香港教育大學
The Education University of Hong Kong

MTH 4902 (02E) Honours Project II

[Capstone Project]
Learning Package for Triangle Centres
-5E Instructional Model

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CONTENT

Background

According to Education Bureau (2020) Explanatory Notes to Junior Secondary Mathematics Curriculum, Education Bureau (2017c) Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Senior Secondary Mathematics and Education Bureau (2017a) Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Junior Secondary Mathematics, I noticed that triangle centers is a very important and complex in DSE curriculum. As it penetrates the whole curriculum from the basic introduction and some basic understanding of triangle in junior form to cross various topics, for example circle, coordinate in senior form. In addition, I observe that in DSE exam, this topic had never been sperate out and always examinate in a cross-topic situation.

Unfortunately, as stated by the Hong Kong Examination and Assessment Authority (HKEAA)(2022) in the students' performance presentation from 2017 to 2021, the four triangle centers questions is close to or worse than wild guess. It shows the struggle from students. Thus, above is the reason for me to construct this capstone topic because I would like to try out a way to learn triangle centers with less struggle.

Project Aims

1. Create an interactive teaching package.
2. Assist students to visualize the concepts about the definition of triangle centres and its properties.
3. Able to help students to connect new knowledge, triangle centres, to prior knowledge.
4. Obtain the ability on eliminating impossible answers in multiple choice questions.

Literature Review -- EduMath 29

As stated by Lee (2010) in EduMath 29 by Hong Kong Association for Mathematics Education on the topic of triangle centers, teachers do not have a unanimous consensus or a clear concept on how to teach this topic. He pointed out that reasoning and making sense from one object to another plays a major part in learning geometry, specify in deductive reasoning. Therefore, referring to Lee (2010) he believes learning triangle centers would focus more on how teachers or learning tools assist students to observe, imagine locate, know, prove, link and apply concepts to concepts.

Literature Review -- Education Bureau Documents

According to Education Bureau (2017b) Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Junior Secondary Mathematics and Education Bureau (2017c) Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Junior Secondary Mathematics. Education Bureau tends to sperate the learning on triangle centers into two different parts. First, the identification on the four triangles, which is suggested to be taught in junior form. Second, cross-topic application.

Literature Review – Florida Atlantic University

According to Yiu, (2001) the logical development of triangle center is start from circle. However, I noticed that in Education Bureau (2020) Explanatory Notes to Junior Secondary Mathematics Curriculum, it clearly stated that in junior form, students are only required to recognize four centers only and not encourage teachers to link up the concept with circle.

In addition, despite in junior form curriculum guide it stated that the concept would be used in senior form but in the senior form curriculum guide the term had never been mentioned again. Therefore, it is a performance of lack of continues on the learning flow.

Literature Review -- Summary

From Johnston-Wilder and Mason (2005) in learning geometry, develop the geometric thinking and able to visualize the concept could make the learning process be more effective. Therefore, during the construction of my capstone project, instead of following the guidelines from the Education Bureau, I would try to develop the stages referring to Yiu's (2001) concept and include the development starting from a circle. Therefore, matches the suggestion Dr. Lee (2010) and Johnston-Wilder and Mason (2005) idea on developing knowledge of geometry, make sense and step by step reasoning.

Theories/ Definition --The BSCS 5E Instructional model

According to Bybee et.al (2006) BSCS 5E instructional includes engage, explore, explain, elaborate and evaluate these five stages.

First, according to Bybee et.al, (2006) it is a well-developed, well-adopted and widely applied instructional model. In addition, Bybee's (2009) personal reflection and contemporary implications research, he agrees that it is still useful on teaching topics

which required reasoning and discovery. Last but not least, as stated in Bybee et.al, (2006) BSCS 5E instructional model is strongly endorse in at least three states in the America. As it is being endorsed by states government, which is a very strong and persuasive point. These three are the main reason I choose 5E-instructional model in my teaching design over other models.

Theories/ Definition -- Mathematical Problem Solving by Alan H. Schoenfeld

The mathematical problem-solving cycle raised by Schoenfeld (1985) is after a problem is received, first is to analysis. Then planning and exploration, in some situation could go back to analysis after exploration. After that, Implementation and last verify the solution before a verified solution is given out. This thinking process matches Lee's (2010) flow of teaching.

On the other hand, the evaluation stage in BSCS 5E instructional model focuses on attempting. The importance of try, explore, experience stands out in the instructional model. However, besides learning the content itself, as we lived in the DSE curriculum, so we need to enhance and strengthen the evaluation stage. According to Schoenfeld (1992) as it concentrates on problem solving aspect and focus on diagnosis the problem while constructing the solution and encourage to review and re-evaluate the solution. This could add one more layer on student's learning process, which guarantee students is fully understand the problem before moving on.

I think the combination of these two-design model could cover each other and let my design of package become complete and efficient.

Package Design: Teaching Schedule

Here would be the teaching schedule, there would be 9 lessons in total, each last for 35 mins in order to included DSE content, this learning package is created for form 5 students.

Learning Objectives:

1. Recognize the definition and properties of four triangle centres, centroid, orthocentre, circumcentre and incentre.
2. Ability to apply the properties in questions.
3. Ability to calculate the coordinate of 4 centres.
4. Eliminate impossible choices and increase the probability of choosing correct answer.

Lesson	Content	Explain and Description	Learning Objective (s)
1	Centroid	Recognize the properties and definition for centroid. Calculate the coordinate of centroid	1, 2, 3
2	Orthocentre	Recognize the properties and definition for orthocentre. Calculate the coordinate of orthocentre	1, 2, 3
3	Circumcentre	Recognize the properties and definition for circumcentre. Calculate the coordinate of circumcentre	1, 2, 3
4	Incentre	Recognize the properties and definition for incentre. Calculate the coordinate of incentre	1, 2, 3
5	Review and Collinearity	Revision. Discover the collinearity of four centres.	1, 4
6	Application: Plane Geometry	Tackle practical problems about four centres in plane geometry.	2
7	Application: Coordinate Geometry	Find the coordinate of four centres with given vertices of triangle	3
8	DSE Past Paper Question	Eliminate the impossible answers by constructing the diagram and utilize the properties learnt.	2, 3, 4
9	Chapter Quiz	Assessment and reflection	2, 3, 4

Package Design: Class Arrangement

According to Bybee (2014) he suggested that for BSCS 5E Instructional model should not impose lesson by lesson or topic by topic. Apply in sub-topic by sub-topic or small cycle would be most effective. The model effectiveness on decrease when impose in both indivisible content and thin content. Therefore, I planned in each lesson there will be 5 stages and support by the learning package created.

Stage	Activity	5E-instructional model	Description	Resources
1	Pre-lesson	Engage	Students are required to observe on their own first and imagine the location of centre. This would give students a brief concept and impression on the definition of "centre"	-Website -Notes -GeoGebra
2	In-class Exploration	Explore	Explore the definition of centres and its properties with teacher's guidance. Through interactive GeoGebra tool, this would help students to locate the centre and have a deeper understanding on its properties.	-Website -Notes -GeoGebra
3	Definition	Explain, Elaborate	Teacher is going to clarify and confirm the location and properties to students. To ensure no misunderstanding or wrong impression on those four centres appears Break down "triangle centres" and pieces of new knowledge to prior knowledge.	-Notes -Diagram

4	In-class Practice	Elaborate, Evaluate	Application on the knowledge learnt in stage 3. Enhance their understanding on the properties.	-Notes -GeoGebra
5	Homework	Evaluate	Textbook exercise to consolidate the knowledge. To clarify the misunderstanding in class or missing part. Question generator for further self-study and consolidation.	-Textbook -GeoGebra

Package Design: Website and Notes

Basically, notes and website function for the same purpose and follow the same design philosophy. Use of various color on website creates an eye-catching e-resource and let students could free from paper and attach their learning anytime and anywhere with their devices only.

In addition, As stated by Lee (2010) on teaching triangle centers is to transfer basic facts to the outcome. By the use of GeoGebra, I try to include engagement and exploration before providing definition to students. After explanation, I would throw some conceptual questions for students to elaborate.

On the other hand, in the DSE past paper question part, I implied Schoenfeld's problem solving cycle in order to increase the depth of the evaluation part. For example, after students had finished the question, before they check the solution, they could go for a GeoGebra tool first or look at the simple reasoning first. They GeoGebra tool would visualize the question or make the question become adjustable. This step is to encourage them to review and re-explore their answer. On the other hand, this could act as a buffer or cushion for students who are less able. Avoiding them once failed to answer anything of the question, besides give up or check the answer they have no other choice. This would provide them courage to try out the questions and learn.

Package Design: GeoGebra Tools

On the other hand, it is important to try and learn from error. For 5E instructional

model, it emphasizes the learning process and exploration of the students but not urge to see to outcome or the solution. I believe adopting this model could create a sandpit for students to explore the concept on their own but with less frustration. Especially at the time dealing with complex cross-topic questions which exam focuses on. Therefore, with the aid from GeoGebra, students can make use of it to assist their learning in different stages.

There are four types of tools I created, observational, explorational, question-based and question generator.

Type of tool:	Aim and explanation
Observational	Aim: Support observation, imagination and location The idea on designing observational tool would be focuses on leading students to observe, imagine and they can try to locate the centre and see if the outcome matches their imagination. In addition, it also supports showing the properties of the triangle centre students imagine and the definition one, students could observe why there would be deviation and make adjustment on their own.
Explorational	Aim: Enhance their impression After knowing the properties, students could deepen their impression and understanding through these tools. The design of these tools is to visualize the properties proof in an interactive way. Therefore, by using them, students could link up their understanding with an experience. Thus, they could have a deeper impression and stronger connection between centre and its properties.
Question-based	Aim: Visualize the answer For cross topic questions, as most of the time there would be more than one path to the outcome. Therefore, the explore stage stood out as an advantage. As “model answer” or “correct way of doing the question” is

	not the necessary and the first priority. Therefore, it would focus on how to utilize the properties of triangle centre to help filter out the impossible answer visually.
Question Generator	Aim: Practice opportunity Question generator just simply give an opportunity for students to have a further practice. Aim at speed their calculating process up and get more familiar with the method of calculating the coordinate of centre or distinguishing the location of centres under different situations.

Package Design: Summary

The idea of design is to help students develop their own understanding of triangle centre. Use of color to highlight and distinguish different terms. Use of various media and method, such as Venn diagram, GeoGebra to stimulate encourage students to develop their own sense of theory. Aim at helping students not just to remember a line of words but remember the process of exploration. Thus, they can connect triangle centre with their prior knowledge.

Package Design: Difficulties and Solution

The major difficulty I faced is to link up triangle centre with its definition up. According to my observations, the change in vocabulary brings a great impact on affecting students on recalling the definition of the triangle centre in this topic, such as student clearly knows what angle bisector is but once the question is given incentre, they will no longer be able to apply angle bisector as a property of incentre in their answer.

The solution I tried is to link two concepts up by image and sequence of arrangement in the package. For example, I created an angle bisector logo and I put it on the right top corner of the notes as an indicator on incentre. By image link up concepts, students think of incentre, the angle bisector logo will pop up in their mind immediately. On the other hand, in the arrangement of wordings I tend to mention angle bisector first, then follow up with “incentre”. This helps students to connect angle bisector and incentre.

Limitation & Suggestion

Resources or technology available to teachers or students, in my progress of this capstone project, I found that learning a new software and creating courseware takes

time. It is difficult to keep the e-resources up to date and tailor made for each question. That would limit the effectiveness of utilizing those e-resources. Suggested that in some certain extend, students could create their own courseware and share among them. Thus, rather than some specific question-based explaining tools or graphs, teachers could focus on building up tools which requires a higher level of understanding on the whole topic.

Enlightenment & Extension

As I mentioned, triangle centre is a tricky topic. It is difficult as it links up various of concepts. In addition, questions about this topic in DSE did not provide the graph for students. It is very discouraging and frustrated topic for students as according to Education Bureau this topic only taught in F3. At that time, they have not obtained adequate concepts and skills to learn the topic fully in a methodical way.

