05$, $^{***}p < .001$