We are stepping in an era of technology, compared to the past only using blackboard and chalk, development of technology gives both teachers and students additional support, which makes the process of learning become easier. In order to utilize various sense of stimulation, e-learning resources become more and more important, in order to make them become easy for students to access at any time and anywhere, I believe constructing website would be a great carrier. As all students have to obtain will be a device and a link, it approaches students a lot easier.

As a mathematics teacher, we should encourage students to explore the history and development of different concepts. Introduce them the work of mathematicians like Euler and Feuerbach. Help students appreciate the historical context of the concept and deepen their appreciation for the beauty of mathematics.

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Appendix A: E-Resources Created

Website for Triangle Centre

https://s1129218.wixsite.com/triangle-centres

GeoGebra Tools:

Exploration Tools:

Centroid: https://www.geogebra.org/m/ve6paayw

Orthocentre: https://www.geogebra.org/m/grwwptr5

Circumcentre: https://www.geogebra.org/m/qyefzpny

Incentre: https://www.geogebra.org/m/p8rjmsdy

Observation Tools:

Centroid: https://www.geogebra.org/m/e3a2crb4

Orthocentre: https://www.geogebra.org/m/c69vtfq7

Circumcentre: https://www.geogebra.org/m/mufn6zm5

Incentre: https://www.geogebra.org/m/d7eab46b

Question Generators:

Centroid: https://www.geogebra.org/m/d8ak25dz

Orthocentre: https://www.geogebra.org/m/sfzfecur

Circumcentre: https://www.geogebra.org/m/rwybd3ws

Incentre: https://www.geogebra.org/m/fcuaexur

Plane Geometrical: https://www.geogebra.org/m/txvedcrk

Coordinate Geometrical: https://www.geogebra.org/m/zxahf5qv

Collinearity:

https://www.geogebra.org/m/vufzjzvh

Visualized DSE Past Paper Questions:

2013: https://www.geogebra.org/m/a4xv88aa

2015: https://www.geogebra.org/m/zf9gsyyf

2017: https://www.geogebra.org/m/phnahj6j

2018: https://www.geogebra.org/m/j2cx3hpa

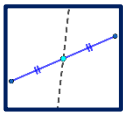
2019: https://www.geogebra.org/m/pqxrs9fh

2020: https://www.geogebra.org/m/q7gw22wu

Appendix B: Teaching Notes

Triangle Centre

Notes for Lesson 1: Centroid (Median)



Name:_____( ) Class:_____ Date:_____

Refered Webpage:

https://s1129218.wixsite.com/triangle-centres/centroid

Pre-Class Exploration:

https://www.geogebra.org/m/ve6paayw

Your Aim: Find where **three medians** (segment bisectors) meet.

Instructions:

1. Guess where three medians intersect.
2. Move "C" to where you think it is.
3. With "Help" try to adjust the location.
4. Click "Centroid" to check if "C" matches C_M.
5. Click "Details" to get more information.

- Drag point X, Y and Z or click randomize to form different triangles.
- Observe the change in the location of triangle centre.
- "Reset" brings you back to an equilateral triangle.

Draw in diagram/ Write in words to describe your exploration:

What is Median?

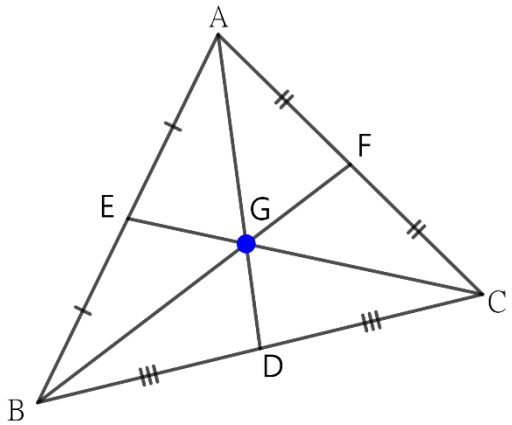
Definition: a line segment that joins a **vertex** to the **mid-point** of the side that is opposite to that vertex

What is Centroid?

Definition: The point of intersection, where all three **medians meet**.

Centroid is the Point in Blue

Properties of Centroid:



- The centroid is always inside the triangle.
- Each median divides the triangle into **two smaller triangles of** equal area**.**
- Centroid divides each median into two parts.

  The ratio of shorter side to longer side is always 1 : 2
  - **GD : GA =** 1 : 2
  - **GF : GB =** 1 : 2
  - **GE : GC =** 1 : 2

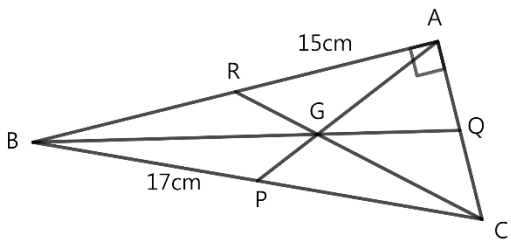
Observe the Properties and See the Reason Behind

https://www.geogebra.org/m/e3a2crb4

Application Questions:

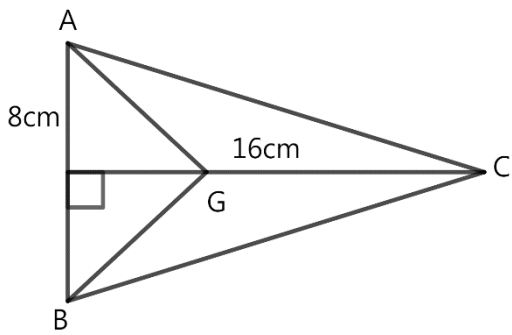
Level 1:

In the figure, G is the centroid of ΔABC. AG, BG and CG are produced to meet BC, AC and AB at P, Q and R respectively. If $\angle BAC = 90°, AR = 15\text{cm}$ and $BP = 17\text{cm}$, find AQ.



Level 2:

In the figure, G is the centroid of ΔABC. CG is produced to meet AB at D. It is given that $AB \perp CD$. If $AD = 8\text{cm}$ and $CG = 16\text{cm}$, find the area of ΔABC.

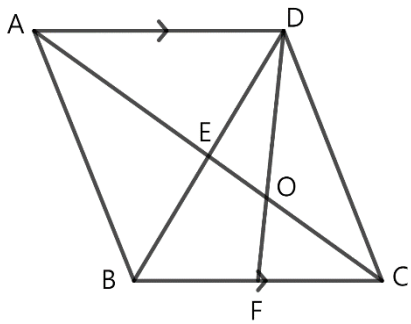


Level 3:

In the figure, O is the centroid of ΔABC. $AEOC, BED, BFC$ and DOF are straight lines. It is given that $AD // BC$.

a.) Prove that $\Delta ADE \cong \Delta CBE$.

b.) Is $ABCD$ a parallelogram? Explain your answer.

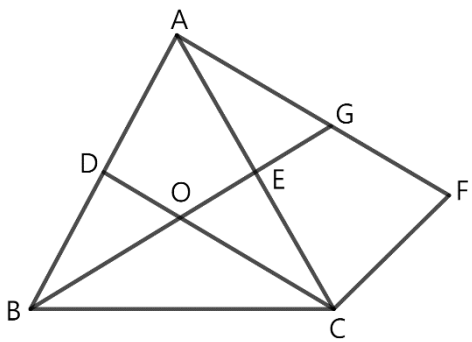


Level 4:

In the figure, O is the centroid of ΔABC. BO is produced to intersect AC at E and meet AF at G. CO is produced to meet AB at D. It is given that $ADCF$ is a parallelogram.

a.) Prove that $\Delta AGE \cong \Delta COE$.

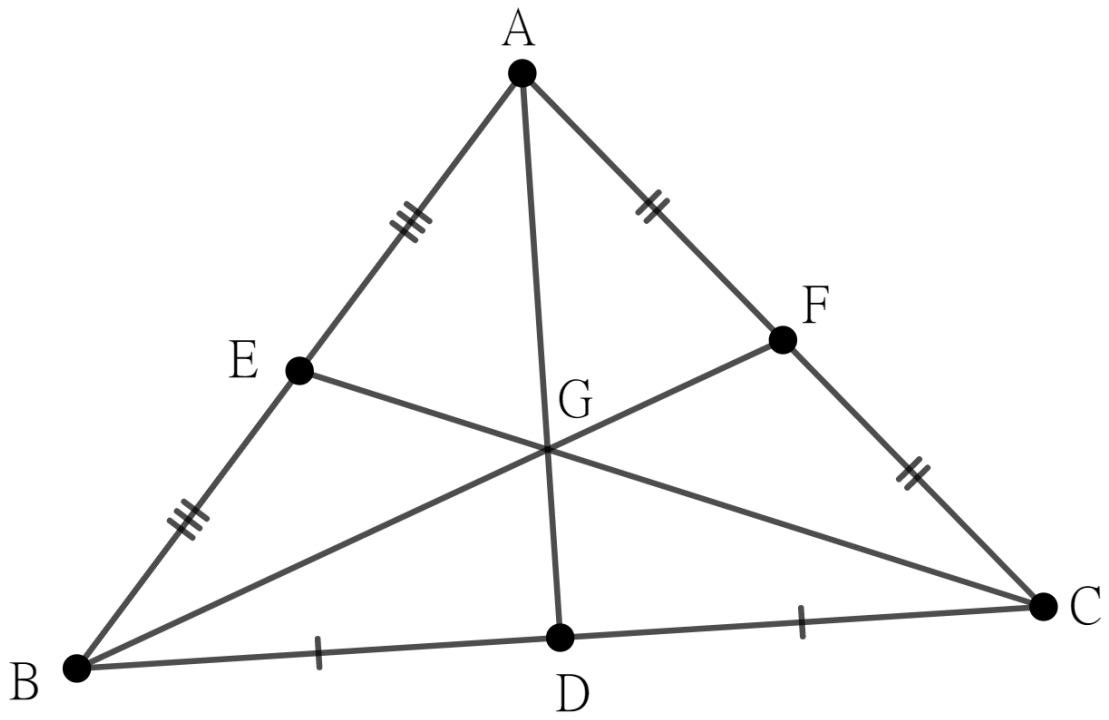
b.) Prove that $OD = GF$.



Centroid in Coordinate Geometry:
How to find the coordinate of centroid

By Centroid Formula

$$G = \left(\frac{x_A + x_B + x_C}{3}, \frac{y_A + y_B + y_C}{3}\right)$$

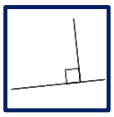


Application: Question Generator
🖫 https://www.geogebra.org/m/d8ak25dz

You can see the Proof of the formula in
🖫 https://s1129218.wixsite.com/triangle-centres/centroid

Triangle Centre

Notes for Lesson 2: Orthocentre (Altitudes)



Name:_____( ) Class:_____ Date:_____

Refered Webpage:

https://s1129218.wixsite.com/triangle-centres/orthocentre

Pre-Class Exploration:

https://www.geogebra.org/m/grwwptr5

Your Aim: Find where **three altitudes** (height/perpendicular line) meet.

Instructions:

6. Guess where three altitudes intersect.
7. Move "C_t" to where you think it is.
8. With "Help" try to adjust the location.
9. Click "Orthocentre" to check if "C_t" matches C_O.
10. Click "Altitudes" to show altitudes information.

- Drag point X, Y and Z or click randomize to form different triangles.
- Observe the change in the location of triangle centre.
- "Reset" brings you back to an equilateral triangle.

Draw in diagram/ Write in words to describe your exploration:

What is Altitudes?

Definition: -the **perpendicular** drawn from the vertex of the triangle to the opposite side.

-As known as the **height**.

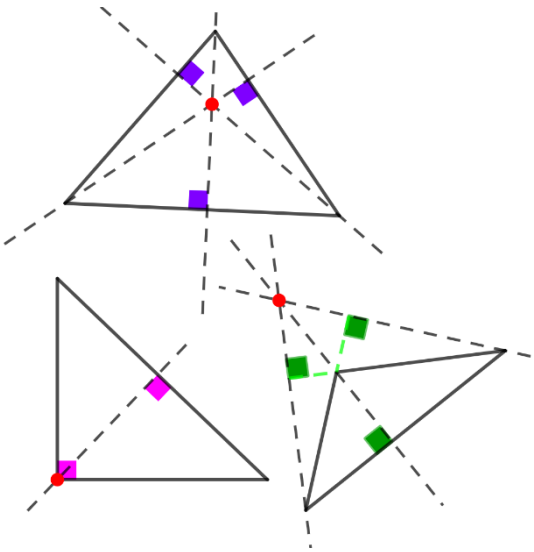
What is Orthocentre?

Definition: The point of intersection, where all three **altitudes meet**.

Orthocentre is the Point in Red

Properties of Orthocentre:

- The Orthocentre is always **inside** the **acute triangle**.
- The Orthocentre is always **outside** the **obtuse triangle**.
- The Orthocentre is always on the **right angle** in **right triangle**.



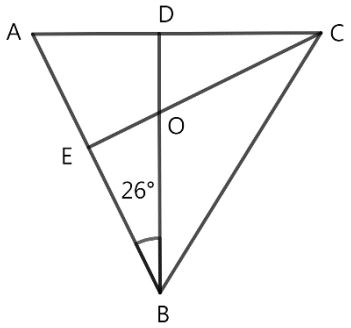
Observe the Properties and See the Reason Behind

🖫 https://www.geogebra.org/m/c69vtfq7

Application Questions:

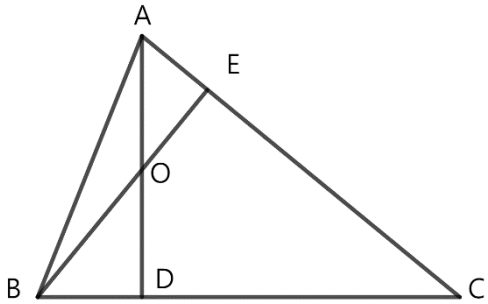
Level 1:

In the figure, O is the orthocentre of ΔABC. BO and CO are produced to meet AC and AB at D and E respectively. If $\angle OBE = 26°$, find $\angle ACO$.



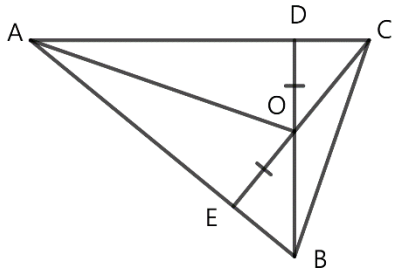
Level 2:

In the figure, O is the orthocentre of ΔABC. AO and BO are produced to meet BC and AC at D and E respectively. Prove that $\Delta AOE \sim \Delta ACD$.



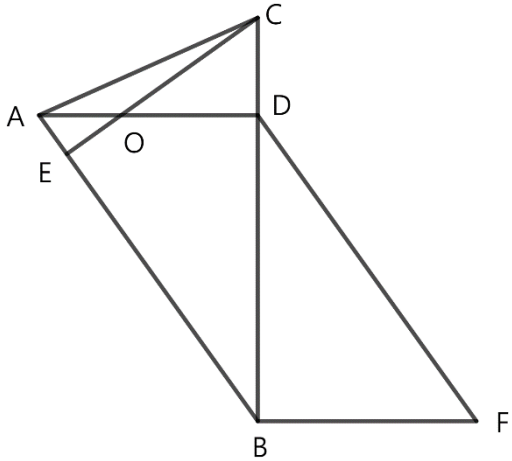
Level 3:

In the figure, O is the orthocentre of ΔABC. BO and CO are produced to meet AC and AB at D and E respectively. It is given that $OD = OE$.
Show that $\angle OAD = \angle OAE$.



Level 4:

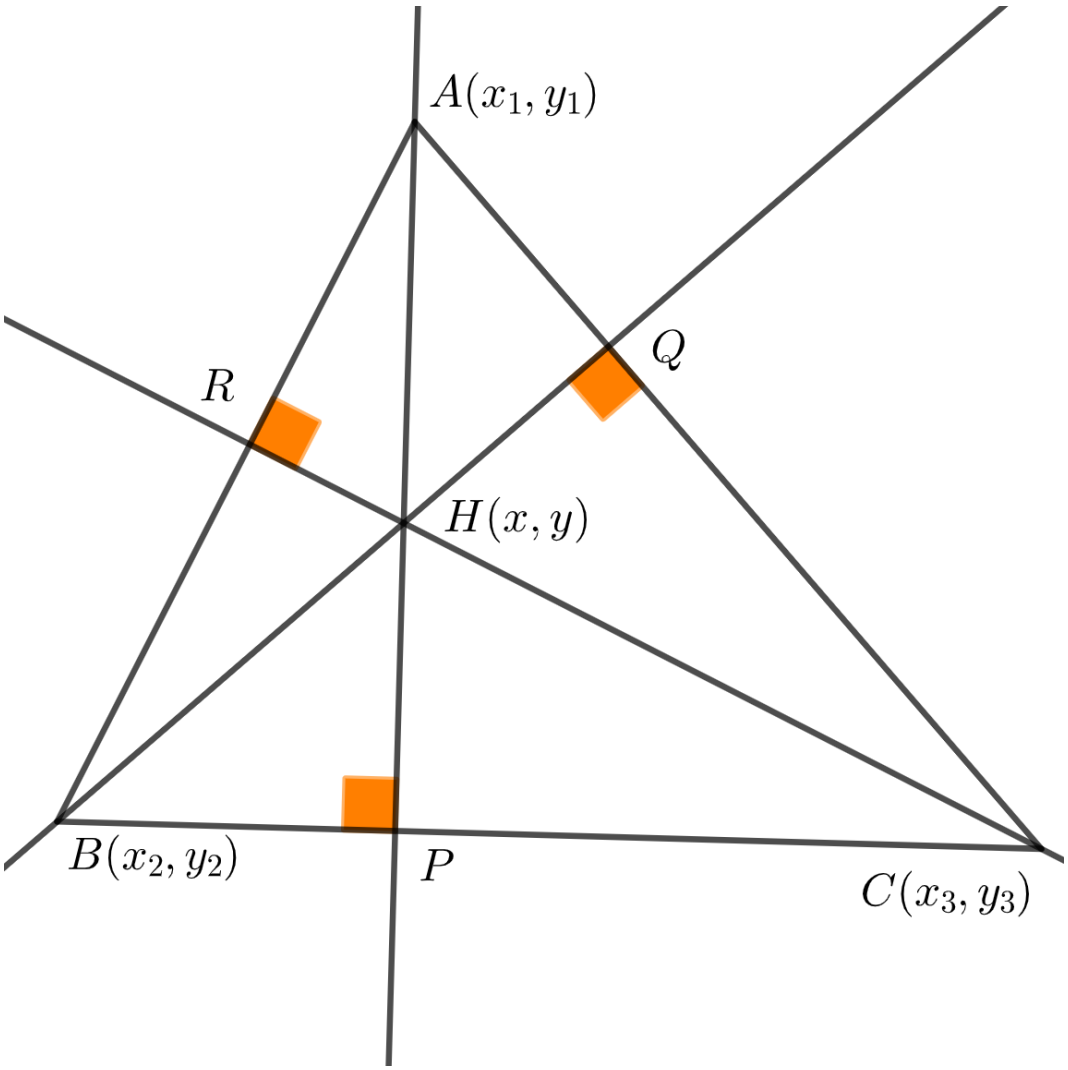
In the figure, O is the orthocentre of ΔABC. AO and CO are produced to meet BC and AB at D and E respectively. It is given that $ABFD$ is a parallelogram.
Prove that $\Delta OCD \sim \Delta DFB$.



Orthocentre in Coordinate Geometry:
How to find the coordinate of Orthocentre

By Setting up Simultaneous Equations

$$\text{Solve for H: } \begin{cases} m_{AH} * m_{BC} = -1 \\ m_{BH} * m_{AC} = -1 \\ m_{CH} * m_{AB} = -1 \end{cases}$$

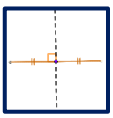


Application: Question Generator
🖫 https://www.geogebra.org/m/sfzfecur

You can see the Proof of the formula in
🖫 https://s1129218.wixsite.com/triangle-centres/orthocentre

Triangle Centre

Notes for Lesson 1: Circumcentre (Perpendicular Bisector)



Name:_____( ) Class:_____ Date:_____

Refered Webpage:

🖫 https://s1129218.wixsite.com/triangle-centres/circumcentre

Pre-Class Exploration:

🖫 https://www.geogebra.org/m/qyefzpny

Your Aim: Find where three **perpendicular bisectors** meet.

Instructions:

11. Guess where three perpendicular bisectors intersect.
12. Move "C_t" to where you think it is.
13. With "Help" try to adjust the location.
14. Click "Circumcentre" to check if "C_t" matches C_C.
15. Click "Details" to get more information.

- Drag point X, Y and Z or click randomize to form different triangles.
- Observe the change in the location of triangle centre.
- "Reset" brings you back to an equilateral triangle.

Draw in diagram/ Write in words to describe your exploration:

What is Perpendicular Bisector?

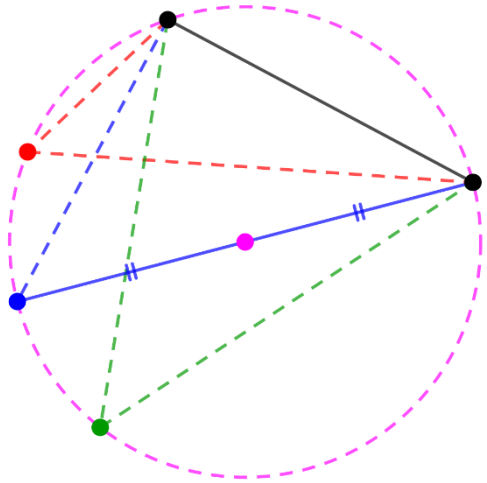
Definition: -A line or a line segment that divides a given line segment into two parts of **equal measurement**.
-Also, making an angle of **90°** at the point of intersection (90° + Line Bisector)

What is Circumcentre?

Definition: The point of intersection, where all three **perpendicular bisectors meet**.

Circumcentre is the Point in Pink

Properties of Circumcentre:



- The Circumcentre is always inside the acute triangle.
- The Circumcentre is always outside the obtuse triangle.
- The Circumcentre is always on the mid-point of the hypotenuse of right triangle.
- The circumcenter is the centre of the circumcircle.
  - A three-points circle form by three vertices of the triangle.
- All the vertices of a triangle are equidistant from the circumcentre.
  - Radii

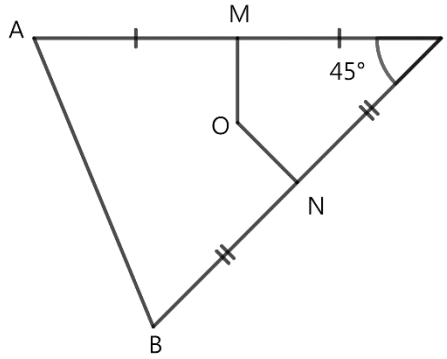
Observe the Properties and See the Reason Behind

🖫 https://www.geogebra.org/m/mufn6zm5

Application Questions:

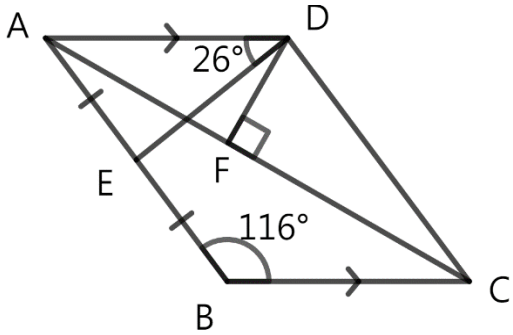
Level 1:

In the figure, M and N are the mid-points of AC and BC respectively. O is the circumcentre of ΔABC. If $\angle ACB = 45°$, find $\angle MON$.



Level 2:

In the figure, E is the mid-point of AB. AC is the angle bisector of $\angle BCD$. F is a point on AC such that $DF \perp AC$. It is given that $AD//BC$, $\angle ABC = 116°$ and $\angle ADE = 26°$. Prove that D is the circumcentre of ΔABC

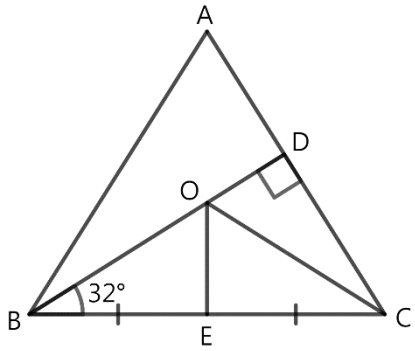


Level 3:

In the figure, O is the circumcentre of ΔABC and E is the mid-point of BC. BOD is a straight line. It is given that $BD \perp AC$, $\angle OBC = 32°$ and the radius of the circumscribed circle of ΔABC is 9 cm.

a.) Find $\angle OCD$.

b.) Find the area of ΔOCD, correct to 3 significant figures.

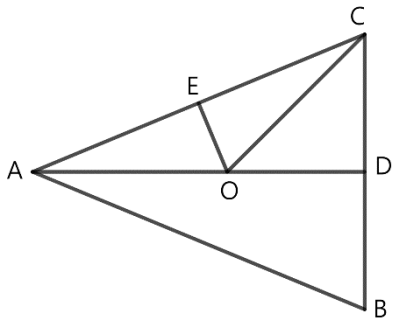


Level 4:

In the figure, ΔABC is an isosceles triangle with $AB = AC$. AO produced intersects BC at D and $BD = CD$. E is a point on AC such that OE is the perpendicular bisector of AC.

a.) Prove that O is the circumcentre of ΔABC.

b.) If $AO : OD = 17 : 12$, find $\angle OCD$, correct to 3 significant figures.

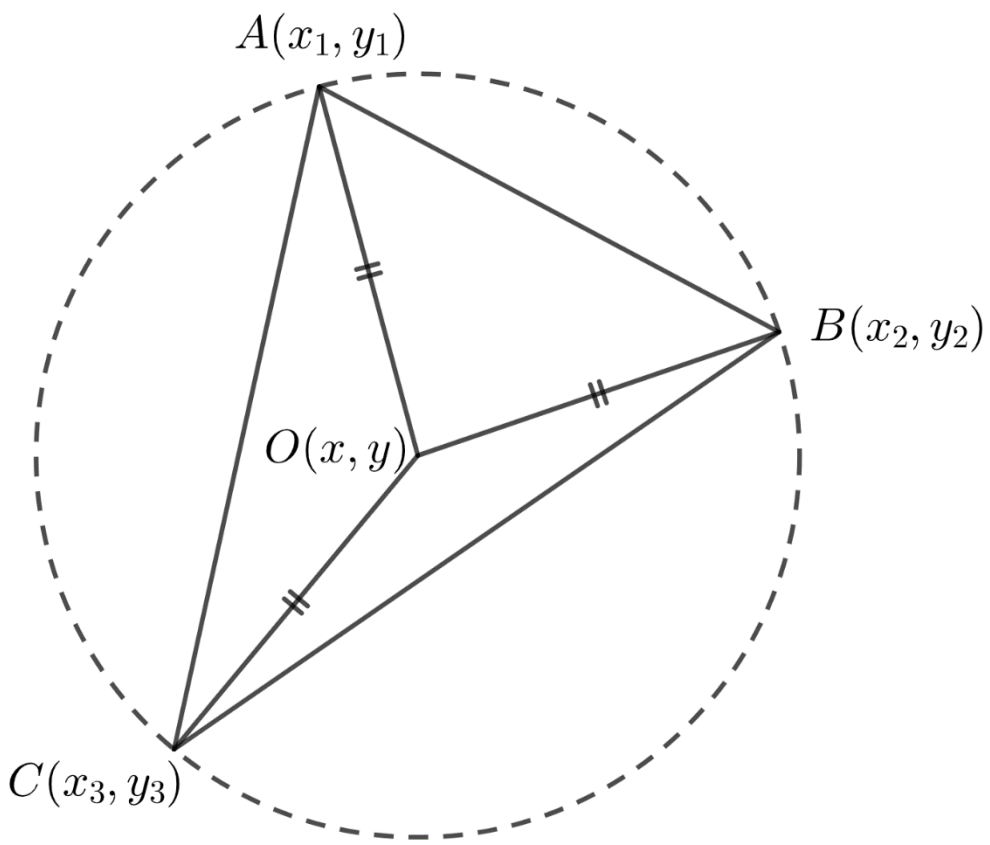


Circumcentre in Coordinate Geometry:
How to find the coordinate of Circumcentre

By Distance Formula
$\because$ Circumcentre is the centre of Circumscribed Circle
$\therefore AO = BO = CO$

Consider simultaneous equations of distance formula:

$$\text{Solve for } O\text{: } \begin{cases} AO = BO \\ BO = CO \\ CO = AO \end{cases}$$

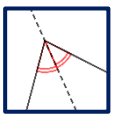


Application: Question Generator
🖫 https://www.geogebra.org/m/rwybd3ws

You can see the details of the method in
🖫 https://s1129218.wixsite.com/triangle-centres/circumcentre

Triangle Centre

Notes for Lesson 4: Incentre (Angle Bisector)



Name:_____( ) Class:_____ Date:_____

Refered Webpage:

🖫 https://s1129218.wixsite.com/triangle-centres/incentre

Pre-Class Exploration:

🖫 https://www.geogebra.org/m/p8rjmsdy

Your Aim: Find where three **angle bisectors** meet.

Instructions:

16. Guess where three angle bisectors intersect.
17. Move "C" to where you think it is.
18. With "Help" try to adjust the location.
19. Click "Circumcentre" to check if "C" matches I_C.
20. Click "Details" to get more information.

- Drag point X, Y and Z or click randomize to form different triangles.
- Observe the change in the location of triangle centre.
- "Reset" brings you back to an equilateral triangle.

Draw in diagram/ Write in words to describe your exploration:

What is Angle Bisector?

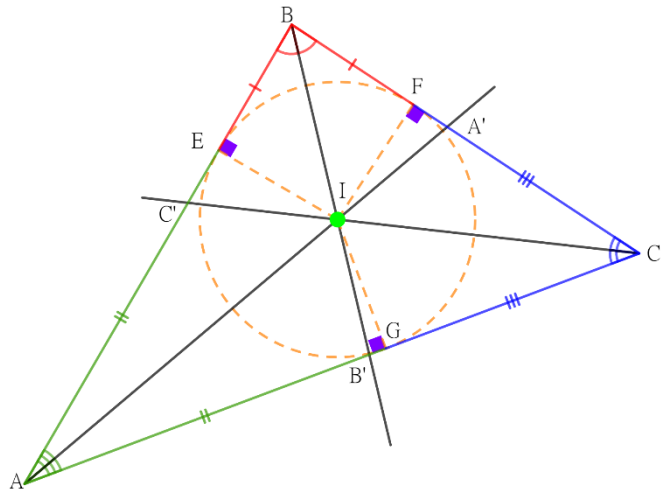
Definition: A line that splits an angle into **two equal angles.**

What is Incentre?

Definition: The point of intersection, where all three **angle bisectors meet**.

Circumcentre is the Point in Green

Properties of Incentre:



- Incentre always lies inside the triangle.
- The sides of the triangle are tangents to the inscribed circle
- I is the incentre (angle bisector)
  - $\angle BAI = \angle CAI$
  - $\angle BCI = \angle ACI$
  - $\angle ABI = \angle CBI$

I is the incentre Equal distance sides

- $AE = AG$
- $CG = CF$
- $BF = BE$

- Properties of angle bisector:
  For any angle bisector cutting a triangle into two:
  - $AB : BC = AB' : B'C$
  - $AB : AC = A'C : A'C$
  - $BC : AC = BC' : AC'$

Observe the Properties and See the Reason Behind

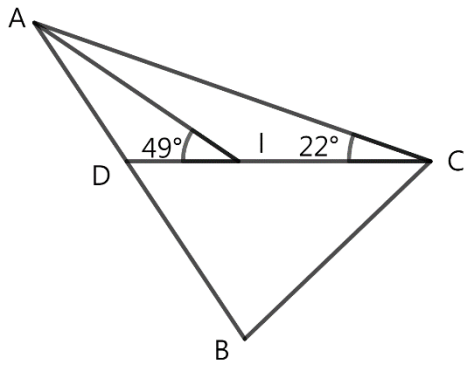
https://www.geogebra.org/m/d7eab46b

Application Questions:

Level 1:

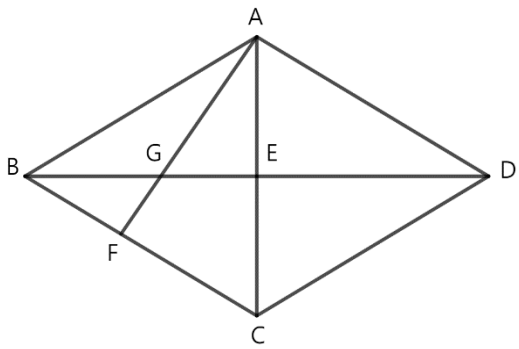
In the figure, I is the incentre of ΔABC. CI Produced intersects AB at D.

If $\angle ACD = 22°$ and $\angle AID = 49°$, find $\angle ADC$.



Level 2:

In the figure, $ABCD$ is a rhombus. F is a point on BC. AC and AF intersect BD at E and G respectively. AF is the perpendicular bisector of BC. Prove that G is the incentre of ΔABC.

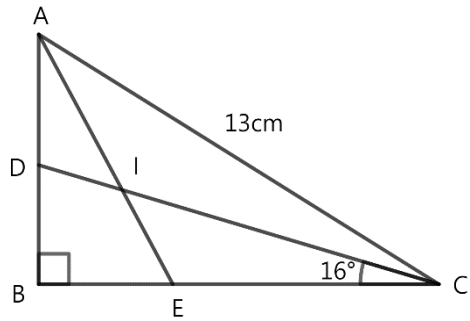


Level 3:

In the figure, I is the incentre of ΔABC. AI and CI are produced to meet BC and AB at E and D respectively. It is given that $\angle ABC = 90°$, $\angle BCD = 16°$ and $AC = 13$cm.

a.) Find AB

b.) Find BE

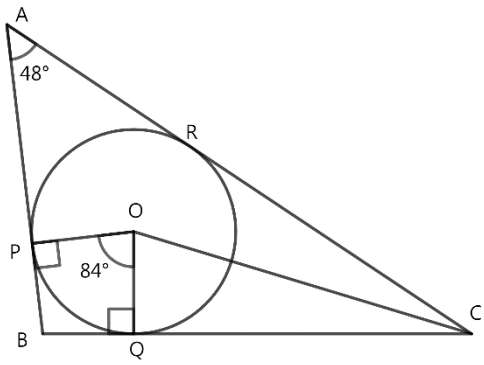


Level 4:

The figure shows a circle centred at O which touches the three sides of ΔABC at P, Q and R. It is given that $\angle BAC = 48°$, $\angle POQ = 84°$, $OP \perp AB$ and $OQ \perp BC$.

a.) Find $\angle OCQ$.

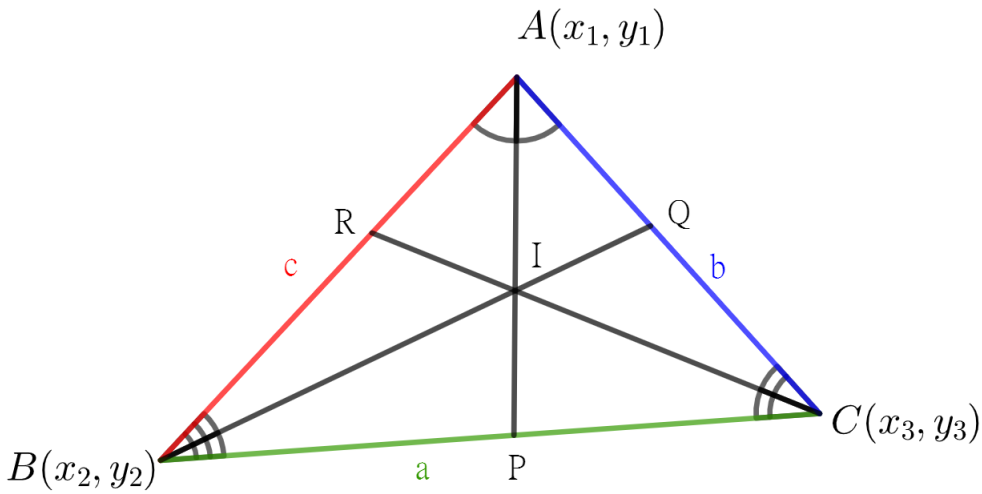
b.) It is given that $OC = 10$cm. Find the area of the inscribed circle of ΔABC, correct to 3 significant figures



Incentre in Coordinate Geometry:
How to find the coordinate of incentre

By Incentre Formula

$$I = \left(\frac{a * x_A + b * x_B + c * x_C}{a + b + c}, \frac{a * y_A + b * y_B + c * y_C}{a + b + c}\right)$$

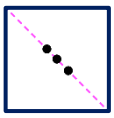


Application: Question Generator
- https://www.geogebra.org/m/fcuaexur

You can see the details of the proof in
- https://s1129218.wixsite.com/triangle-centres/incentre

Triangle Centre

Notes for Lesson 5: Review and Collinearity

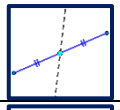
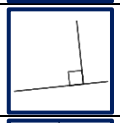
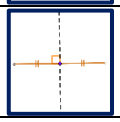
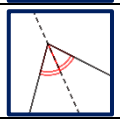


Name:_____( ) Class:_____ Date:_____

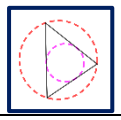
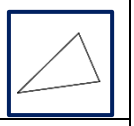
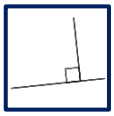
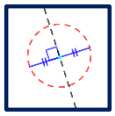
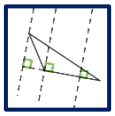
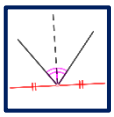
Refered Webpage:

🖫 https://s1129218.wixsite.com/triangle-centres/collinearity

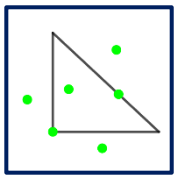
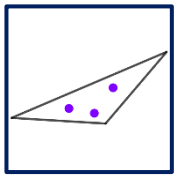
Review of Four Centres:

Lesson	Centre	Intersection of
1	Centroid	Medians 
2	Orthocentre	Altitudes 
3	Circumcentre	Perpendicular Bisectors 
4	Incentre	Angle Bisectors 

Classification:

	Relates to Circle 	**Not Relates to Circle** 
Relates to Altitudes 	Circumcentre 	Orthocentre 
Relates to Bisectors attaching vertices 	Incentre 	Centroid 

Possible Location of the Centre:

Relates to Height (Altitude) 	**Acute Angle Triangle:**	**Inside**
	Obtuse Angle Triangle:	**Outside**
	Right Angle Triangle:	**On**
Relates to Bisector (Through Vertices) 	**Acute Angle Triangle:**	**Inside**
	Obtuse Angle Triangle:	**Inside**
	Right Angle Triangle:	**Inside**

Question Generation on the location of Centres:

🖫 https://www.geogebra.org/m/txvedcrk

Collinearity:

Discover Section:

🖫 https://www.geogebra.org/m/vufzjzvh

● Observe and Imagine

Draw in diagram/ Write in words to describe your discovery:

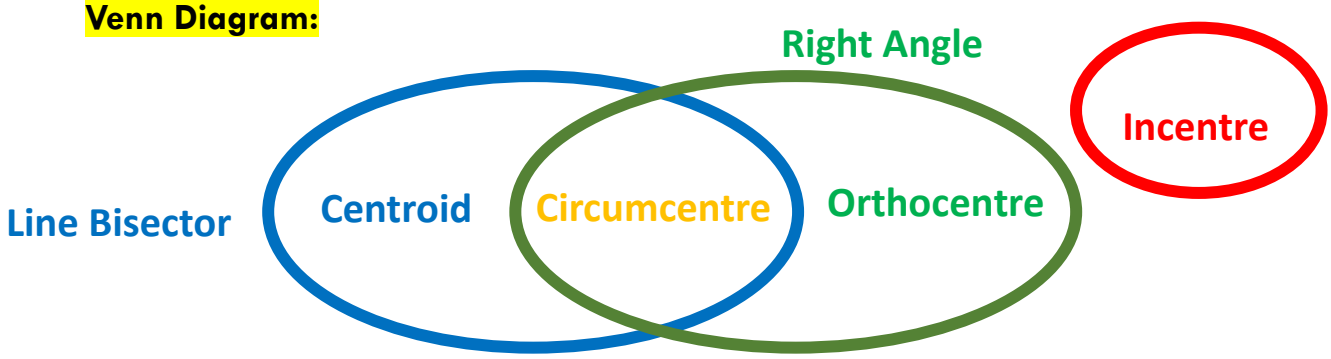
<u>What is Collinearity?</u>

Definition: A set of points **lying on a single line**.

<u>Four Centre Collinearity</u>

Characteristic: Only **Centroid, Orthocentre and Circumcentre** must be collinear.
Incentre is **not guaranteed** to be collinear with other centres

Venn Diagram:



- In **Equilateral Triangle**, Four Centres **Overlaps**.
  - All on the **same position**.

Triangle Centre

Notes for Lesson 6: Application to Plane Geometry

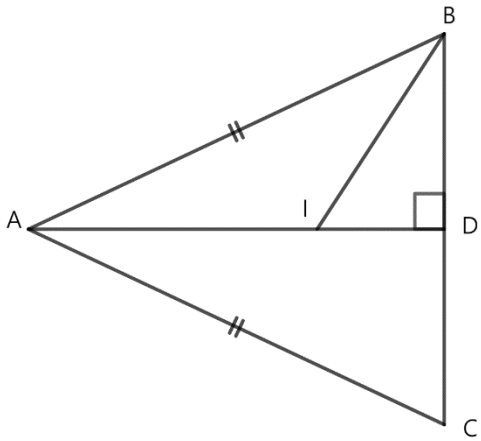
Name:_____( ) Class:_____ Date:_____

Level 1:

In the figure, ΔABC is an isosceles triangle with $AB = AC$. D is a point on BC such that $AD \perp BC$. I is a point on AD such that CI is the angle bisector of $\angle ACB$.

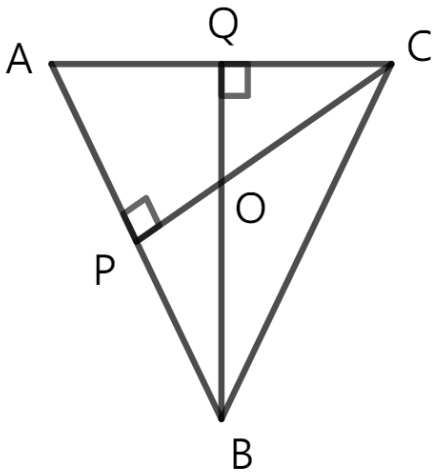
a.) Prove that I is the incentre of ΔABC.

b.) It is given that $AI : ID = 25 : 11$. Find $\angle BAC$, correct to 3 significant figures.



Level 2

In the figure, ΔABC is an equilateral triangle. O is the circumcentre of ΔABC. P and Q are points on AB and AC respectively, such that COP and BOQ are straight lines. If $\angle BPC = \angle BQC = 90°$, prove that $OP = OQ$



Level 3

In the figure, ΔABC is an equilateral triangle. D and E are the mid-points of BC and AC respectively. AFD and BFE are straight lines.

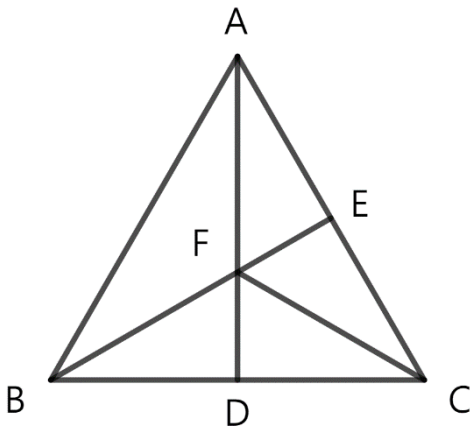
a.) Prove that F is the circumcentre of ΔABC.

b.) It is given that the radius of the circumscribed circle of ΔABC is 8 cm.

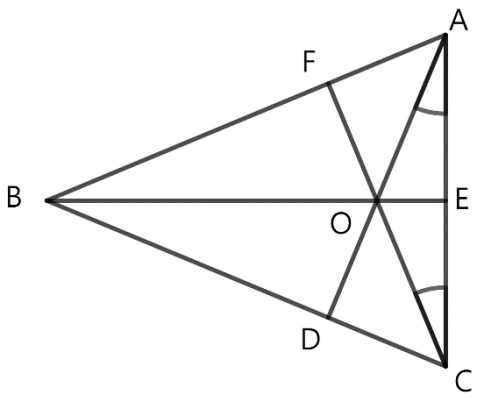
i) Find $\angle BCF$.

ii) Find the area of ΔBFC,

correct to 3 significant figures.



Level 4:

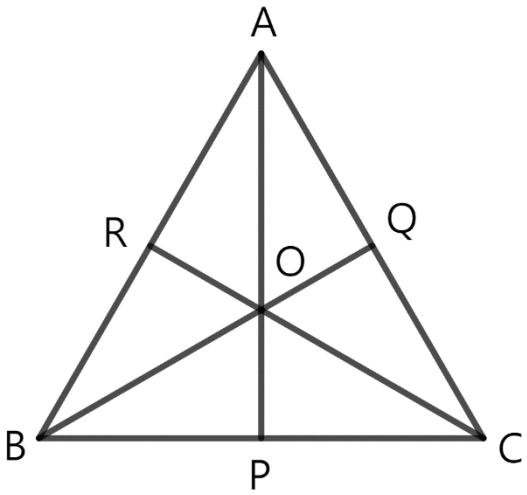


In the figure, O is the orthocentre of $\Delta ABC. AO, BO$ and CO are produced to meet BC, AC and AB at D, E and F respectively. It is given that $\angle OAE = \angle OCE$.

a.) Prove that $OD = OF$.

b.) If I is the incentre of ΔABC, are B, I and O collinear? Explain your answer.

Level 5:



In the figure, O is both the orithocentre and the centroid of $\Delta ABC. AO, BO$ and CO are produced to meet BC, AC and AB at P, Q and R respectively.

a.) Prove that O is also the circumcentre of ΔABC.

b.) Is O also the incentre of ΔABC? Explain your answer

Triangle Centre

Notes for Lesson 8: DSE Past Paper Questions

Name:_____( ) Class:_____ Date:_____

For Full Solution and Explanation:

https://s1129218.wixsite.com/triangle-centres/dse-question

[2013-DSE-Paper II-Q43] [30%]

Let O be the origin. If the coordinates of the points A and B are (0,12) and (30,12) respectively, then the y-coordinate of the circumcentre of ΔOAB is

A. 6
B. 8
C. 12
D. 15

Hints & Explanation:

➢ https://www.geogebra.org/m/a4xv88aa

[2015-DSE-Paper II-Q42] [30%]

Let O be the origin. The coordinates of the points P and Q are (0,60) and (96,48) respectively. The x-coordinate of the orthocentre of ΔOPQ is

A. 6
B. 32
C. 45
D. 48

Hints & Explanation:

➢ https://www.geogebra.org/m/zf9gsyyf

[2017-DSE-Paper II-Q41] [21%]

Let O be the origin. The coordinates of the points P and Q are $(p, 0)$ and $(0, q)$ respectively, where p and q are positive numbers. If the incentre of ΔOPQ lies on the straight line $3x + 4y = 3p$, then $p : q =$

A. 2 : 3
B. 4 : 3
C. 4 : 9
D. 7 : 24

Hints & Explanation:

➢ https://www.geogebra.org/m/phnahj6j

[2018-DSE-Paper II-Q40] [20%]

It is given that a is a positive constant. The straight line $2x + 5y = a$ cuts the x-axis and the y-axis at the points P and Q respectively. Let R be a point lying on the y-axis such that the x-coordinates of the orthocentre of ΔPQR is 10. Find the y-coordinates of R

A. -25
B. -4
C. 4
D. 25

Hints & Explanation:

- https://www.geogebra.org/m/j2cx3hpa

[2019-DSE-Paper II-Q41] [44%]

If ΔABC is a right-angled triangle with $\angle ABC = 90°$, which of the following is/are true?

I. The orthocentre of ΔABC lies on AC
II. The centroid of ΔABC lies inside ΔABC
III. The incentre of ΔABC lies outside ΔABC

A. I only
B. II only
C. I and III only
D. II and III only

Hints & Explanation:

- https://www.geogebra.org/m/pqxrs9fh

[2020-DSE-Paper II-Q40] [23%]

The equations of three sides of a triangle are $4x + 3y = 24$, $4x - 3y = 24$ and $x = a$, where a is a constant. If the x-coordinates of the incentre of the triangle is 31, then $a =$

A. 15
B. 31
C. 45
D. 51

Hints & Explanation:

- https://www.geogebra.org/m/q7gw22wu

Appendix C: Teaching Design and Lesson Plan

Topic: Triangle Centres	Teaching Time: 9 Classes (Each class 35mins)
Prior Knowledge: 1. Basic geometry concepts 2. Properties of triangle 3. Properties of circle 4. Medians 5. Perpendicular bisectors 6. Angle bisectors 7. Altitudes	**Learning Objective** 1. Able to eliminate options in multiple choice questions 2. Analyzing and comparing the properties of different triangle centres. 3. Applying the properties of the four centres to solve problems related to triangles, such as finding the area, perimeter, and angles. 4. Building proficiency in algebraic manipulation and problem-solving skills through the application of coordinate geometry and trigonometry to problems involving triangle centres. 5. Identifying the location of the four centres in a given triangle. 6. Understanding the definitions and properties of four centres.
Definition of the Topic: Triangle centres, in geometry means a point in the triangle's plane that is in some sense in the middle of the triangle. In DSE curriculum, only centroid, circumcentre, incentre and orthocentre is included. ● Centroid: The centroid of a triangle is the point of intersection of the medians of the triangle. ● Circumcentre: The circumcentre of a triangle is the point of intersection of the perpendicular bisectors of the sides of the triangle. ● incentre: The incentre of a triangle is the point of intersection of the angle bisectors of the triangle. ● Orthocentre: The orthocentre of a triangle is the point of intersection of the altitudes of the triangle.	
Foresee Learning Difficulties: 1. Algebraic manipulation: Finding the coordinates of four centres might be difficult for students. 2. Identification: Students may be unable to connect the definition to the name of centre.	

3. Overlapping Concepts: The concepts of centroid, circumcentre, incentre, and orthocentre are related but distinct concepts. Students may find it difficult to distinguish between these concepts and understand how they are different. 4. Prior Knowledge: Four centres require students to be familiar with geometry concepts they learnt before and utilize them skillfully. If students do not have a solid based of prior knowledge, it will be a burden for them to learn. 5. Proof: There are questions about proof on these topics, students may find it difficult on constructing and understanding mathematical proofs. 6. Visualization: On doing DSE questions, if diagram is not provided, students may find it difficult to apply the properties they learnt. 7. Vocabulary: For specific vocabulary, such as incentre, centroid, circumcentre and orthocentre. Student may struggle on using them correctly.
Other Abilities: ● Problem-sloving ● Self-learning

Lesson Plan

Topic: Triangle Centres		
Lesson: Centroid	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Points, lines, and angles
2. Properties of triangle
3. Medians
4. Coordinate system
5. Algebraic manipulation

Learning Objectives:

1. Understand the definition of centroid and its properties.
2. Knows the method of calculating the coordinate of centroid.
3. Able to utilize the properties of centroid in application problems.
4. Able to calculate the location of the centroid of a given triangle.

Resources:

- Notes (Appendix B)
- Quiz
- GeoGebra Tools (Appendix A)
- Website: https://s1129218.wixsite.com/triangle-centres

Learning Difficulties:

- Difficulty on visualizing the centroid.
- Unable to utilize the properties of centroid in application problems

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
Pre-Class	◆ A set of notes is given to student in advance, students are required to finish the self-exploration section on the first page of the notes. ◆ Students are required to go to website/GeoGebra to try out the tool and observe what is centroid. ◆ Students are required to draw/write down their observation down.	✓ Raise interest. ✓ have an idea.	✓ Encourage students to take ownership of their learning and explore the concept of the centroid of a triangle on their own. This can help to solidify their understanding of the concept and promote active engagement in the learning process. ✓ Provide students with a dynamic and interactive way to explore the concept of the centroid of a triangle. By using a digital tool, students can visualize the concept and develop a deeper understanding of the properties of the centroid.	✧ Engage ✧ Explore	➢ Notes ➢ GeoGebra ➢ Website

			✓ Encourage students to reflect on their learning and solidify their understanding of the concept. By drawing or writing down their observations, students can reinforce their learning and identify any areas where additional support may be needed.		
5 mins	◆ An activity on showing students' observation written/ drawn on their notes. ◆ Define median. ◆ Define centroid as a point of intersection of the median of a triangle.	✓ Recall ✓ Compare	✓ Encourage students to share their observations with their peers and promote class discussion. By discussing their observations, students can gain a deeper understanding of the concept of the centroid of a triangle and identify any areas where additional support may be needed. On the other hand, this could create help students to remember better by linking the knowledge to their classmates ✓ Ensure that all students have a clear understanding of the concept of median before learning about the centroid of a triangle. As understanding medians is a prerequisite to understanding the concept of the centroid. ✓ Introduce the concept of the centroid of a triangle.	✧ Engage ✧ Evaluate	➢ Notes ➢ Blackboard
10 mins	◆ Introduce the properties of centroid. ● Show a triangle on the blackboard. ● Locates mid-points and create median on three sides. ● Show centroid is the intersection of three medians (Point out intersection of two can be enough	✓ Assemble ✓ Identify ✓ Define	✓ Help students understand the properties of the centroid of a triangle, which can be used to solve problems related to the centroid. This can be achieved by showing a triangle on the blackboard, locating midpoints, and creating medians on three sides. The intersection of the three medians can	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

	too) ◆ Show the properties of centroid. ◆ Let students to explore the properties on GeoGebra tool.		be pointed out as the centroid of the triangle. ✓ Introduce the properties of the centroid of a triangle, such as the fact that the centroid divides each median into two segments with a 2:1 ratio. This can help students understand the properties of the centroid and how to use them to solve problems related to the centroid. ✓ The purpose of this is to provide students with an interactive and dynamic way to explore the properties of the centroid of a triangle. By using the GeoGebra tool, students can visualize the properties of the centroid and develop a deeper understanding of how they can be used to solve problems related to the centroid.		
15 mins	◆ Questions are classified into level 1-4 ◆ Teacher is going to demonstrate level 1 question and remind students on how the answer should be presented. ◆ Let students to work on the remaining questions. ◆ Teacher walks around and see any support have to be provided. ◆ Give hints on blackboard on how to tackle the problems.	✓ Practice ✓ Consolidate ✓ Application	✓ Differentiated by level of difficulty. This can help to challenge students at different levels and ensure that all students are appropriately challenged or go up step by step. ✓ Ensure that students understand the format of the question and how to present their answers. By demonstrating a level 1 question, the teacher can provide an example for students to follow. ✓ Allowing students to work independently on the questions and practice their problem-solving skills. This can also provide an opportunity for students to work collaboratively and	✧ Elaborate ✧ Evaluate	➢ Notes ➢ Blackboard

			support each other. ✓ Provide individualized support to students who may be struggling with the questions. ✓ Provide additional guidance and support to students who may be struggling with the questions.		
7 mins	◆ Introduce the centroid formula. ◆ Demonstrate how to apply it.	✓ Define	✓ Provide students with a way to calculate the coordinates of the centroid of a triangle using the coordinates of its three vertices. ✓ Demonstrating how to apply the formula, students can gain a deeper and clear understanding of the formula.	✧ Explain ✧ Elaborate	➢ Notes ➢ Blackboard
3 mins	◆ Recap the main points of the lesson, emphasizing the definition and properties of the centroid. ◆ Encourage students to practice more problems on their own and to ask questions if they need help. ◆ Encourage students to check out the proof on their own. ◆ Give homework.	✓ Conclude	✓ Reinforce the main points of the lesson and ensure that all students have a clear understanding of the concept of the centroid of a triangle. ✓ By exploring the proof on their own, students can develop their critical thinking and problem-solving skills.	✧ Elaborate ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook ◆ GeoGebra: Question generator on practicing the centroid formula.	✓ Consolidate ✓ Practice	✓ Consolidate their knowledge and practice the application of the knowledge they learnt in-class.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ GeoGebra ➢ Website

Topic: Triangle Centres		
Lesson: Orthocentre	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Points, lines, and angles
2. Properties of triangle
3. Altitudes
4. Coordinate system
5. Algebraic manipulation
6. Slope of a line
7. Product of slope of a pair of perpendicular lines

Learning Objectives:

1. Understand the definition of orthocentre and its properties.
2. Knows the method of calculating the coordinate of orthocentre.
3. Able to utilize the properties of orthocentre in application problems.
4. Able to calculate the location of the orthocentre of a given triangle.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on visualizing the orthocentre.
- Unable to utilize the properties of centroid in application problems

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
Pre-Class	◆ A set of notes is given to student in advance, students are required to finish the self-exploration section on the first page of the notes. ◆ Students are required to go to website/GeoGebra to try out the tool and observe what is orthocentre. ◆ Students are required to draw/write down their observation down.	✓ Raise interest. ✓ have an idea.	✓ Encourage students to take ownership of their learning and explore the concept of the orthocentre of a triangle on their own. This can help to solidify their understanding of the concept and promote active engagement in the learning process. ✓ Provide students with a dynamic and interactive way to explore the concept of the orthocentre of a triangle. By using a digital tool, students can visualize the concept and develop a deeper understanding of the properties of the orthocentre. ✓ Encourage students to reflect on their	✧ Engage ✧ Explore	➢ Notes ➢ GeoGebra ➢ Website

			learning and solidify their understanding of the concept. By drawing or writing down their observations, students can reinforce their learning and identify any areas where additional support may be needed.		
5 mins	◆ An activity on showing students' observation written/ drawn on their notes. ◆ Define altitudes. ◆ Define orthocentre as a point of intersection of the altitudes of a triangle.	✓ Recall ✓ Compare	✓ Encourage students to share their observations with their peers and promote class discussion. By discussing their observations, students can gain a deeper understanding of the concept of the orthocentre of a triangle and identify any areas where additional support may be needed. On the other hand, this could create help students to remember better by linking the knowledge to their classmates. ✓ Ensure that all students have a clear understanding of the concept of altitudes before learning about the orthocentre of a triangle. As understanding altitude is a prerequisite to understanding the concept of the orthocentre. ✓ Introduce the concept of the orthocentre of a triangle.	✧ Engage ✧ Evaluate	➢ Notes ➢ Blackboard
10 mins	◆ Introduce the properties of orthocentre. ● Show a triangle on the blackboard. ● Draw altitudes in acute, obtuse and right triangle. ● Show orthocentre is the intersection of three altitudes (Point out intersection of two can	✓ Assemble ✓ Identify ✓ Define	✓ Help students understand the properties of the orthocentre of a triangle, which can be used to solve problems related to the orthocentre. ✓ Introduce the properties of the orthocentre of a triangle.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

	be enough too) ◆ Show the properties of orthocentre. ◆ Let students to explore the properties on GeoGebra tool.		✓ The purpose of this is to provide students with an interactive and dynamic way to explore the properties of the orthocentre of a triangle. By using the GeoGebra tool, students can visualize the properties of the orthocentre and develop a deeper understanding of how they can be used to solve problems related to the orthocentre.		
15 mins	◆ Questions are classified into level 1-4 ◆ Teacher is going to demonstrate level 1 question and remind students on how the answer should be presented. ◆ Let students to work on the remaining questions. ◆ Teacher walks around and see any support have to be provided. ◆ Give hints on blackboard on how to tackle the problems.	✓ Practice ✓ Consolidate ✓ Application	✓ Differentiated by level of difficulty. This can help to challenge students at different levels and ensure that all students are appropriately challenged or go up step by step. ✓ Ensure that students understand the format of the question and how to present their answers. By demonstrating a level 1 question, the teacher can provide an example for students to follow. ✓ Allowing students to work independently on the questions and practice their problem-solving skills. This can also provide an opportunity for students to work collaboratively and support each other. ✓ Provide individualized support to students who may be struggling with the questions. ✓ Provide additional guidance and support to students who may be struggling with the questions	✧ Elaborate ✧ Evaluate	➢ Notes ➢ Blackboard
7 mins	◆ Introduce the method of calculating orthocentre.	✓ Define	✓ Provide students with a way to calculate the coordinates of the orthocenter of a	✧ Explain ✧ Elaborate	➢ Notes ➢ Blackboard

	◆ Demonstrate the operation.		triangle using the product of slope of two perpendicular lines. ✓ By demonstrating how to calculate the orthocenter, students can gain a deeper understanding of the concept and develop the skills needed to apply it in problem-solving situations.		
3 mins	◆ Recap the main points of the lesson, emphasizing the definition and properties of the orthocentre. ◆ Encourage students to practice more problems on their own and to ask questions if they need help. ◆ Encourage students to check out more on their own ◆ Give homework.	✓ Conclude	✓ Reinforce the main points of the lesson and ensure that all students have a clear understanding of the concept of the orthocentre of a triangle. ✓ By exploring the proof on their own, students can develop their critical thinking and problem-solving skills. ✓	✧ Elaborate ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook ◆ GeoGebra: Question generator on practicing how to find the coordinate of orthocentre	✓ Consolidate ✓ Practice	✓ Consolidate their knowledge and practice the application of the knowledge they learnt in-class.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ GeoGebra ➢ Website

Topic: Triangle Centres		
Lesson: Circumcentre	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Points, lines, and angles
2. Properties of triangle
3. Perpendicular bisectors
4. Coordinate system
5. Algebraic manipulation
6. Slope of a line
7. Product of slope of a pair of perpendicular lines

Learning Objectives:

1. Understand the definition of circumcentre and its properties.
2. Knows the method of calculating the coordinate of circumcentre.
3. Able to utilize the properties of circumcentre in application problems.
4. Able to calculate the location of the circumcentre of a given triangle.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on visualizing the circumcentre.
- Unable to utilize the properties of centroid in application problems

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
Pre-Class	◆ A set of notes is given to student in advance, students are required to finish the self-exploration section on the first page of the notes. ◆ Students are required to go to website/GeoGebra to try out the tool and observe what is circumcentre. ◆ Students are required to draw/write down their observation down.	✓ Raise interest. ✓ have an idea.	✓ Encourage students to take ownership of their learning and explore the concept of the circumcentre of a triangle on their own. This can help to solidify their understanding of the concept and promote active engagement in the learning process. ✓ Provide students with a dynamic and interactive way to explore the concept of the circumcentre of a triangle. By using a digital tool, students can visualize the concept and develop a deeper understanding of the properties of the circumcentre. ✓ Encourage students to reflect on their	✧ Engage ✧ Explore	➢ Notes ➢ GeoGebra ➢ Website

			learning and solidify their understanding of the concept. By drawing or writing down their observations, students can reinforce their learning and identify any areas where additional support may be needed.		
5 mins	◆ An activity on showing students' observation written/ drawn on their notes. ◆ Define perpendicular bisector. ◆ Define circumcentre as a point of intersection of the perpendicular bisectors of a triangle.	✓ Recall ✓ Compare	✓ Encourage students to share their observations with their peers and promote class discussion. By discussing their observations, students can gain a deeper understanding of the concept of the circumcentre of a triangle and identify any areas where additional support may be needed. On the other hand, this could create help students to remember better by linking the knowledge to their classmates. ✓ Ensure that all students have a clear understanding of the concept of perpendicular bisector before learning about the circumcentre of a triangle. As understanding perpendicular bisector is a prerequisite to understanding the concept of the circumcentre. ✓ Introduce the concept of the circumcentre of a triangle.	✧ Engage ✧ Evaluate	➢ Notes ➢ Blackboard
10 mins	◆ Introduce the properties of circumcentre. ● Show a triangle on the blackboard. ● Draw perpendicular bisectors of the triangle. ● Show circumcentre is the intersection of three perpendicular bisectors (Point out intersection of two can be enough	✓ Assemble ✓ Identify ✓ Define	✓ Help students understand the properties of the circumcentre of a triangle, which can be used to solve problems related to the circumcentre. ✓ Introduce the properties of the circumcentre of a triangle. ✓ The purpose of this is to provide students	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

	too) ◆ Show the properties of circumcentre. ◆ Let students to explore the properties on GeoGebra tool.		with an interactive and dynamic way to explore the properties of the circumcentre of a triangle. By using the GeoGebra tool, students can visualize the properties of the circumcentre and develop a deeper understanding of how they can be used to solve problems related to the circumcentre.		
15 mins	◆ Questions are classified into level 1-4 ◆ Teacher is going to demonstrate level 1 question and remind students on how the answer should be presented. ◆ Let students to work on the remaining questions. ◆ Teacher walks around and see any support have to be provided. ◆ Give hints on blackboard on how to tackle the problems.	✓ Practice ✓ Consolidate ✓ Application	✓ Differentiated by level of difficulty. This can help to challenge students at different levels and ensure that all students are appropriately challenged or go up step by step. ✓ Ensure that students understand the format of the question and how to present their answers. By demonstrating a level 1 question, the teacher can provide an example for students to follow. ✓ Allowing students to work independently on the questions and practice their problem-solving skills. This can also provide an opportunity for students to work collaboratively and support each other. ✓ Provide individualized support to students who may be struggling with the questions. ✓ Provide additional guidance and support to students who may be struggling with the questions	✧ Elaborate ✧ Evaluate	➢ Notes ➢ Blackboard
7 mins	◆ Introduce the method of calculating circumcentre. ◆ Demonstrate the operation.	✓ Define	✓ Provide students with a way to calculate the coordinates of the circumcentre of a triangle using the product of slope of two perpendicular lines, which intersects mid-	✧ Explain ✧ Elaborate	➢ Notes ➢ Blackboard

			points. ✓ By demonstrating how to calculate the circumcentre, students can gain a deeper understanding of the concept and develop the skills needed to apply it in problem-solving situations.		
3 mins	◆ Recap the main points of the lesson, emphasizing the definition and properties of the circumcentre. ◆ Encourage students to practice more problems on their own and to ask questions if they need help. ◆ Encourage students to check out more on their own ◆ Give homework.	✓ Conclude	✓ Reinforce the main points of the lesson and ensure that all students have a clear understanding of the concept of the circumcentre of a triangle. ✓ By exploring the proof on their own, students can develop their critical thinking and problem-solving skills.	✧ Elaborate ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook ◆ Question generator on practicing how to find the coordinate of circumcentre.	✓ Consolidate ✓ Practice	✓ Consolidate their knowledge and practice the application of the knowledge they learnt in-class.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ GeoGebra ➢ Website

Topic: Triangle Centres		
Lesson: Incentre	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Points, lines, and angles
2. Properties of triangle
3. angle bisectors
4. Coordinate system
5. Algebraic manipulation

Learning Objectives:

1. Understand the definition of incentre and its properties.
2. Knows the method of calculating the coordinate of incentre.
3. Able to utilize the properties of incentre in application problems.
4. Able to calculate the location of the incentre of a given triangle.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on visualizing the incentre.
- Unable to utilize the properties of centroid in application problems

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
Pre-Class	◆ A set of notes is given to student in advance, students are required to finish the self-exploration section on the first page of the notes. ◆ Students are required to go to website/GeoGebra to try out the tool and observe what is incentre. ◆ Students are required to draw/write down their observation down.	✓ Raise interest. ✓ have an idea.	✓ Encourage students to take ownership of their learning and explore the concept of the incentre of a triangle on their own. This can help to solidify their understanding of the concept and promote active engagement in the learning process. ✓ Provide students with a dynamic and interactive way to explore the concept of the incentre of a triangle. By using a digital tool, students can visualize the concept and develop a deeper understanding of the properties of the incentre. ✓ Encourage students to reflect on their learning and solidify their understanding of the concept. By	✧ Engage ✧ Explore	➢ Notes ➢ GeoGebra ➢ Website

			drawing or writing down their observations, students can reinforce their learning and identify any areas where additional support may be needed.		
5 mins	◆ An activity on showing students' observation written/ drawn on their notes. ◆ Define angle bisector. ◆ Define incentre as a point of intersection of the angle bisectors of a triangle.	✓ Recall ✓ Compare	✓ Encourage students to share their observations with their peers and promote class discussion. By discussing their observations, students can gain a deeper understanding of the concept of the incentre of a triangle and identify any areas where additional support may be needed. On the other hand, this could create help students to remember better by linking the knowledge to their classmates. ✓ Ensure that all students have a clear understanding of the concept of angle bisector before learning about the incentre of a triangle. As understanding angle bisector is a prerequisite to understanding the concept of the incentre. ✓ Introduce the concept of the incentre of a triangle.	✧ Engage ✧ Evaluate	➢ Notes ➢ Blackboard
10 mins	◆ Introduce the properties of incentre. ● Show a triangle on the blackboard. ● Draw angle bisectors of the triangle. ● Show incentre is the intersection of three angle bisectors (Point out intersection of two can be enough too)	✓ Assemble ✓ Identify ✓ Define	✓ Help students understand the properties of the incentre of a triangle, which can be used to solve problems related to the incentre. ✓ Introduce the properties of the incentre of a triangle. ✓ The purpose of this is to provide students with an interactive and	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

	◆ Show the properties of incentre. ◆ Let students to explore the properties on GeoGebra tool.		dynamic way to explore the properties of the incentre of a triangle. By using the GeoGebra tool, students can visualize the properties of the incentre and develop a deeper understanding of how they can be used to solve problems related to the incentre.		
15 mins	◆ Questions are classified into level 1-4 ◆ Teacher is going to demonstrate level 1 question and remind students on how the answer should be presented. ◆ Let students to work on the remaining questions. ◆ Teacher walks around and see any support have to be provided. ◆ Give hints on blackboard on how to tackle the problems.	✓ Practice ✓ Consolidate ✓ Application	✓ Differentiated by level of difficulty. This can help to challenge students at different levels and ensure that all students are appropriately challenged or go up step by step. ✓ Ensure that students understand the format of the question and how to present their answers. By demonstrating a level 1 question, the teacher can provide an example for students to follow. ✓ Allowing students to work independently on the questions and practice their problem-solving skills. This can also provide an opportunity for students to work collaboratively and support each other. ✓ Provide individualized support to students who may be struggling with the questions. ✓ Provide additional guidance and support to students who may be struggling with the questions	✧ Elaborate ✧ Evaluate	➢ Notes ➢ Blackboard
7 mins	◆ Introduce the incentre formula. ◆ Demonstrate the operation.	✓ Define	✓ Provide students with a way to calculate the coordinates of the incentre of a triangle using the product of slope of two perpendicular lines, which intersects mid-points.	✧ Explain ✧ Elaborate	➢ Notes ➢ Blackboard

			✓ By demonstrating how to calculate the incentre, students can gain a deeper understanding of the concept and develop the skills needed to apply it in problem-solving situations.		
3 mins	◆ Recap the main points of the lesson, emphasizing the definition and properties of the incentre. ◆ Encourage students to practice more problems on their own and to ask questions if they need help. ◆ Encourage students to check out more on their own. ◆ Give homework.	✓ Conclude	✓ Reinforce the main points of the lesson and ensure that all students have a clear understanding of the concept of the incentre of a triangle. ✓ By exploring the proof on their own, students can develop their critical thinking and problem-solving skills.	✧ Elaborate ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook ◆ Question generator on practicing how to find the coordinate of incentre.	✓ Consolidate ✓ Practice	✓ Consolidate their knowledge and practice the application of the knowledge they learnt in-class.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ GeoGebra ➢ Website

Topic: Triangle Centres		
Lesson: Review and Collinearity	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Centroid
2. Orthocentre
3. Circumcentre
4. Incentre
5. Collinearity

Learning Objectives:

1. Review all four centres at once.
2. Help students to classify the non-overlapping part
3. The overlapping situation of four centres in equilateral triangle
4. Collinearity of centroid, orthocentre and circumcentre

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on distinguish four centres at once.
- Unable to utilize the properties of four centres in application problems.
- Confuse with the collinearity.

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
Pre-Class	◆ A set of notes is given to student in advance, students are required to finish the collinearity section. ◆ Students are required to go to website/GeoGebra to try out the tool and observe the collinearity. ◆ Students are required to draw/write down their observation down.	✓ Raise interest. ✓ have an idea.	✓ Encourage students to take ownership of their learning and explore the concept of collinearity on their own. This can help to solidify their understanding of the concept. ✓ Provide students with a dynamic and interactive way to explore the concept of collinearity. By using a digital tool, students can visualize the concept and develop a deeper understanding of the properties of collinear points.	✧ Engage ✧ Explore	➢ Notes ➢ GeoGebra ➢ Website
15 mins	◆ Review on four centres' characteristic. ◆ Table of classification by relating centres to height, bisectors through vertices, circle and non-circle. ◆ Point out centres' location is always	✓ Recall ✓ Comparison	✓ Reinforce students' understanding of the properties of each center and how they relate to the triangle. ✓ Help students understand the relationships between the centers and different aspects of the triangle. By organizing the information into	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard

	inside if not relates to heights, such as orthocentre and circumcentre.		a table, students can compare and contrast the different centers and develop a deeper understanding of their properties. ✓ Emphasize the importance of understanding the relationship between the centers and the different aspects of the triangle. By understanding the location of the centers, students can develop a deeper understanding of their properties and how they relate to the triangle.		
7 mins	◆ Explore collinearity on GeoGebra. ◆ Share students' observation in class.	✓ Identify	✓ Provide students with an interactive and dynamic way to explore the concept of collinearity. By using GeoGebra, students can manipulate the vertices of a triangle and observe the properties of collinear points. ✓ Provide students with an opportunity to share their observations and insights with their peers. By sharing their observations, students can deepen their understanding of the concept and identify any areas where additional support may be needed.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra
10 mins	◆ Define what is collinearity. ◆ Explain the collinearity in Venn diagram. ◆ Mention the overlapping situation in equilateral triangle.	✓ Define ✓ Consolidate	✓ Provide students with a clear understanding of the concept of collinearity. Collinearity refers to the property of three or more points lying on a straight line. ✓ Help students visualize the concept of collinearity and understand its relationship with other concepts. In a Venn diagram, collinearity would be represented as a subset of the set of all possible points in a plane, with the subset consisting of all collinear points. ✓ Clarify any misconceptions that students may have about collinearity and overlapping in	✧ Elaborate ✧ Evaluate	➢ Notes ➢ Blackboard

			triangles. In an equilateral triangle, the three medians, altitudes, and angle bisectors are all concurrent and intersect at the same point, which is also the center of the circle inscribed in the triangle. This is an example of overlapping points in a triangle, which only occurs in equilateral triangles.		
8 mins	◆ Emphasize the table of distinguishment and location of the four centres. ◆ Remind and emphasize the collinearity and overlapping only exist on Equilateral triangle.	✓ Conclude	✓ Reinforce students' understanding of the properties and locations of the four centers of a triangle. By emphasizing the table and reviewing the information, students can deepen their understanding of the concept and its applications. ✓ Clarify any misconceptions that students may have about collinearity and overlapping in triangles. By emphasizing that these properties only exist in equilateral triangles, students can develop a deeper understanding of the concept and its limitations.	✧ Elaborate ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook ◆ Question generator on practicing the location of four centres	✓ Consolidate ✓ Practice	✓ provide students with a range of practice problems related to the concept of the centers of a triangle. By practicing these problems, students can develop their problem-solving skills and reinforce their understanding of the concept. ✓ provide students with an opportunity to practice identifying the location of the four centers of a triangle. By using a question generator, students can access a range of practice problems and develop their skills in identifying the location of the centers.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ GeoGebra ➢ Website

Topic: Triangle Centres		
Lesson: Further Application -- Plane Geometry	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Centroid
2. Orthocentre
3. Circumcentre
4. Incentre

Learning Objectives:

1. Review all four centres at once.
2. Practice on the properties application.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- ◆ Difficulty on distinguish four centres at once.
- ◆ Unable to utilize the properties of four centres in application problems.

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
10 mins	◆ Review on the properties of four centres. ◆ List out all properties and classify them in table and put it on the blackboard for students' reference	✓ Review ✓ Recall ✓ Consolidate	✓ Reinforce students' understanding of the properties of each center and how they relate to the triangle. ✓ Help students organize and visualize the properties of each center. By creating a table that lists and classifies the properties of each center, students can compare and contrast the different centers and develop a deeper understanding of their properties.	✧	➢ Notes ➢ Blackboard
23 mins	◆ 5 levels of questions tackle them one by one. ◆ Guide students to finish them through use of questions and hints form the properties table on the blackboard.	✓ Apply ✓ Consolidate	✓ Challenge students at different levels of understanding and reinforce their knowledge of the properties of the four centers of a triangle. The questions should start with easier ones and gradually become more difficult. ✓ Encourage students to apply their knowledge of the properties of the four centers of a triangle to problem-solving situations. By guiding students through the use of questions and hints from the properties table on the blackboard,	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard

			students can develop their problem-solving skills and deepen their understanding of the concept.		
7 mins	◆ Clarify any common mistakes from students	✓ Review ✓ Consolidate	✓ Identify and clarify any misconceptions that students may have about the concept of the centers of a triangle. By addressing common mistakes and clarifying any misunderstandings, students can deepen their understanding of the concept and avoid making similar mistakes in the future.	✧ Explain ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook	✓ Consolidate ✓ Practice	✓ provide students with a range of practice problems related to the concept of the centers of a triangle. By practicing these problems, students can develop their problem-solving skills and reinforce their understanding of the concept.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes

Topic: Triangle Centres		
Lesson: Further Application -- Coordinate Geometry	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Centroid
2. Orthocentre
3. Circumcentre
4. Incentre

Learning Objectives:

1. Practice on the use of formulas and methods.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on distinguish four centres at once.
- Unable to utilize the properties of four centres in application problems.

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
15 mins	◆ Review on the method on finding the coordinates of four centres. ◆ List out all the methods and formula on blackboard for students' reference	✓ Review ✓ Recall ✓ Consolidate	✓ Reinforce students' understanding of the methods for finding the coordinates of the four centers of a triangle. ✓ Provide students with a visual aid to help them remember the formulas and methods for finding the coordinates of the four centers of a triangle. By listing them out on the blackboard, students can refer to them as needed and deepen their understanding of the concept.	✧	➢ Notes ➢ Blackboard
18 mins	◆ Showing Questions randomly generated ◆ Grouping up students into groups to give four different answers, four centres respectively. ◆ Change duty of each group after each question by randomly drawn.	✓ Apply ✓ Consolidate	✓ Challenge students and reinforce their knowledge of the concept. By generating questions randomly, students can be challenged at different levels of understanding and develop their problem-solving skills. ✓ Encourage collaboration and teamwork among students. By working in groups, students can share their knowledge and learn from each other.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

			✓ Ensure that all students have equal opportunities to participate and contribute to the group. By randomly drawing the duty of each group after each question, students can take turns in different roles and develop a range of skills.		
7 mins	◆ Clarify any common mistakes from students	✓ Review ✓ Consolidate	✓ The purpose of this is to identify and clarify any misconceptions that students may have about the concept of the centers of a triangle. By addressing common mistakes and clarifying any misunderstandings, students can deepen their understanding of the concept and avoid making similar mistakes in the future.	✧ Explain ✧ Evaluate	➢ Blackboard
Homework	◆ Exercise from textbook	✓ Consolidate ✓ Practice	✓ provide students with a range of practice problems related to the concept of the centers of a triangle. By practicing these problems, students can develop their problem-solving skills and reinforce their understanding of the concept.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes

Topic: Triangle Centres		
Lesson: Further Application -- DSE Question	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Centroid
2. Orthocentre
3. Circumcentre
4. Incentre

Learning Objectives:

1. Able to avoid the wrong options.

Resources:

- Notes
- GeoGebra Tools

Learning Difficulties:

- Difficulty on distinguish the impossible answers.
- Unable to utilize the properties of four centres in application problems.

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
40 mins	◆ Focus on Dse past paper questions, aim on eliminating impossible answers. ◆ Assist by the GeoGebra tools to visualize the answers. ◆ Observe various situation or changes throughout choices. ◆ Construct diagram and through the diagram to distinguish impossible answers.	✓ Consolidate	✓ Practice their problem-solving skills and reinforce their knowledge of the concept. By focusing on past paper questions, students can become familiar with the types of questions that may appear on the exam, and by eliminating impossible answers, they can increase their chances of selecting the correct answer. ✓ Provide students with an interactive and dynamic way to visualize the answers to the questions. By using GeoGebra, students can manipulate the vertices of the triangle and observe the properties of the centers, which can help them eliminate impossible answers and select the correct one. ✓ encourage students to think critically and apply their knowledge of the concept to problem-solving situations. By observing different situations and changes throughout the choices, students can identify the properties of the	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Blackboard ➢ GeoGebra

			centers and use them to construct a diagram to distinguish impossible answers.		
Homework	◆ After knowing the answers, check the full solution.	✓ Consolidate ✓ Practice	✓ Help students develop a deeper understanding of the concept and reinforce their problem-solving skills. By checking the full solution on their own, students can identify any mistakes they may have made, clarify any misunderstandings, and improve their problem-solving strategies for similar questions in the future. ✓ Help students develop a sense of independence and self-confidence in their ability to solve problems. By learning from their mistakes and using the correct methods and formulas, students can deepen their understanding of the concept and improve their performance on future assessments.	✧ Explore ✧ Explain ✧ Evaluate	➢ Notes ➢ Website

Topic: Triangle Centres		
Lesson: Further Application -- DSE Question	**Time: 40mins**	**Class: F5**

Prior Knowledge:

1. Centroid
2. Orthocentre
3. Circumcentre
4. Incentre

Learning Objectives:

1. Able to avoid the wrong options.
2. Able to apply the properties correctly.

Resources:

- Quiz

Learning Difficulties:

- Difficulty on distinguish the impossible answers.
- Unable to utilize the properties of four centres in application problems.
- Time management

Time	Class Arrangement	Purpose		5E-Instructional Model	Resources
40 mins	◆ Quiz	✓ Consolidate	✓ Assess students' understanding of a particular concept or topic	✧ Evaluate	➢ Quiz
Homework	◆ Quiz Correction	✓ Consolidate	✓ Provide students with feedback on their performance and help them identify areas where they may need additional support. By correcting quizzes, students can learn from their mistakes and improve their problem-solving skills. ✓ Clarify any misunderstandings that students may have and reinforce their understanding of the concept.	✧ Evaluate	➢ Quiz